Theory of Spin Noise in Nanowires

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We develop a theory of spin noise in semiconductor nanowires considered as prospective elements for spintronics. In these structures, spin-orbit coupling can be realized as a random function of a coordinate correlated on a spatial scale of the order of 10 nm. By analyzing different regimes of electron transport and spin dynamics, we demonstrate that the spin relaxation can be very slow, and the resulting noise power spectrum increases algebraically as the frequency goes to zero. This effect makes spin phenomena in nanowires best suitable for studies by rapidly developing spin-noise spectroscopy.

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Nanostructures are the promising hardware elements for spintronics [1]—a rapidly developing branch of physics and technology aiming at studies and application of spin-dependent phenomena in the charge transport and information processing. The quest for systems with ultralong spin relaxation times [2] is one of the main challenges in this field. Since the dynamical spin fluctuations [3], characterized by correlations on the spin relaxation time scale, are seen as spin noise in the frequency domain, this search can be done with recently developed highly accurate low-frequency spin-noise spectroscopy [4] aimed at the measurement of intrinsic equilibrium spin dynamics. Spin-noise spectroscopy allows one to study the slow spin dynamics in (110)-grown quantum wells [5] and in quantum dots [6]. A theoretical background of this method is given, e.g., in Refs. [7–9].

An interesting class of semiconductor nanostructures demonstrating peculiar and slow spin dynamics are the quantum wires [10–12], where, e.g., InAs, InSb, as well as GaAs/AlGaAs systems are the prospective realizations. The effects of spin-orbit (SO) coupling on the transport were clearly demonstrated there [13,14], and the nanowire-based qubits were introduced [15,16]. A SO coupling induced effective magnetic field acting on electron spins in nanowires is directed parallel or antiparallel to a certain axis [17–21], resulting in a giant spin relaxation anisotropy similar to that expected in some two-dimensional systems [22]. Since the SO coupling is a structure- and material-dependent property, all sorts of disorders (random doping [23–26], interface fluctuations [27], random variations in the shape, etc.) which cause electron scattering and nonzero resistivity can cause local variations in the coupling. As a result, in addition to the regular SO coupling, caused by the lack of bulk (Dresselhaus term) or structure (Rashba term) inversion symmetry, all low-dimensional structures inevitably have the random contribution in it. The spatial scale of the fluctuations is of the order of 10 nm, as determined by the characteristic distances in nanostructure. The fluctuations were shown to give rise to a number of fascinating phenomena [28–30]. However, their role in quantum wires was not studied so far.

Here, we address theoretically the electron spin dynamics in ballistic and diffusive semiconductor nanowires aiming at the study of the spin-noise spectrum. Different regimes of electron spin relaxation are determined, and the crossovers between them are analyzed in detail. In particular, we demonstrate that, when the electron motion is diffusive and the dominant contribution to the SO interaction is random, the spin relaxation becomes algebraic rather than exponential and the spin-noise power spectrum diverges at low frequencies ω as $1/\omega^{1/2}$, showing colored noise [31–33] well-suited for the studies by the spin-noise spectroscopy. A very slow spin dynamics resulting in the low-frequency noise divergence makes nanowires an exception among semiconductor systems.

The spin-noise spectroscopy, reviewed in Ref. [4], is based on the optical monitoring of the spin fluctuations [34] in Faraday, Kerr, or ellipticity signals measured with a weak linearly polarized probe beam incident on a single wire or a wire array sample; see Fig. 1. It can be shown similarly to Refs. [4,9,35] that, for the probe tuned to the fundamental absorption edge, the Kerr rotation angle $\theta_K \propto s_z$ [36]; hence, its autocorrelation function is directly related to the spin noise: $\langle \theta_K(t)\theta_K(t')\rangle \propto \langle s_z(t)s_z(t')\rangle$, where $s_z(t)$ is the density of the z component of the total electron spin. As a result, this optical technique measures long-range correlations of equilibrium spin fluctuations occurring in the illuminated spot.

