Nonclassical Behavior of an Intense Cavity Field Revealed by Quantum Discord

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We investigate the quantum-to-classical crossover of a dissipative cavity field by measuring the correlations between two noninteracting atoms coupled to the cavity mode. First, we note that there is a time window in which the mode shows a classical behavior, which depends on the cavity decay rate, the atom-field coupling strength, and the number of atoms. Then, considering the steady state of two atoms inside the cavity, we note that the entanglement between the atoms disappears while the mean number of photons of the cavity field ($\bar{n}$) rises. However, the quantum discord reaches an asymptotic nonzero value even in the limit of $\bar{n} \to \infty$, whether $\bar{n}$ is increased coherently or incoherently. Therefore, the cavity mode always preserves some quantum characteristics in the macroscopic limit, which is revealed by the quantum discord.

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Although the quantum theory predicts many nonclassical and intriguing phenomena, such as quantum superposition of states and quantum nonlocality [1,2], such phenomena can be observed only with difficulty in the macroscopic world, as classical physics is recovered from quantum mechanics for large excitation numbers and many-particle systems [3]. The emergence of classical physics from quantum mechanics is therefore actively studied and suggested explanations include decoherence due to interaction with the environment [4], impossibility of macroscopic superposition of distinct states [5], and restrictions due to imprecise measurements [6]. However, to determine the quantum or classical behavior of a given system we need to introduce the meter to observe its properties, which in quantum theory is not a simple task. The simple interaction of a given system and a meter modifies its dynamics in such a way that the quantum-to-classical crossover may depend on the meter involved. For cavity fields one usually employs a single atom as the meter for the cavity-field properties, as, for example, in Refs. [7–9]. In Ref. [8] the quantum-to-classical crossover was investigated by raising gradually the effective temperature of a circuit QED system. At low temperatures, vacuum Rabi oscillations and mode splitting were observed, revealing the quantum nature of the light field. However, these effects disappear when the effective temperature is raised, increasing the mean number of photons $\bar{n}$ of the cavity mode. Naturally, the conditions needed for a bosonic mode to show a classical behavior depend on the system parameters and even a cavity mode with a very small $\bar{n}$ may behave classically [9]. Normally, however, the increasing of $\bar{n}$ destroys its quantum properties, as shown in Refs. [7,8]. In Ref. [10] the quantumness of an ideal cavity field interacting with a single ideal two-level atom was also investigated, showing that the antibunching is still present for an initial $\bar{n}$ up to three photons. However, from the analytical results in [10], one can see that the antibunching (quantumness) disappears in the limit of macroscopic field ($\bar{n} \to \infty$).

Differently from the studies above, here we investigate the behavior of a dissipative cavity field interacting with N atoms instead of just one. As in [9], we assume that the cavity mode is pumped by a classical (external) field that controls $\bar{n}$. It can be shown that a purely classical field [11] is not able to generate any kind of correlation between the atoms, since such fields perform only local operations on the atoms. On the other hand, the correlations can be generated only when the cavity mode has some quantum behavior, owing of the indistinguishability of paths in the exchange of a photon between the cavity mode and atomic system. Hence, the presence of any kind of correlations (quantum [12,13] or classical [14]) between the atoms can be taken as a sign of the nonclassical (granular) behavior of a macroscopic cavity field, just as Brownian motion of a pollen grain is a sign of the behavior of moving particles composing a macroscopic fluid.

