Measurement of the Abraham Force and Its Predicted QED Corrections in Crossed Electric and Magnetic Fields

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(Received 2 July 2011; published 17 October 2011)

We report the observation by a new method of mechanical momentum transferred to gas phase atoms and molecules upon application of crossed oscillating electric and static magnetic fields. We identify this momentum as the microscopic analogue of the classical Abraham force. Two QED predictions of additional magnetoelectrically induced mechanical momentum are addressed. One of them is experimentally refuted; the other is found to be currently below our experimental detection.

DOI: 10.1103/PhysRevLett.107.170401 PACS numbers: 03.50.De, 42.50.Nn, 42.50.Wk

It has been shown that in crossed electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, the optical and electrical properties of matter become anisotropic along the axis $\mathbf{E}\times\mathbf{B}$ [1–3]. As this anisotropy manifests itself in the dispersion law and thus in the momentum of the photons and the charge carriers, respectively, one may wonder whether a similar anisotropy can exist in the mechanical momentum of particles in crossed fields.

When we impose invariance under time, charge, and parity reversal, we see that it is symmetry allowed for a particle to acquire a mechanical momentum $\mathbf{p}$ upon applying a crossed electric and magnetic field $\mathbf{E}$ and $\mathbf{B}$

$$\mathbf{p} = a\mathbf{E}\times\mathbf{B}. \tag{1}$$

If we assume the particles to be in a gaseous phase in a container, and the collisions between the particles to be elastic, the momentum imparted to each of the particles by the application of the fields will be conserved within the gas as a whole, and ultimately transferred to the wall perpendicular to $\mathbf{E}\times\mathbf{B}$. If we apply a harmonically oscillating electric field $\mathbf{E}(t) = \mathbf{E}\sin\omega t$ and a static magnetic field $\mathbf{B}$, each particle will contribute a force on this wall given by

$$\mathbf{F}_p = \frac{d\mathbf{p}}{dt} = a\omega\mathbf{E}\times\mathbf{B}\cos\omega t. \tag{2}$$

Such a force exerted on individual atoms would be the microscopic equivalent of the so-called Abraham force density which was first formulated for macroscopic media, and has been the subject of a long-standing controversy [4–7].

The classical Maxwell equations lead to a momentum conservation law of the type $\partial_t\mathbf{G} + \boldsymbol{\nabla}\cdot\mathbf{T} = -\mathbf{f}$, with $\mathbf{G}$ the electromagnetic momentum density, $\mathbf{T}$ the electromagnetic stress tensor, and $\mathbf{f}$ the force density [8]. However, the exact form of each term is not unequivocally defined. In the so-called Minkowski version [4] one adopts $\mathbf{G}_M \propto \mathbf{D}\times\mathbf{B}$, and no term bilinear in $\mathbf{E}$ and $\mathbf{B}$ is present in the force density $\mathbf{f}_M$. In the Abraham version [4] one insists on $\mathbf{G}_A \propto \mathbf{E}\times\mathbf{H}$, and the force density has a contribution (in SI units) $\mathbf{f}_A = \varepsilon_0(\varepsilon_r - 1/\mu_r)\partial_t(\mathbf{E}\times\mathbf{B})$. Finally, in the Nelson version [5] one takes $\mathbf{G}_N \propto \mathbf{E}\times\mathbf{B}$ which leads to $\mathbf{f}_N = \varepsilon_0(\varepsilon_r - 1)\partial_t(\mathbf{E}\times\mathbf{B})$ (ε_r and μ_r are the relative dielectric permittivity and magnetic permeability, respectively). This can be compared to the quantum-mechanical conserved pseudomomentum of a neutral atom in a homogeneous magnetic field, $\mathbf{K} = \sum_i m_i\dot{\mathbf{r}}_i + \sum_i q_i\mathbf{B}\times\mathbf{r}_i$ [9]. An additional electric field creates a finite polarization $\langle\mathbf{P}\rangle = \langle\sum_i q_i\mathbf{r}_i\rangle = \alpha\mathbf{E}$ (α is the static electric polarizability of the particle with SI units $\mathrm{C\,m^2/V}$) in the ground state so that $0 = \dot{\mathbf{K}} = \sum_i m_i\ddot{\mathbf{r}}_i - \alpha\partial_t(\mathbf{E}\times\mathbf{B})$. This would lead to a force density $\mathbf{f} = N\alpha\partial_t(\mathbf{E}\times\mathbf{B})$ (where N is the particle density) and which is consistent with the Nelson version, since $\epsilon_0(\epsilon_r - 1) = N\alpha$, and we deduce $a = \alpha$ in Eq. (1). Note that the pseudomomentum $\mathbf{K}$ in this model equals neither the *conjugated* momentum $\mathbf{P} = \sum_i m_i\dot{\mathbf{r}}_i + \frac{1}{2}\sum_i q_i\mathbf{B}\times\mathbf{r}_i$ nor the kinetic momentum $\mathbf{P}_{\mathrm{kin}} = \sum_i m_i\dot{\mathbf{r}}_i$. Both were proposed by Barnett [7] to solve the Abraham-Minkowski controversy.

