

Objectively Discerning Autler-Townes Splitting from Electromagnetically Induced Transparency

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Autler-Townes splitting (ATS) and electromagnetically induced transparency (EIT) both yield transparency in an absorption profile, but only EIT yields strong transparency for a weak pump field due to Fano interference. Empirically discriminating EIT from ATS is important but so far has been subjective. We introduce an objective method, based on Akaike's information criterion, to test ATS vs EIT from experimental data for three-level atomic systems and determine which pertains. We apply our method to a recently reported induced-transparency experiment in superconducting-circuit quantum electrodynamics.

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Coherent processes in atoms and molecules yield many interesting and practical phenomena such as coherent population trapping [1], lasing without inversion [2], and electromagnetically induced transparency (EIT) [3]. Pioneering EIT experiments employed alkali metals due to their simple electronic level structure and long-lived coherence; but recently, coherent processes have been investigated in other systems such as quantum dots [4], nanoplasmonics [5], superconducting circuits [6], metamaterials [7,8], and optomechanics [9]. EIT is also observed for a classical coupled oscillator, e.g., inductively or capacitively coupled electrical resonator circuits [10,11]. EIT systems could enable new practical applications of coherent processes, but the lack of time-scale separations characteristic of alkalis [12] obfuscates the signatures of coherent processes.

Here we focus on EIT, where transparency is induced coherently by a pump field even if the pump is arbitrarily weak. EIT is crucial for optically controlled slowing of light [13] and optical storage [14] and is achieved by Fano interference [15] between two atomic transitions. Without Fano interference, the transparency is simply due to a doublet structure in the absorption profile caused by electromagnetic (EM) pumping and known as Autler-Townes splitting (ATS) [16]. Here we introduce an objective test for use on empirical data to discern EIT from ATS in any experiment involving a three-level atom (TLA). This test is based on Akaike weights for the models [17] and reveals whether EIT or ATS has been observed or whether the operating conditions make the data inconclusive.

Fano's seminal study of two nearly resonant modes decaying via a common channel differed from the prevalent normal-mode analyses at the time: He showed that this shared decay channel yields additional cross coupling between modes mediated by the common reservoir, which explained the anomalous asymmetric line shape for electrons scattering from helium [15]. In fact, any response that combines multiple modes can have Fano interference,

which can be extremely sharp and highly sensitive to variability in the system [18].

Harris and Imamoğlu showed that hybrid "atom + field" modes in the dressed-state formalism interact with the same reservoir and hence, readily satisfy the Fano interference conditions [19], thereby producing a transparency window in the absorption profile $A(\delta)$ for δ the two-photon detuning frequency. This effect was originally demonstrated for a Λ -type TLA with energy levels $|a\rangle$, $|b\rangle$, and $|c\rangle$ and judiciously chosen rates as shown in Fig. 1(a). Dressed-state frequency separation is proportional to the pump-field Rabi frequency Ω , and this separation yields ATS in the absence of Fano interference. Fano interference is negligible for large Ω but must transition smoothly from ATS to EIT as Ω decreases and the dressed-states try to merge, thereby strengthening the Fano interference effect. Under EIT conditions, complete transparency holds even in the weak-pump limit.

There are four TLAs: Λ , V , and two ladder (Ξ) cascade systems with upper- and lower-level driving, respectively. Only Λ - and upper-level-driven Ξ TLAs exhibit Fano interference-induced suppression of absorption [20]. For simplicity, we focus on the Λ TLA to show how the decaying-dressed states formalism yields distinctive absorption profiles characteristic of EIT and ATS [21,22], but our approach to discern EIT from ATS is independent of the choice of TLA and so directly applicable to upper-level-driven Ξ -type TLA.

We use a semiclassical description with decay and dephasing rates manually inserted. The electromagnetic response to the probe is proportional to the probe-induced excited coherence corresponding to the off-diagonal TLA density matrix element σ_{ab} . The steady-state solution to linear order of the probe electric field has all the population in $|b\rangle$, so excited coherence at the probed transition depends only on dephasing rates Γ_{ab} and Γ_{bc} : $\sigma_{ab} = \alpha / [\delta + \Delta - i\Gamma_{ab} - \Omega^2 / (\delta - i\Gamma_{bc})]$, with Δ the one-photon detuning and α the probe Rabi frequency [21].