Consider a cavity mode interacting resonantly with N identical two-level atoms ($|g\rangle =$ ground state, $|e\rangle =$ excited state) and simultaneously driven by a resonant classical field. Such a system is described by the total Hamiltonian ($\hbar = 1$)

$$H = \frac{\omega_0}{2} S_z + \omega a^\dagger a + H_P + H_I, \tag{1}$$

where ω is the cavity mode frequency and a ($a^\dagger$) its annihilation (creation) operator; $S_z = \sum_{j=1}^{N} \sigma_z^j$, $\omega_0 = \omega$, and σ_z^j ($= |e\rangle_j\langle e| - |g\rangle_j\langle g|$) are the atomic transition frequency and the z-Pauli matrix of atom j, respectively; $H_P = \varepsilon(a e^{i\omega_L t} + \mathrm{H.c.})$ describes the pumping field on the cavity mode, ε and $\omega_L = \omega$ being the strength and

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frequency of the driving field, respectively; $H_I = g\sqrt{N}(aS_+ + \text{H.c.})$ is the interaction Hamiltonian between the cavity mode and atoms, with $S_+ = S_-^\dagger = \frac{1}{\sqrt{N}}\sum_{j=1}^N \sigma_+^j$, $\sigma_+^j = |e\rangle_j\langle g|$, and g the atom-field coupling. Writing the Hamiltonian in a frame rotating at the laser frequency, by applying the unitary transformation $U = \exp[-i\omega t(a^\dagger a + S_z/2)]$, we have $V_L = (g\sqrt{N}S_+ a + \varepsilon a + \text{H.c.})$. The dynamics of this system, assuming a leaking cavity, is governed by the master equation

$$\dot{\rho} = -i[V_L, \rho] + \kappa(n_{\text{th}} + 1)\mathcal{L}[a]\rho + \kappa n_{\text{th}}\mathcal{L}[a^\dagger]\rho, \qquad (2)$$

n_{th} being the mean number of thermal photons, κ the dissipation rate of the cavity mode, and $\mathcal{L}[A]\rho = (2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A)$. As in [7], we neglect the atomic decay as the atoms act as a meter to monitor the behavior of the cavity mode. To observe the action of the driven cavity field on the atoms, first we apply a time-independent unitary transformation which consists of a displacement operation $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$, i.e., $\tilde{\rho} = D^\dagger(\alpha)\rho D(\alpha)$. Setting $\alpha = -i\varepsilon/\kappa$ and $T = 0K$, we finally obtain

$$\frac{d\tilde{\rho}}{dt} = -i[H_{\text{JC}} + H_{\text{SC}}, \tilde{\rho}] + \kappa\mathcal{L}[a]\tilde{\rho}, \qquad (3)$$

with $H_{\text{JC}} = g\sqrt{N}(aS_+ + \text{H.c.})$, $H_{\text{SC}} = (\Omega S_+ + \text{H.c.})$, and $\Omega = g\sqrt{N}\alpha$. We must observe that the chosen value for α is exactly the amplitude of the asymptotic coherent field of the cavity mode for $\varepsilon \gg g\sqrt{N}$. Under this condition we find, in all the numerical calculations carried out below, that the cavity field exhibits the statistical properties of a coherent field [i.e., correlation function $g^{(2)}(0) = 1$, Mandel factor $Q = 0$, and mean number of photons $\bar{n} = |\alpha|^2$]. Looking at the atoms, it is clear from Eq. (3) that, in this displaced picture, we have two kinds of atomic dynamics: one governed by a classical field and the other by a quantum field. Now we proceed to investigate what happens to the dynamics of the N two-level atoms when the cavity mode dissipates strongly.

First, we consider the weak coupling limit, such that $\kappa \gg g_{\text{eff}}\sqrt{\bar{n}_D + 1}$, with $g_{\text{eff}} = g\sqrt{N}$ and $\bar{n}_D$ the intracavity mean number of photons in the displaced representation. For $t \gg 1/\kappa$, we can adiabatically eliminate the field variables [15], resulting in a reduced master equation for the atoms, which in the interaction picture is given by

$$\dot{\rho}_a = -i[H_{\text{SC}}, \rho_a] + \Gamma_{\text{eff}}\mathcal{L}[S_-]\rho_a, \qquad (4)$$

where $\Gamma_{\text{eff}} = g^2 N/\kappa$.