The observation of the Abraham force due to a crossed oscillating electric field and a static magnetic field was reported by James [10] and by Walker *et al.* [11,12] in solid dielectrics. Interestingly, and against all expectations, experiments have failed to observe the Abraham force due to a time varying magnetic field and a static electric field [13–15]. (We do not address this latter point here and we refer the reader to [4,6].)

Feigel was the first to consider the interaction of a macroscopic magnetoelectric material with the quantum vacuum [16]. The so-called Feigel effect implies that momentum from the vacuum fluctuations can be transferred to matter by the intermediary of the optical magnetoelectric anisotropy and that therefore a QED contribution $\partial p_F/\partial t$ exists to the classical Abraham force, corresponding to a "Feigel" momentum

$$p_F = \frac{1}{32N\pi^2}\Delta n_{\mathrm{MEA}}\hbar\left(\frac{2\pi}{\lambda_c}\right)^4, \tag{3}$$

where $\Delta n_{\mathrm{MEA}} \equiv \chi_{\mathrm{MEA}}EB$ is the magnetoelectric optical anisotropy [1,2]. In order to avoid the notorious UV catastrophe, Feigel was obliged to introduce an empirical cutoff wavelength λ_c for the material's response. In particular, this cutoff procedure was contested by several groups, since it is widely believed that the UV catastrophe should somehow be absorbed in the parameter values attributed to bulk media [17]. It was shown [18–20] that such a transfer would then only occur in a geometry of finite size, similar to that of the Casimir effect, albeit with much smaller values than obtained by Feigel. Obukhov and Hehl [21] also argued that no net momentum transfer from vacuum fluctuations to bulk media can exist. However, very recently, Croze has put forward new theoretical support in favor of Feigel's claim [22], correcting in the process a minor numerical error. Also very recently, Kawka and van Tiggelen have proposed a nonrelativistic quantum theory of a harmonic oscillator in crossed electric and magnetic fields [9], in which the UV catastrophe was shown to be absorbed in a mass renormalization of the oscillator. Applying this model to a hydrogen atom predicts a reduction of a by 2%.

For the ratio between the Feigel and Abraham momenta, we find

$$\frac{p_F}{p_A} = \frac{\pi^2}{2}\frac{\chi_{\mathrm{MEA}}}{N\alpha}\left(\frac{1}{\lambda_c}\right)^4\hbar. \tag{4}$$

Feigel proposed $\lambda_c = 0.1$ nm as the limit of the matter response to the vacuum fluctuations. Using the experimental results of Roth and Rikken [1] for large organometallic molecules, Croze predicts $p_F/p_A \approx 7$; i.e., the magnetoelectrically induced particle momentum would be dominated by the contribution from the quantum vacuum.