We consider a single-channel quantum wire extended along the x axis and represent the SO Hamiltonian as

$$\mathcal{H}_{\mathrm{SO}} = \frac{1}{2}[\alpha(x)k_x + k_x\alpha(x)]\sigma_\lambda. \tag{1}$$

Here, $k_x = -i\partial/\partial x$ is the electron wave vector component along the wire axis, and $\alpha(x)$ is the coordinate-dependent SO coupling strength. In Eq. (1), we assumed that the spin

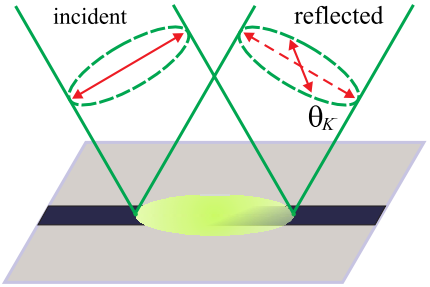

FIG. 1 (color online). Schematic plot of the experimental configuration: a quantum wire (dark stripe) is illuminated by a linearly polarized beam, and the Kerr rotation angle of its polarization plane θ_K is measured. Polarizations of the beams are marked by double-headed arrows. The dashed arrow corresponds to the polarization of the reflected beam in the absence of the Kerr effect.

quantization axis, λ, is fixed, and σ_λ is the component of the spin operator along this axis. The specific form of the SO Hamiltonian Eq. (1) implies that the effective field acting on electron spin points either parallel or antiparallel to the axis λ. This is obvious for a constant $\alpha(x)$ [17–21] and holds true, provided that the microscopic symmetry of the fluctuations forming the SO coupling randomness is the same as the overall symmetry of the system.

The SO coupling is assumed to be the sum of the coordinate-independent contribution, α_0, and the Gaussian random function with zero average, $\alpha_r(x)$, such as $\alpha(x) = \alpha_0 + \alpha_r(x)$ with the correlation function [29]:

$$\langle \alpha_r(x)\alpha_r(x')\rangle = \langle \alpha_r^2\rangle F_{\rm corr}(x - x'), \tag{2}$$

where $\langle \alpha_r^2\rangle$ is the mean square of SO coupling fluctuations and the range function $F_{\rm corr}(x - x')$. We introduce also the typical correlation length of the SO coupling

$$l_d = \int_0^\infty F_{\rm corr}(x)dx, \tag{3}$$

characterizing the size of the correlated domain of the random SO coupling. Details of the models of random SO coupling can be found in Ref. [29].

We begin with the semiclassical regime, where the SO coupling disorder is smooth on the scale of electron wavelength, $l_d \gg \lambda_{\rm F}$, where the wavelength of the Fermi-level electrons $\lambda_{\rm F} = 2\pi/k_{\rm F}$, with $k_{\rm F}$ being the Fermi wave vector for the degenerate electron gas. The Hamiltonian (1) implies that the spin rotation angle around the λ axis during the motion from the point x_0 to x_1 is

$$\theta(x_1, x_0) = \frac{2m}{\hbar^2}\int_{x_0}^{x_1} \alpha(x')dx', \tag{4}$$

where m is the electron effective mass. Equation (4) shows that the angle is solely determined by electron initial and final positions and does not depend on the history of the motion between these points. This result, being well-established for the systems with regular SO coupling [20,37–40], holds also for the nanowires with the SO coupling disorder. As follows from Eq. (1), the spin precession rate is proportional to the electron velocity and given coordinate-dependent function. Hence, it does not matter whether the electron starting from the point x_0 reached the point x_1 ballistically or diffusively: all contributions to the spin precession of the closed paths, where an electron passes the same configuration of $\alpha(x)$ in the opposite directions, cancel each other.

The temporal evolution of electron spin is directly related to the electron motion along the wire. We consider here spin projections at a given z axis, perpendicular to the spin quantization axis λ. The time dependence of the electron spin z component averaged over its random spatial motion and over the random precession caused by the field $\alpha(x)$ can be most conveniently characterized by the correlator $\langle s_z(t)s_z(0)\rangle = \langle s_z^2(0)\rangle \mathcal{C}_{ss}(t)$ with the normalized correlation function

$$\mathcal{C}_{ss}(t) = \int_{-\infty}^{\infty} dx p(x, t)\langle \cos[\theta(x, 0)]\rangle, \tag{5}$$