Note that Eq. (4) describes a set of N atoms driven by a classical field with an effective Rabi frequency $|\Omega|$ and interacting with a common effective reservoir with an effective decay rate Γ_{eff}. For $|\Omega| \gg \Gamma_{\text{eff}}$, i.e., for $\varepsilon \gg g\sqrt{N}$, and for interaction time $t \ll 1/\Gamma_{\text{eff}} = \kappa/g^2 N$, according to Eq. (4) the dynamics of the system will be governed mainly by a free evolution. Then, as the reduced master equation above was derived for $t \gg 1/\kappa$, we can see that, for interaction times limited to the time window

$$1 \ll \kappa t \ll \frac{\kappa}{\Gamma_{\text{eff}}} = \left(\frac{\kappa}{g\sqrt{N}}\right)^2, \qquad (5)$$

the master equation can be approximated by $\dot{\rho}_a \simeq -i[H_{\text{SC}}, \rho_a]$, which represents an atomic system interacting with a classical electromagnetic field (a similar classical time window is presented in [9]). Preparing the atoms A and B initially in the separable state $\rho(0) = \rho_A \otimes \rho_B$, we will have $\rho(t) = \rho_A(t) \otimes \rho_B(t)$, clearly showing that the interaction Hamiltonian H_{SC} is unable to generate any kind of correlation between the atoms. Moreover, if an initial atomic state is pure, it will remain pure for any time. Thus, in this case the atomic purity is a good parameter to validate the semiclassical approximation, since outside the time window (5) the dynamics of the system is governed by Eq. (4), where the presence of the effective dissipative term introduces decoherence into the atomic system.

Looking within the time window, we see that, for a fixed g/κ, the interaction time for which the semiclassical regime is still valid decreases with $1/N$; in words, the bigger the number of atoms inside the cavity, the smaller is the time window in which the semiclassical regime is valid. In Fig. 1(a) we show the atomic purity for 1, 2, and 3 atoms inside the cavity, for $g = 0.01\kappa$, $\varepsilon = \kappa$ (which results in a maximum mean number of photons $\bar{n}_{\max} = |\varepsilon/\kappa|^2 = 1$) and with the atoms initially prepared in the excited state $|e\rangle$ and the cavity mode in the vacuum $|0\rangle$. For atoms initially prepared in the ground state $|g\rangle$, the graph is qualitatively the same. We can see in this figure that the purity of the system falls quickly when it leaves the time window that defines the semiclassical regime.

The next step consists in determining the correlations between two atoms. The measure of total quantum correlations used here is the quantum discord (QD) [12]. Nonzero QD in a bipartite system implies that it is

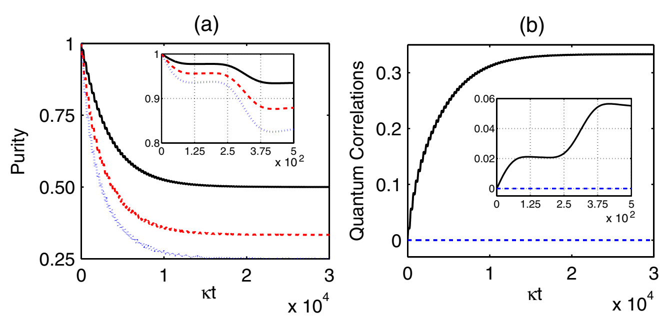

FIG. 1 (color online). Evolution in time of (a) atomic purity for one (full line), two (dashed line), and three (dotted line) atoms within the cavity and (b) quantum correlations for two atoms: QD (full line) and EoF (dashed line). We set $g = 0.01\kappa$, $\varepsilon = \kappa$, and all atoms initially in the excited state $|e\rangle$ and the cavity mode in the vacuum $|0\rangle$. Inset: start of evolution (expanded).