Linear absorption $A \propto \text{Im}(\sigma_{ab})$, shown in Figs. 1(b)–1(d), has spectral poles $\delta_{\pm} = -\Delta/2 + i(\Gamma_{ab} + \Gamma_{bc})/2 \pm [\Omega^2 + (\Delta - i\Gamma_{ab} + i\Gamma_{bc})^2/4]^{1/2}$, which produce resonant contributions to atomic response, $A_{\pm} = S_{\pm}/(\delta - \delta_{\pm})$, with strengths $S_{\pm} = \pm(\delta_{\pm} - i\Gamma_{bc})/(\delta_{+} - \delta_{-})$. These resonant contributions can be attributed to “decaying-dressed states” [21] with frequencies and dephasing rates given by $\text{Re}(\delta_{\pm})$ and $\text{Im}(\delta_{\pm})$, respectively. Decaying-dressed states arise from the interaction between dressed states with eigenenergies $-\Delta/2 \pm (\Omega^2 + \Delta^2/4)^{1/2}$ and two reservoirs with decay rates Γ_{ab} and Γ_{bc} . This interaction is affected by the pump in two ways: separating dressed states and exciting the $|a\rangle \leftrightarrow |c\rangle$ transition needed for destructive Fano interference with the $|a\rangle \leftrightarrow |b\rangle$ reservoir. Unfortunately, the excited $|a\rangle \leftrightarrow |c\rangle$ transition interacts with the $|b\rangle \leftrightarrow |c\rangle$ reservoir, which is always positive and thus negates absorption suppression. Finally, one-photon detuning further separates dressed states thereby weakening Fano interference.

Strong Fano interference, hence strong EIT, occurs for resonant driving ($\Delta = 0$) where the spectral poles exist in three Ω regions: (i) dressed states share a reservoir $\Omega \leq \Omega_{\text{EIT}} \equiv (\Gamma_{ab} - \Gamma_{bc})/2$, (ii) dressed states decay into distinct reservoirs $\Omega \gg \Gamma_{ab}$, and (iii) an intermediate regime where the dressed-state reservoirs are only partially distinct. In Ω region (i) $\text{Re}(\delta_{\pm}) = 0 = \text{Im}(S_{\pm})$ so the absorption profile comprises two Lorentzians centered at the origin, one broad and positive and the other narrow

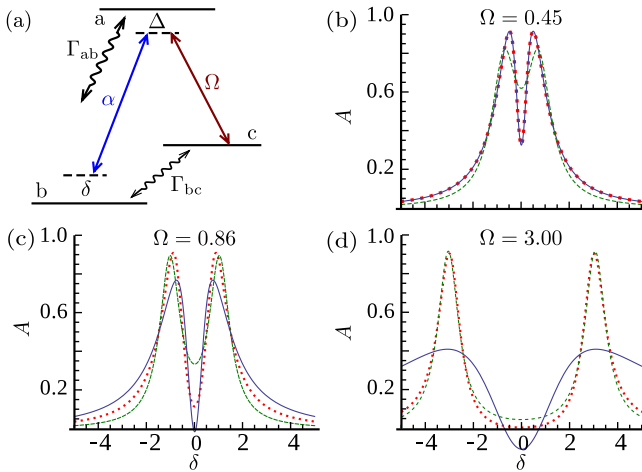


FIG. 1 (color online). (a) Λ -type TLA with a probe (pump) driving field with Rabi frequency α (Ω), which probes (drives) the $|a\rangle \leftrightarrow |b\rangle$ ($|a\rangle \leftrightarrow |c\rangle$) transition. (b)–(d) Absorption A vs two-photon detuning δ (red dotted lines) for the resonant ($\Delta = 0$) pump with $\Gamma_{ab} = 1$, $\Gamma_{bc} = 0.1$, and various Ω with best fits to $A_{\text{EIT}}(C_+, C_-, \gamma_+, \gamma_-)$ (blue solid lines) and $A_{\text{ATS}}(C, \gamma, \delta_0)$ (green dashed lines) models calculated for (b) weak Ω with a good fit to $A_{\text{EIT}}(0.532, 0.633, 0.712)$ and a poor fit to closest $A_{\text{ATS}}(0.472, 0.512, 1.03)$ as well as $A_{\text{EIT}}(88.3, 88.3, 0.75, 0.752)$, and (d) strong Ω with a poor fit to closest $A_{\text{EIT}}(1.3 \times 10^3, 1.3 \times 10^3, 2.92, 2.92)$ and a good fit to $A_{\text{ATS}}(0.499, 0.521, 3.05)$.

and negative: $A_{\text{EIT}} = C_+^2/(\gamma_+^2 + \delta^2) - C_-^2/(\gamma_-^2 + \delta^2)$. Hence, low-power pump-induced transparency, where Fano interference dominates, has a transparency window without splitting [21]. For strong-pump Ω region (ii) $\delta_{\pm} \approx \pm\Omega + i(\Gamma_{ab} + \Gamma_{bc})/2$ and $S_{\pm} \approx 1/2$ so $A_{\text{ATS}} = C^2[1/(\gamma^2 + (\delta - \delta_0)^2) + 1/(\gamma^2 + (\delta + \delta_0)^2)]$, corresponding to the sum of two equal-width Lorentzians shifted from the origin by $\delta_0 \approx \pm\Omega$.