Clearly the magnetoelectrical momentum, in spite of its long history, still poses fundamental problems. In this Letter we will describe a new method to accurately measure the momentum transferred to atoms or molecules in crossed oscillating electric fields and static magnetic fields. We experimentally confirm the prediction of the classical Abraham force. More specifically, we do not observe any deviations from the Abraham prediction for media where the predicted contribution from the Feigel effect should be observable.

Since the work by James and by Walker *et al.*, no new experiments to measure the Abraham force have been reported. Very recently, a proposition was made to measure it at optical frequencies using whispering gallery modes [23]. The method used here directly measures the pressure exerted by an atomic or molecular gas on the wall of the container if it is exposed to crossed electric magnetic and magnetic fields, $E\hat{\mathbf{x}}\sin\omega t$ and $B\hat{\mathbf{y}}$ respectively. If we define the effective length of the $\mathbf{E}\times\mathbf{B}$ region as $L = \int E(z)B(z)dz/EB$, the momentum change due to the Abraham force exerts an oscillating pressure P on the wall perpendicular to $\mathbf{E}\times\mathbf{B}$ given by

$$P(t) = \alpha\omega NLEB\cos\omega t. \tag{5}$$

Such a pressure can be detected by a microphone located at the wall. By tuning ω to a longitudinal acoustic resonance of the system, the pressure can by multiplied by the Q factor of the resonance. Using values of $N = 2.7\times 10^{25}$ m^{-3} (1 bar ideal gas), $E = 10^5$ V/m, $B = 1$ T, $\omega = 3\times 10^4$ s^{-1}, $L = 2$ cm, $Q = 10$, and $\alpha = 2.2\times 10^{-41}$ C m^2/V, we find $P = 4\times 10^{-7}$ Pa and a velocity of 0.3 nm/s (values for He, [24]). The typical sensitivity of an electret microphone is $S = 10$ mV/Pa, so microphone signal voltages of around 5 nV can be expected, which are within experimental reach when using a lock-in amplifier (LIA). Figure 1 shows schematically the setup used. It consists of a 3 mm diameter, 5 cm long glass tube, with commercial electret microphones butt coupled to its ends, carefully shielded in thick-walled copper housings. The electric field was supplied by a high voltage amplifier (HV amp), generating voltages up to 1000 V, and the magnetic field was provided by an electromagnet, with fields up to 1.5 T. The Q factor was determined from the acoustic resonance line shape. The systematic inaccuracy of our setup is estimated to be 3%, mostly due to the inaccuracy of the microphone sensitivity calibration.

Typical results for nitrogen gas are shown in Fig. 2, confirming the linear dependence of the magnetoelectrically induced pressure on magnetic field strength, gas pressure ($\propto$ particle density), and electric field oscillation frequency. The linear dependence on electric field strength is intrinsic because of the phase sensitive detection of the pressure signal. The dashed lines in the two top panels are the theoretical predictions, based on Eq. (5). Within the experimental accuracy, the experimental results agree with the theory. The slope in the bottom panel of Fig. 2 allows us

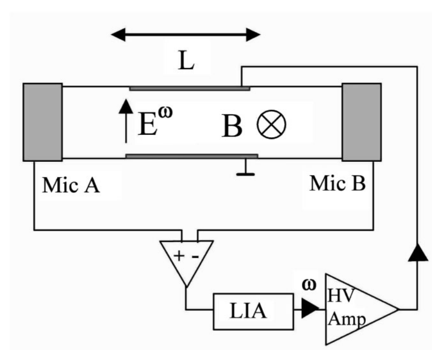

FIG. 1. Schematic setup of the experiment. A magnetic field B is applied perpendicular to the drawing, and high voltage at frequency ω is applied to the electrodes of the cell. The pressures at the cell ends are detected by microphones Mic A and Mic B. The pressure difference is detected by the lock-in amplifier LIA. The effective interaction length L between the gas and the fields is defined in the text.