impossible to extract all information about one subsystem without perturbing its complement. In all cases investigated here the reduced density matrix for the atomic system ρ_{AB} in the basis $\{|e,e\rangle, |e,g\rangle, |g,e\rangle, |g,g\rangle\}$ has an X structure defined by its elements $\rho_{12} = \rho_{13} = \rho_{24} = \rho_{34} = 0$, with real coherences and $\rho_{22} = \rho_{33}$. For this class of density matrix, the QD can be calculated analytically [16]: $\mathrm{QD}(\rho_{AB}) = S(\rho_A) - S(\rho_{AB}) - \max\{D_1, D_2\}$, where $D_1 = \sum_{i=1,3}\rho_{ii}\log_2(\frac{\rho_{ii}}{\rho_{ii}+\rho_{i+1,i+1}}) + \sum_{i=2,4}\rho_{ii}\log_2(\frac{\rho_{ii}}{\rho_{ii}+\rho_{i-1,i-1}})$ and $D_2 = \sum_{i=0,1}(\frac{1+(-1)^i\beta}{2})\log_2(\frac{1+(-1)^i\beta}{2})$, with $\beta^2 = (\rho_{11} - \rho_{44})^2 + 4(|\rho_{23}| + |\rho_{14}|)^2$. Here $S(\cdot)$ denotes the von Neumann entropy [17] and $\rho_A = \mathrm{Tr}_B\rho_{AB}$. The entanglement, a kind of quantum correlation, is computed through entanglement of formation (EoF) [13]. For an X-form density matrix the EoF is [16,18]: $\mathrm{EoF}(\rho_{AB}) = -\eta\log_2\eta - (1-\eta)\log_2(1-\eta)$, with $\eta = \frac{1}{2}(1+\sqrt{1-C^2})$ with $C = 2\max\{0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\}$ being the concurrence [18]. Although for pure states the QD is equal to EoF, there are some mixed correlated states with null entanglement and nonzero quantum discord.

Now, for two atoms initially prepared in the state $|e,e\rangle$, we see that the EoF is always zero (for initial state $|e,g\rangle$ or $|g,g\rangle$ the EoF is null for long interaction times and almost zero for very short interaction times), as we see in Fig. 1(b). Therefore, the EoF is not useful to distinguish the quantum and classical character of a cavity-field in the steady state of the system. However, the QD has very small values within the time window (5), so that the correlations generated by the cavity field in the atoms are negligible, which confirms the classical character of the field within this time window. Moreover, the QD grows continuously until it reaches a stationary value. This appreciable value ($= 1/3$) for the QD for an initial state $|e,e\rangle$ (or $|g,g\rangle$) shows that the quantum correlation between atoms, generated by interaction of the atoms with the cavity mode, is significant for long interaction times and reveals the quantum nature of this cavity field. Thus, to determine the classical or quantum behavior of the field, we calculate the correlations between atoms in the steady state.

Entanglement and Quantum Discord in the stationary regime.—Here we analyze the stationary behavior of the QD and EoF as a function of the ratios g/κ and ε/κ, by numerical solution of Eq. (2), with $n_{\mathrm{th}} = 0$, without any approximation, and assuming the cavity mode initially in the vacuum $|0\rangle$ (the results are the same for any initial coherent or Fock state). In Figs. 2(a) and 2(b), for initial atomic state $|e,g\rangle$ (or $|g,e\rangle$), and 2(c) and 2(d), for initial atomic state $|g,g\rangle$ (or $|e,e\rangle$), we see that EoF always goes to zero for $\varepsilon \gg g$, being different from zero only for $\varepsilon \ll g$, i.e., for a small $\bar{n}$. This result seems to confirm the equivalence principle since, for a very high $\bar{n}$, we expect an agreement between quantum and semiclassical description, which means no quantum correlations between the atoms. Surprisingly, the QD is always nonzero, reaching a significant value in the limit of $\varepsilon \gg g$ ($\mathrm{QD}_{\mathrm{ss}} \simeq 1/8$ for the