Figures 1(b)–1(d) demonstrate how well these EIT and ATS models fit calculated absorption profiles, but an objective criterion is needed to discern the best model or whether the data are inconclusive. Akaike’s information criterion (AIC) identifies the most informative model based on Kullback-Leibler divergence (relative entropy), which is the average logarithmic difference between two distributions with respect to the first distribution. AIC quantifies the information lost when model A_i with K_i fitting parameters is used to fit actual data: $I_i = -2 \log \mathcal{L}_i + 2K_i$ for \mathcal{L}_i the maximum likelihood for model A_i with penalty $2K_i$ for fitting parameters [17].

We demonstrate AIC-based testing by fitting an absorption data set $D = \{A(\delta_j); |\delta_j| \leq 5\}$, incrementing in steps $\Delta\delta_j = 0.05$, for the TLA in Fig. 1(a) to models A_{EIT} and A_{ATS} using the `NONLINEARMODELFIT` function in `MATHEMATICA`, which can calculate AIC. The relative likelihood of model A_i out of n models is its Akaike weight $w_i = e^{-I_i/2} / \sum_{k=1}^n e^{-I_k/2}$ depicted in Fig. 2(a). This figure shows that, based on AIC, the EIT model explains data with 100% likelihood for all $\Omega < \Omega_{\text{AIC}} = 0.86$. Figure 2(b) shows that increasing Γ_{bc} reduces the EIT threshold Ω_{AIC} and guides devising EIT experiments.

Testing for EIT is affected by the fact that experiments have additional complexities such as one-photon detuning or more than three energy levels, but these complexities do not negate the validity of our test; rather, these complications just make it harder to *pass* the EIT test. Consequently, one can construct and test more general models that accommodate these extra features because AIC allows relative testing between any number of models. The corresponding signatures of Fano interference in generalized models can be identified, thus revealing genuine EIT effects.

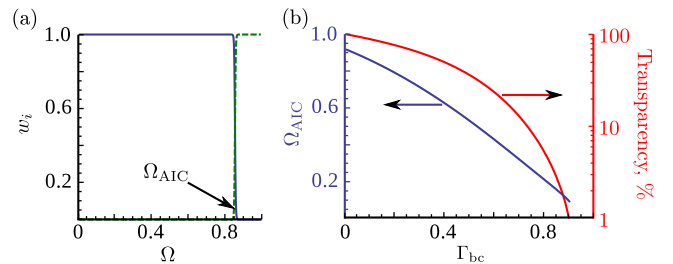


FIG. 2 (color online). (a) Akaike weights vs the Rabi frequency for the TLA in Fig. 1 showing a sharp transition at Ω_{AIC} from EIT model (blue solid line) to ATS model (green dashed line); (b) Transition boundary Ω_{AIC} with corresponding transparency values vs Γ_{bc} .

A more important issue of working with experimental data sets $D = \{A(\delta_j)\}$ is that experiments are noisy so each run produces a different data set, say D_ℓ , with many data points measured. In turn, the Akaike weight reveals the likelihood of describing a data set D_ℓ that becomes binary (0 or 1), hence conclusive, for large data sets as shown in Fig. 2(a). Consequently, one will conclusively say which model pertains after each run, but, because of noise, this conclusion could vary from run to run. Intuitively, the best model should be picked more often, however, experimental data are not reported on per run basis. Experimental data are typically reported as mean values with error bars representing the confidence interval for the data. Hence, we need to adapt the AIC-based testing to the way experimental data are reported.

Akaike's information according to the least-squares analysis is $I = N \log(\hat{\sigma}^2) + 2K$ for $\hat{\sigma}^2 = \sum_{j=1}^N \hat{\epsilon}_j^2/N$ and $\hat{\epsilon}_j^2$ for the estimated residuals from the fitted model [17]. Technical noise, however, blurs the distinction between models $\{A_i\}$ causing Akaike's information to become $I = N \log(\hat{\sigma}^2 + \hat{\sigma}_{\text{exp}}^2) + 2K$ with the aforementioned consequences. Hence, we propose a fitness test for Akaike's information obtained from reported experimental data.