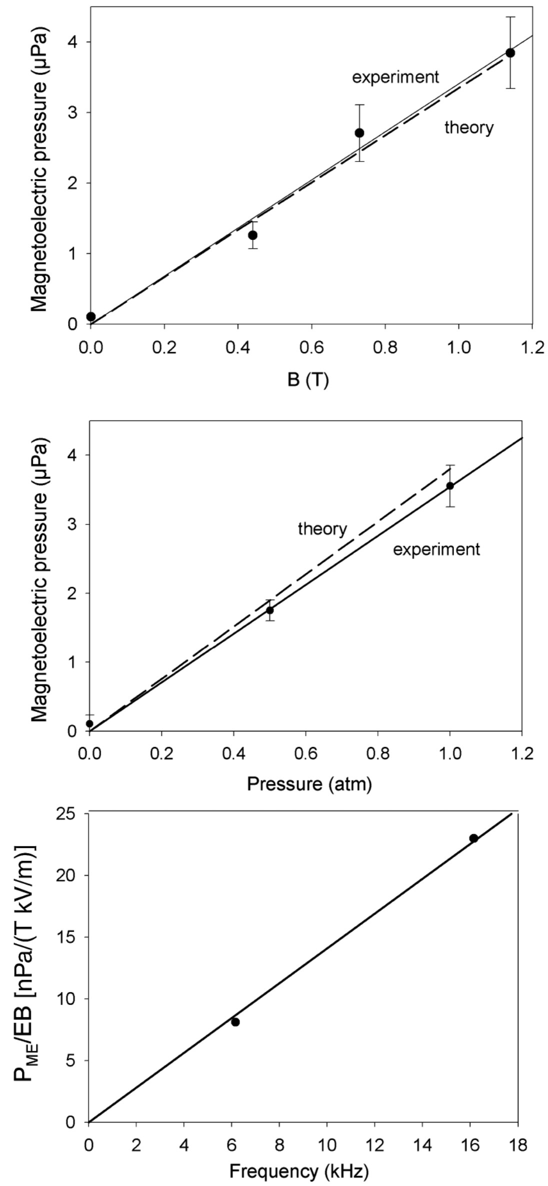

FIG. 2. Magnetoelectric acoustic pressure observed in nitrogen gas. Top panel: 6.15 kHz, 1 atm, $E = 370$ kV/m. Middle panel: 6.15 kHz, 1 atm, $B = 1.14$ T. Bottom panel: 1 atm. Solid lines are linear fits to the data, dashed lines theoretical predictions.

to determine α, using Eq. (5). Figure 3 shows the results for α obtained this way for several gases, as a function of the literature value for α. All gases were measured at room temperature and atmospheric pressure, except furan, which was measured at its room temperature vapor pressure. Table I summarizes these results, and also shows the calculated contribution of the Feigel momentum, expressed as a fraction of the Abraham momentum, and based on experimental or theoretical values for χ_{MEA}. Only the value for nitrogen is experimental [26], but it is in good agreement with the calculated value [25], giving confidence in the other values calculated by the same authors. For the two molecules in the table with the highest

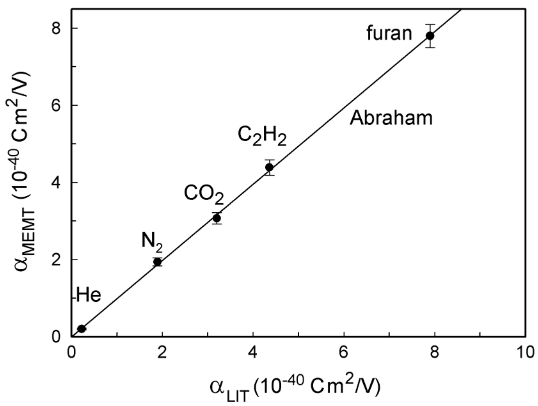

FIG. 3. Polarizability for different atoms and molecules as deduced from magnetoelectrically generated pressure, as a function of the literature values. Solid line corresponds to the classical Abraham force prediction.