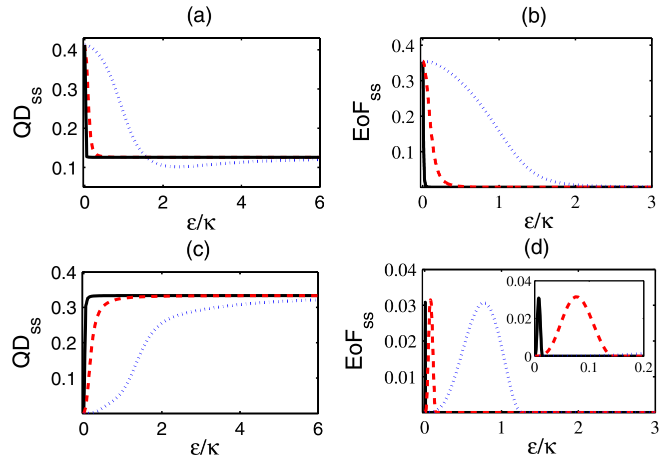

FIG. 2 (color online). Stationary quantum correlations versus ε/κ for initial atomic states $|e,g\rangle$ [(a) and (b)] and $|g,g\rangle$ [(c) and (d)] and cavity mode in the vacuum $|0\rangle$. The atom-field coupling was fixed as $g = 0.01\kappa$ (full line), 0.1κ (dashed line), and 1.0κ (dotted line).

initial state $|e,g\rangle$ and $\mathrm{QD}_{\mathrm{ss}} = 1/3$ for initial state $|g,g\rangle$). This means that the cavity mode is able to generate correlations between the atoms for any finite $\bar{n}$, even for extremely intense fields. Thus, the cavity mode, which is quantum by construction, has its quantum character revealed through the QD between the atoms.

The origin of the quantum character is the indistinguishability of paths in the exchange of a photon between atoms and the cavity mode. When a photon is absorbed from the field by an atom one generates a superposition of possibilities, either the photon is absorbed by the first atom or is absorbed by the second one, with these two possibilities happening simultaneously. One could argue that the origin of this quantum character could be in the coherence of the driving field, which generates a coherent field inside the cavity, as in [19]. However, being this the case, an incoherent pumping of photons into the cavity would not generate correlations between the atoms in the stationary regime. To analyze this point more carefully, we have neglected the driving field (i.e., $\varepsilon = 0$) and assumed that the cavity mode is at a finite temperature T, which implies in an incoherent injection of photons into the cavity. Assuming that the cavity mode has a strong decay rate, i.e., $\kappa \gg g\sqrt{N}$, it quickly reaches its steady state (ρ_c^{ss}), so that, for $t \gg 1/\kappa$, we can approximate $\rho \simeq \rho_c^{\mathrm{ss}} \otimes \rho_a$, ρ_a being the atomic density matrix. With this assumption and adiabatically eliminating the field variables [15], we obtain the effective master equation for the atomic system

$$\dot{\rho}_a = \Gamma_{\mathrm{eff}}(n_{\mathrm{th}} + 1)\mathcal{L}[S_-]\rho_a + \Gamma_{\mathrm{eff}}n_{\mathrm{th}}\mathcal{L}[S_+]\rho_a,$$

which, for $n_{\mathrm{th}} \gg 1$, simplifies to $\dot{\rho}_a \approx \Gamma_{\mathrm{eff}}n_{\mathrm{th}}(\mathcal{L}[S_-]\rho_a + \mathcal{L}[S_+]\rho_a)$. For atoms prepared initially in the state $|g,g\rangle$, the atomic steady state will be $\rho_a^{\mathrm{ss}} = \frac{1}{3}(|\Phi^+\rangle\langle\Phi^+| + |\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+|)$, giving us EoF $= 0$ and