where $p(x, t)$ is the probability that an electron travels the distance x during the time t. Note that $\mathcal{C}_{ss}(t)$ can be understood as the disorder-averaged electron spin z component found with the initial condition $s_z(0) = 1$. It results from the linearity of the spin dynamics equations: the correlators $\langle s_i(t)s_j(0)\rangle$ satisfy exactly the same equations as average values $\langle s_i(t)\rangle$ $(i, j = x, y, z)$. In derivation of Eq. (5), we assumed also that the scattering of electrons, which determines $p(x, t)$, is not correlated with the random SO field $\alpha_r(x)$; hence, the averaging over the realizations of $\alpha_r(x)$ denoted by the angular brackets and over the trajectories can be considered independently. This can occur in nanowires where random Rashba fields are induced by doping, while the momentum scattering is due to the wire width fluctuations. If the same local disorder determines the electron scattering and random SO fields, in relatively clean systems, the electron mean free path l exceeds by far the disorder correlation length l_d in Eq. (3). Hence, spatial scales of two random processes—l for the electron backward scattering in the random potential and l_d for the spin precession—are strongly different. As a result, on the l scale, the memory of the short-range correlations is lost, and Eq. (5) holds. Although Eq. (5) is presented for the smooth SO coupling disorder, where the electron motion is semiclassical, $l_d/\lambda_{\rm F} \gg 1$, a general Green's function approach confirms it for arbitrary $l_d/\lambda_{\rm F}$ values.

Our next step is to perform the averaging of $\cos[\theta(x, 0)]$ in Eq. (5) over the random realizations of the $\alpha(x)$ field. For this purpose, we recast

$$\cos[\theta(x, 0)] = {\rm Re}\left\{\exp\left(i\frac{2m\alpha_0}{\hbar^2}x\right)\exp[i\vartheta_r(x)]\right\}, \tag{6}$$

where

$$\vartheta_r(x) = 2m/\hbar^2\int_0^x \alpha_r(x')dx' \tag{7}$$

is the contribution of the random SO coupling to the spin rotation angle. We expand the last exponent in the series in ϑ_r, assuming the Gaussian SO coupling disorder. In the averaging, the odd powers of the spin rotation angle vanish, $\langle\theta_r^{2n+1}(x)\rangle = 0$, for integer n, and even powers can be expressed solely with $\langle\theta_r^2(x)\rangle$ as

$$\langle\theta_r^{2n}(x)\rangle = \left\langle\left[\frac{2m}{\hbar^2}\int_0^x \alpha_r(x')dx'\right]^{2n}\right\rangle = (2n-1)!!\langle\theta_r^2(x)\rangle^n. \tag{8}$$

A direct calculation shows that the mean square $\langle\theta_r^2(x)\rangle$ caused by the random SO interaction is given by

$$\langle\theta_r^2(x)\rangle = 2\left(\frac{2m}{\hbar^2}\right)^2\langle\alpha_r^2\rangle\int_0^x dx'\int_0^{x'} dy F_{\rm corr}(y). \tag{9}$$

Finally, Eq. (5) reduces to

$$\mathcal{C}_{ss}(t) = \int_{-\infty}^{\infty} dx p(x,t)\cos\left(\frac{2m\alpha_0}{\hbar^2}x\right)\exp[-\langle\theta_r^2(x)\rangle/2]. \tag{10}$$

When $\langle\theta_r^2(x)\rangle$ becomes considerably larger than 1, spins are completely dephased. Equation (10) is our central result: it relates temporal average spin dynamics with electron motion along the wire. The distribution function of electron displacements, $p(x, t)$, presented for different regimes of electron motion below, enables us to calculate spin evolution by Eq. (10). The spin-noise power spectrum is given by the transform of $\mathcal{C}_{ss}(t)$ [9]:

$$\langle s_z^2\rangle_\omega = 2\int_0^\infty \mathcal{C}_{ss}(t)\cos(\omega t)dt. \tag{11}$$