magnetoelectric anisotropy, the predicted contributions for the Feigel effect are much larger than the experimental uncertainties on α_{MEMT}, up to 7.5 times for furan. As α_{MEMT} and α_{LIT} agree within the experimental uncertainties, we conclude from these results that the prediction for the Feigel effect as expressed by Eq. (3) is not observed. Note that the Feigel prediction contains one adjustable parameter, the response cutoff wavelength λ_c, and that increasing its value to 0.17 nm decreases the prediction of the Feigel momentum contribution to below our experimental uncertainty. However, strong magnetoelectric anisotropy was still reported at 0.16 nm wavelength [29], the shortest wavelength at which its observation was ever attempted. Our experimental results therefore unambiguously contradict Feigel's prediction. Recent theoretical work on simple models suggests that χ_{MEA} decays algebraically as ω^{-2} at high frequencies, in much the same way as the dynamic electrical polarizability [30]. This makes the UV catastrophe in the macroscopic description as proposed by Feigel unavoidable and unrepairable.

In another QED version of the Feigel effect by Kawka and van Tiggelen [9], this UV catastrophe was removed by mass regularization and they have predicted a correction to the Abraham force of the order of 2%. Our current

TABLE I. Polarizabilities deduced from magnetoelectric momentum transfer (α_{MEMT}), the corresponding literature values (α_{LIT}), the experimental or calculated magnetoelectric anisotropy (χ_{MEA}), and the calculated ratio of Feigel and Abraham momenta ($\mathbf{p}_F/\mathbf{p}_A$) for the gases studied.

Gas	α_{MEMT} (10^{-40} C m^2/V)	α_{LIT} (10^{-40} C m^2/V)	χ_{MEA} (10^{-22} m/V T)	$\mathbf{p}_F/\mathbf{p}_A$
He	0.20 ± 10%	0.22 [24]	0.017 [25]	1.5%
N_2	1.9 ± 5%	1.89 [24]	0.47 [26]	4.8%
C_2H_2	4.4 ± 5%	4.4 [24]	3.7 [25]	16%
Furan	7.8 ± 4%	7.9 [27]	12 [28]	29%

experimental accuracy does not allow us to make statements concerning this prediction, but our setup could be improved to attain the $\leq 1\%$ accuracy necessary for the confirmation of this correction. We hope that this perspective will stimulate realistic calculations of this regularization, beyond the harmonic oscillator approximation and in a relativistic context.

Our experiment invalidates the Minkowski version of the force density, and is consistent with both the Abraham and Nelson versions of electromagnetic momentum, as well as with the conserved pseudomomentum introduced in quantum mechanics. What remains to be tested for both versions is the dependence of the force density on the time derivative of the magnetic field. In order to make a further contribution to this debate on electromagnetic momentum, our experiment would have to detect the difference between the Abraham and Nelson versions, i.e., between 1 and $1/\mu_r$. The gas with the largest μ_r to our knowledge is oxygen, with $\mu_r - 1 = 3.4 \times 10^{-3}$ at room temperature and 1 atm [24]. Attaining such a precision is a considerable experimental challenge, but going to lower temperatures or higher pressures can increase $\mu_r - 1$ to values within our experimental resolution.

In summary, we have reported the observation of mechanical momentum transferred to atoms and molecules in the gas phase by applying crossed oscillating electric and static magnetic fields. We quantitatively identify this momentum as the microscopic analogue of the classical Abraham force. We exclude an additional force, related to quantum vacuum fluctuations, proposed by Feigel. Another predicted contribution to the Abraham force, resulting from mass renormalization as predicted by Kawka and van Tiggelen, is currently beyond our experimental resolution, as is the discrimination between the Abraham and Nelson versions of the force density, but the new method described in this Letter has potential to successfully address these issues.

This work was supported by the ANR contract PHOTONIMPULS ANR-09-BLAN-0088-01. We gratefully acknowledge helpful discussions and calculations by Antonio Rizzo.

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