Our fitness test uses a per-point (mean) AIC contribution $\bar{I} = I/N$ to calculate a per-point weight for the i -th model: $\bar{w}_i = \exp(-\bar{I}_i/2) / \sum_{k=1}^n \exp(-\bar{I}_k/2)$. These unnormalized per-point weights $\exp(-\bar{I}_i/2)$ converge to $1/\sqrt{\hat{\sigma}_i^2}$ for large data sets; for noisy data this yields equal per-point weights for all models, as expected, intuitively.

We simulate a noisy absorption profile by generating data D_ℓ according to $\langle A(\delta_j) \rangle = (1 + \xi)A(\delta_j)$ for ξ randomly chosen from the normal distribution $\exp[-x^2/2\sigma^2]/\sqrt{2\pi}\sigma$. Figure 3(a) shows our per-point weights for generated data with no noise, small noise, and moderate noise for the conditions of Fig. 1. In the no noise case and $\Omega < \Omega_{\text{EIT}} = 0.45$, the ATS model fails and has per-point weight: $\bar{w}_2 = 0$; beyond the EIT threshold Ω_{EIT} , the per-point weight for ATS starts to increase with both models describing the absorption profile equally well at $\Omega_{\text{AIC}} = 0.86$. This agrees with intuition about fitting models, especially a continuous trade-off between models in the intermediate regime. It is also intuitive to expect that under noisy conditions and a weak pump, $\Omega^2 < \Omega_\sigma^2 = 2\sigma\Gamma_{\text{ab}}\Gamma_{\text{bc}}/(1 - 2\sigma)$, induced transparency is buried in noise, $1 - \text{Im}[\sigma_{\text{ab}}(\delta = 0, \Omega)] / \text{Im}[\sigma_{\text{ab}}(\delta = 0, \Omega = 0)] < 2\sigma$, and both models account for the absorption profile equally well [see Fig. 3(b)]. Consequently, at $\Omega = 0$ and any amount of noise, per-point weights are equal to 0.5 and results are inconclusive. Increasing the pump field, however, favors the EIT model until it gives way to ATS dominance for pump strength greater than Ω_{AIC} . Therefore, a convincing EIT demonstration requires suppression of technical noise to the point that our per-point weights become well separated.

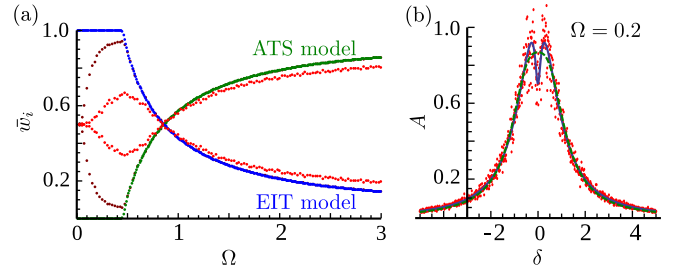


FIG. 3 (color online). (a) Per-point weights \bar{w}_i for the conditions of Fig. 1 as a function of the pump-field Rabi frequency Ω illustrate three distinct regions: $\Omega < \Omega_{\text{EIT}} = 0.45$, where the EIT model (blue line) dominates unconditionally; $0.45 < \Omega < 0.86$, where the ATS model (green line) shows nonzero likelihood; $\Omega > 0.86$, where the ATS model dominates. The presence of Gaussian noise with standard derivation $\sigma = 0.1$ (red dots) [$\sigma = 0.01$ (burgundy dots)] affects the per-point weights for EIT and ATS models leading to the absence of unconditional dominance by the EIT model. (b) In the weak-pump limit and a poor signal-to-noise ratio, both models are equally likely to fit data (red dots).

We apply our theory to the recent observation of induced transparency (i.e., transparency), reported as EIT, for an open transmission line of a superconducting circuit with a single flux-type artificial atom (“flux qubit”) [23]. In contrast to the TLA system discussed here, a flux qubit driven or probed by microwave fields, which are polarized and confined to one dimension, presents a nearly lossless upper-pumped Ξ system. Nevertheless, EIT testing of this observation is straightforward, with absorption being effectively replaced by reflection, since their analysis shows that the transmission coefficient agrees with the electromagnetic response for a TLA: $t = 1 - (\gamma_{\text{ab}}/2)/[\Gamma_{\text{ab}} + i\delta + \Omega^2/(\Gamma_{\text{bc}} + i\delta)]$ with our Rabi frequency Ω being half their Rabi frequency [23].