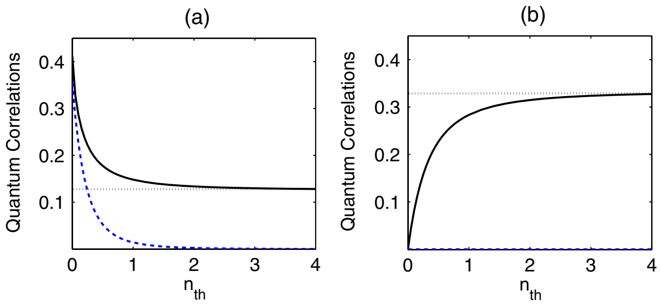

FIG. 3 (color online). Stationary quantum correlations versus n_{th}: QD (full line) and EoF (dashed line). The dotted lines represent the analytical predictions for QD. We have assumed atom-field coupling $g = 0.01\kappa$, initial atomic state (a) $|e, g\rangle$ and (b) $|g, g\rangle$. In (b), the EoF is zero for all n_{th}.

QD $= 1/3$. For the initial state $|e, g\rangle$, the atomic steady state will be $\rho_a^{ss} = \frac{1}{6}\mathbf{1} + \frac{1}{3}|\Psi^-\rangle\langle\Psi^-|$, which give us EoF $= 0$ and QD $\simeq 1/8$. Here, $|\Psi^\pm\rangle = 1/\sqrt{2}(|e, g\rangle \pm |g, e\rangle)$ and $|\Phi^\pm\rangle = 1/\sqrt{2}(|g, g\rangle \pm |e, e\rangle)$ are the Bell states.

For one atom interacting with a cavity mode and in the limit of large photon numbers, the classical limit derived in [8] requires $n_{th} > (g/\kappa)^2$. In this limit, the statistical properties of the field are those of a thermal one. In Fig. 3 we have assumed $g = 0.01\kappa$, so that $n_{th} = 1$ is already much bigger than $(g/\kappa)^2$, and then numerically solved the Eq. (2). We see that the EoF goes to zero in the steady state as the temperature rises. However, this does not happen to the QD, showing us that the cavity mode is still able to generate quantum correlations between the atoms even when it is interacting with a thermal reservoir, as we see in Fig. 3. Taking into account the initial atomic state $|e, g\rangle$ we note that both the QD and the EoF decay as we increase n_{th}. The EoF goes to zero while the QD reaches the asymptotic value $\simeq 1/8$, as in the case of a coherent injection of photons. For the initial atomic state $|g, g\rangle$ we see that the EoF is always zero and for low temperatures the QD is negligible. But, increasing n_{th}, the QD increases, reaching the predicted asymptotic value $\mathrm{QD}_{ss} = 1/3$, as when we have a coherent driving field. In this case, exactly the same pattern of quantum correlations is obtained for any value of atom-field coupling. This happens due the thermalization of the system; i.e., the cavity mode thermalizes with the reservoir and the atoms effectively thermalize with the cavity mode. The atom-field coupling merely determines the interaction time required for the thermalization of the atoms with the cavity field. Therefore, the stronger this coupling, the shorter the interaction time required for the system to reach the stationary state. Thus we see that the cavity field is always able to generate correlations, irrespective of the temperature of the reservoir, revealing the quantum character of the field at any temperature.

In conclusion, we have shown that a nonzero QD between atoms can be taken as a signature of the nonclassical behavior of the cavity mode. It is important to realize that, for a high $\bar{n}$, entanglement is not present in the atomic system, so that the quantum character of the cavity mode can be revealed by QD. Also, the QD is different from zero for any value of atom-field coupling and even in the limit of a very strong driving field or high temperatures. We can thus affirm that, although the statistical properties of a field show that by increasing $\bar{n}$ in a dissipative cavity mode we reach the classical limit, the nonzero correlations between the atoms shows that the quantum character of this field is still there, even for a macroscopic field. Moreover, there is a time window during which the mode effectively shows classical behavior, which depends on the cavity decay rate, the strength of the atom-field coupling, and the number of atoms interacting with it.

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