To get a better insight into the problem, we begin with the key limits [41]. First, for the ballistic electron dynamics, $p(x, t) = \delta(x - v_F t)$, where $v_F = \hbar k_F/m$ is the Fermi velocity. The ballistic motion is realized on the temporal scale $t \ll \tau = l/v_F$, with τ being the momentum relaxation time. We are interested in the spin dynamics on the time scale $t \gg \tau_d = l_d/v_F$, where τ_d is the time during which an electron passes the correlated interval of the SO coupling fluctuations. Using Eq. (10), we obtain damped oscillations of the spin z component:

$$\mathcal{C}_{ss}(t) \approx \cos(\Omega_0 t)\exp(-t/\tau_{s,r}), \tag{12}$$

with the frequency $\Omega_0 = 2m\alpha_0 v_F/\hbar^2$ determined by the averaged SO coupling and the decay time caused by the SO coupling fluctuations

$$\frac{1}{\tau_{s,r}} = \left(\frac{2mv_F}{\hbar^2}\right)^2\langle\alpha_r^2\rangle\tau_d. \tag{13}$$

Equation (13) for the spin relaxation time $\tau_{s,r}$ is a result of random spin precession [29]. The spin-noise power spectrum calculated using Eqs. (11) and (12) reads

$$\langle s_z^2\rangle_\omega = 2\tau_{s,r}\,\mathrm{Re}\frac{1 - i\omega\tau_{s,r}}{\Omega_0^2\tau_{s,r}^2 + (1 - i\omega\tau_{s,r})^2}, \tag{14}$$

with the result presented in Fig. 2.

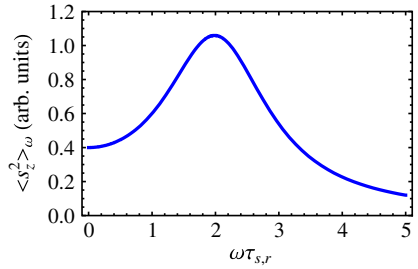

FIG. 2 (color online). The spin-noise power spectrum, $\langle s_z^2\rangle_\omega$, for ballistic propagation, $\Omega_0\tau_{s,r} = 2$. Because of the exponential decay in Eq. (12), it is finite at $\omega = 0$, with the width determined by the spin relaxation time $\tau_{s,r}$. The spectrum peaks at the frequency $\approx \Omega_0$, since the average electron spin rotates in the SO field at the rate Ω_0 and asymptotically decays as ω^{-2}, in accordance with the fluctuation-dissipation theorem.

This ballistic regime of spin dynamics, however, can be realized only in very clean systems, where $\Omega_0\tau \gg 1$. Otherwise, electron spin evolution occurs at the time scale, where an electron moves diffusively (Fig. 3, upper panel), i.e.,

$$p(x, t) = \frac{1}{2\sqrt{\pi Dt}}e^{-x^2/4Dt}, \tag{15}$$

where $D = v_F^2\tau$ is the diffusion coefficient. If the SO coupling is uniform and the condition $\Omega_0\tau \ll 1$ is satisfied, spin relaxation is exponential. The relaxation time is given by the Dyakonov-Perel formula $\tau_{s,\rm DP} = 1/(\Omega_0^2\tau)$

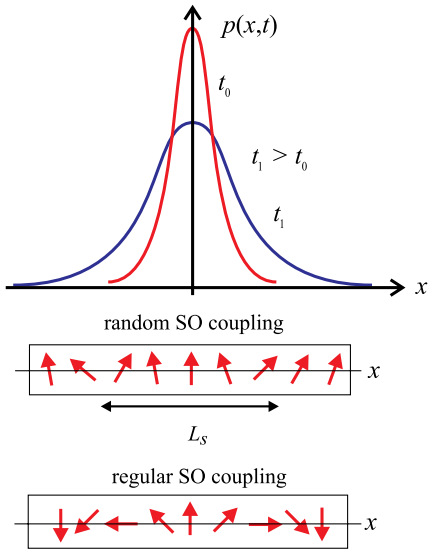

FIG. 3 (color online). Upper panel: schematic illustration of the displacement distribution $p(x, t)$ for two different time moments: $t_0 < t_1$. Lower panels: the quantum wire and spins of a diffusing electron for the random and regular SO couplings, respectively. For the random SO coupling, if the electron is within the L_s distance from its initial point [see Eq. (17)], its spin is preserved; when it leaves this interval, the spin dephases.

[17,21]. The spin-noise spectrum has a Lorentzian form $\langle s_z^2\rangle_\omega = 2\tau_{s,\mathrm{DP}}/(1+\omega^2\tau_{s,\mathrm{DP}}^2)$ with the width determined by the relaxation time.