Induced transparency is evident from calculating $\text{Re}(t)$ for the probe field in the presence of the control field. Their system has a population relaxation rate $\gamma_{\text{ab}}/2\pi = 11$ MHz and dephasing rates $\Gamma_{\text{ab}}/2\pi = 7.2$ MHz and $\Gamma_{\text{bc}} = 0.96\Gamma_{\text{ab}}$. Therefore, the transparency window appears for a control-field amplitude of $\Omega/2\pi = 6$ MHz, which exceeds $\Omega_{\text{EIT}}/2\pi = 0.15$ MHz, so the experiment operates in a region where demonstrating Fano interference must be inconclusive.

In fact, the theoretical transmission curve based on the reported parameters, shown in Fig. 4(a), is indistinguishable from the best-fit ATS model and clearly distinct from the EIT model. This is further corroborated by our per-point weight that yields $\bar{w}_1 = 0.03$, implying that the result is far from EIT. Whereas the reported induced transparency suffices for switching of propagating waves in a superconducting circuit [23], our objective test shows conclusively that they demonstrated ATS and not EIT.

Because of noise, however, the actual experimental data shown in Fig. 4(b) differ from the theoretical predictions

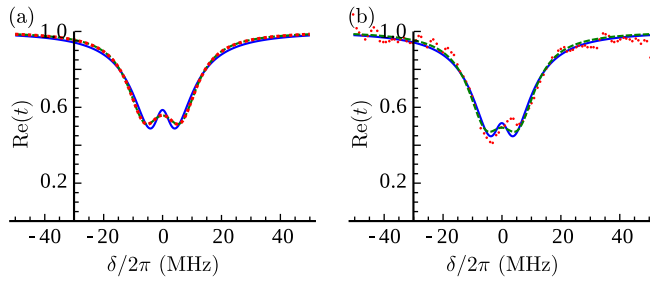


FIG. 4 (color online). Transmission $\text{Re}(t)$ vs two-photon detuning δ for (a) a theoretical curve (red dotted line) with parameters taken from Ref. [23] and control-field amplitude $\Omega = 6$ MHz compared to the best-fit $A_{\text{EIT}}(25.4, 24.2, 6.36, 6.15)$ (blue solid line) and $A_{\text{ATS}}(4.42, 7.1, 6.1)$ (green dashed line) and (b) actual experimental data from Ref. [23] (red dots) vs best-fit $A_{\text{EIT}}(11.8, 9.08, 6.77, 5.66)$ (blue solid line) and $A_{\text{ATS}}(4.59, 7.29, 5.49)$.

discussed above and shown in Fig. 4(a), so a reported data set does not conclusively show EIT nor rule it out. That is, optimal choices of A_{EIT} and A_{ATS} seem to fit the data equally well. Yet, there is a slight preference for ATS according to our per-point weight criterion, $\bar{w}_1 = 0.48$ and $\bar{w}_2 = 0.52$, in the weak-field limit with the obvious favoring of ATS in the strong-field regime.

In conclusion, we propose an objective way to discern ATS vs EIT from experimental data obtained from TLA systems. Our test exhibits a smooth transition from ATS to EIT through three qualitative regions as the strength of the driving field Ω decreases. The sought-after EIT signal is due to Fano interference manifested as a narrow negative Lorentzian in the absorption data for TLA accompanied by the absence of splitting, which ATS lacks. Akaike's information criterion objectively finds this evidence of Fano interference and ascertains from each data set whether EIT or ATS pertains. We have introduced a per-point weight that accommodates experimental noise and readily produces a conclusion of whether EIT or ATS pertain as well as provides a "do not know" alternative for inconclusive experiments. Furthermore, Akaike's information criterion, combined with our per-point weights, is not limited to our simple EIT and ATS models, but allows the inclusion of additional models. Hence, data can be tested against more complicated models that take care of additional levels, one-photon detunings, as well as inhomogeneous broadenings and may have a greater likelihood to return a definite answer with otherwise inconclusive experiments.

The EIT vs ATS distinction is especially important for applications, such as sensing, where the weak-pump regime is necessary, but transparency and sensitivity are required. Nowadays EIT demonstrations are attempted in a multitude of experimental systems, and a versatile test is needed that can unambiguously reveal whether the requisite conditions have been met. We have provided such a test that, provided data from adequate experiments, objectively discerns whether ATS or EIT pertains for a given

experiment without the need of prior knowledge or disposition about the system.

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