New physical features arise when the SO coupling fluctuations dominate over the regular contribution. From now on, we put $\alpha_0 = 0$, consider the system where SO coupling is purely random, and concentrate on the long-time ($t \gg \tau_d, \tau, \tau_{s,r}$) dynamics. At these times, the system in Eq. (10) is characterized by two length parameters. One parameter is the diffusion length $\sqrt{Dt}$ in Eq. (15); the other one,

$$L_s = \int_0^\infty dx \exp[-\langle\theta_r^2(x)\rangle/2], \tag{16}$$

characterizes spin randomization. At sufficiently long times, when $\sqrt{Dt} \gg L_s$, one can take $p(0, t)$ instead of $p(x, t)$ and immediately obtain from Eq. (10) that the relaxation is algebraic rather than exponential:

$$\mathcal{C}_{ss}(t) \approx p(0,t)\int_{-\infty}^{\infty} dx \exp[-\langle\theta_r^2(x)\rangle/2] = \frac{L_s}{\sqrt{\pi Dt}}. \tag{17}$$

Equation (17) predicts extremely long spin decoherence described by the inverse square root law: $\langle s_z(t)\rangle \propto 1/\sqrt{t}$. This surprising result has a transparent physical interpretation (see Fig. 3): Indeed, if an electron is displaced from its initial position by a sufficiently large distance, $x \gtrsim L_s$, its spin rotation angle becomes so large that it does not contribute to the total spin polarization, owing to $\exp[-\langle\theta_r^2(x)\rangle/2]$ in Eq. (10). As a result, the spin polarization is supported by the electrons located in the vicinity of their initial positions, mainly due to the return after multiple scatterings by the random potential. The fraction of such electrons, in agreement with the diffusion distribution, decays as $p(0, t) \propto 1/\sqrt{t}$, resulting in the same behavior in the spin polarization. It is interesting to mention that this qualitative argument does not work for the constant SO coupling despite the fact that the spin of the electron is also restored here upon the return to the origin. The reason is that, due to the oscillations of the spin on the spatial scale of the order of $\hbar^2/m\alpha_0$ (see Fig. 3, lower panel) in Eq. (10), the diffusive return of electrons to the origin is insufficient for formation of the algebraic relaxation tail.

Another realization of the $1/\sqrt{t}$ spin decay can be achieved for the very strong random SO couping where the spin relaxation occurs within one nanosize domain of the SO coupling that is, at the electron displacement, much less than l_d. In this case, the spin relaxation rate is due to the Dyakonov-Perel mechanism and is determined by the local value of $\alpha(x)$ inside the domain. The spins of electrons located in the intervals with large $\alpha(x)$ will relax quickly, while the spins of those experiencing weak $\alpha(x)$ will relax slowly.

Slow nonexponential spin relaxation, described by Eq. (17), manifests itself in the low-frequency spin-noise spectrum. From Eq. (11), it follows that $\langle s_z^2\rangle_\omega \propto 1/\sqrt{\omega}$, i.e., the spin noise diverges at $\omega \to 0$. Such a nontrivial behavior is inherent to the quantum wires with random SO coupling, where the spin restores upon its return to the origin: in multichannel wires for sufficiently fast interchannel scattering [42] and in two-dimensional systems, spin relaxation is exponential [29] and $\langle s_z^2\rangle_{\omega=0}$ is finite.

To conclude, we studied theoretical spin noise in a semiconductor nanowire for different regimes of the electron propagation. We demonstrated that, if the spin relaxation is determined by the randomness in the SO coupling, spin relaxation becomes algebraic, being closely related to the high probability for an electron to stay close to its initial position as a result of a multiple scatterings in the random potential. This behavior can appear in at least two possible regimes: (i) when the electron motion is diffusive and (ii) when the spin relaxation occurs on a small spatial scale of the order of 10 nm. In any of these cases, the spin-noise power spectrum shows colored $1/\sqrt{\omega}$ noise. In addition, this observation shows that low-frequency optical spin-noise spectroscopy is an excellent tool for studying spin phenomena in semiconductor nanowires and the characterization of random potential and SO coupling there.

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[42] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.107.156602 for the analysis of spin noise in multichannel wires.