Mode-Coupling-Induced Dissipative and Thermal Effects at Long Times after a Quantum Quench

Aditi Mitra[1] and Thierry Giamarchi[2]

[1]Department of Physics, New York University, 4 Washington Place, New York, New York 10003, USA
[2]DPMC-MaNEP, University of Geneva, 24 Quai Ernest-Ansermet, CH-1211 Geneva, Switzerland
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An interaction quench in a Luttinger liquid can drive it into an athermal steady state. We analyze the effects on such an out of equilibrium state of a mode coupling term due to a periodic potential. Employing a perturbative renormalization group approach we show that even when the periodic potential is an irrelevant perturbation in equilibrium, it has important consequences on the athermal steady state as it generates a temperature as well as a dissipation and hence a finite lifetime for the bosonic modes.

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The high degree of tunability and control associated with cold-atomic gases make them an exciting test bed for studying a host of phenomena related to interacting quantum particles [1]. Among the topics of great current interest are the nonequilibrium physics of quantum quenches for which these systems are particularly well adapted (see [2] and references therein) and other classes of steady state nonequilibrium phenomena such as systems subjected to a time dependent noise [3].

Crucial questions in all these out of equilibrium phenomena is the relaxation mechanism by which the system reaches a steady state and the properties of the steady state, in particular, whether the latter is athermal or thermal and hence described by a Gibbs distribution. In the case of a quench where nontrivial time evolution is triggered after a sudden change in system parameters, many models mainly in one-dimension [4] and involving simple effective theories [5,6], are found to reach an athermal steady state characterized by a generalized Gibbs ensemble (GGE). However the generality of the GGE remains under debate since certain observables do not obey it [5,7–10]. More generally, the relaxation mechanism and the nature of the steady state in more complicated field theories, as well as the role of a finite system size in numerical studies, are still largely unknown [11,12].

It is thus important to have theoretical models for which such nonequilibrium questions can be reliably studied. One good candidate for such an analysis is a one dimensional system of interacting bosons, leading to the so called Luttinger liquid physics [13]. The excitations of such a system can be represented by density modes which are essentially independent. On such a system, quenches corresponding to a change of the interaction reveal a steady state which still has independent modes, but these are now characterized by a nonequilibrium distribution that does not relax to a thermal state [5].

In this paper we examine the effect of a mode coupling term on the above system, where the mode coupling is due to an externally imposed optical lattice. We address explicitly the question of thermalization and asymptotic relaxation of such a system. We use the Keldysh technique [14] and a controlled renormalization analysis and show that even in cases for which the lattice potential would be irrelevant in equilibrium, it leads for the out of equilibrium situation to the appearance of finite dissipation, as well as a finite temperature for the low energy modes. We thus explicitly obtain a mechanism, which we argue should be generic, in which thermalization and dissipation arises due to the transfer of energy from the long wavelength modes to the short wavelength modes, the latter thus acting as a bath.

We consider interacting one dimensional bosons in the continuum. The low energy properties of such a system can be efficiently represented by a Luttinger liquid [13]

$$H_i = \frac{u_0}{2\pi}\int dx\left[K_0[\pi\Pi(x)]^2 + \frac{1}{K_0}[\partial_x\phi(x)]^2\right] \quad (1)$$

where ϕ is related to the long wavelength part of the density by $\rho(x) = -\nabla\phi(x)/\pi$, while Π is the canonically conjugate variable to ϕ. The eigenmodes of the Hamiltonian are the sound waves of density with a dispersion $\omega = u_0 q$. The information about the interactions is contained in the two Luttinger parameters u_0 the velocity of density oscillations, and K_0 a dimensionless parameter controlling the decay of correlation functions.

The bosons are driven out of equilibrium via a sudden interaction quench which for the effective Luttinger liquid theory, simply implies a sudden change of the Luttinger parameter from $K_0 \to K$, and the velocity from $u_0 \to u$. To satisfy Galilean invariance we choose $u = v_F/K$ and $u_0 = v_F/K_0$. The time evolution of the initial state is therefore due to $H_f = H_i(K_0 \to K, u_0 \to u)$.

We first give here the full solution for a quench $K_0 \to K$ with arbitrary interactions. In the Keldysh formalism [14] it is convenient to define classical $[\phi_{\rm cl} = (\phi_- + \phi_+)/\sqrt{2}]$ and quantum $[\phi_q = (\phi_- - \phi_+)/\sqrt{2}]$ fields where $\phi_{-(+)}$ are the time (antitime) ordered fields on the Keldysh contour. In terms of these fields, the action that describes the steady state behavior at long times ($t + t' \to \infty$) after the quench when transients related to oscillations of $e^{-iu|q|(t+t')}$ have averaged out to zero, is

$$S_0 = \frac{1}{\pi K u} \sum_{q\neq 0,\omega} (\phi^*_{\rm cl} \quad \phi^*_q) \times \begin{pmatrix} 0 & (\omega - i\delta)^2 - u^2 q^2 \\ (\omega + i\delta)^2 - u^2q^2 & 4i|\omega|\delta \frac{K_0}{2K}\left(1 + \frac{K^2}{K_0^2}\right) \end{pmatrix} \begin{pmatrix} \phi_{\rm cl} \\ \phi_q \end{pmatrix}. \tag{2}$$

Equation (2) implies that the retarded propagator $-i\langle \phi_{\rm cl}\phi^*_q\rangle = G^R(q, \omega) = \frac{\pi K u}{(\omega + i\delta)^2 - u^2 q^2}$ is identical to that in the ground state of H_f, while the Keldysh propagator $G^K = -i\langle\phi_{\rm cl}\phi^*_{\rm cl}\rangle$ which is sensitive to the occupation of the bosonic modes is,

$$G^K(q, \omega) = \frac{K_0}{2K}\left(1 + \frac{K^2}{K_0^2}\right)\mathrm{sgn}(\omega)[G_R - G_A]. \tag{3}$$

Thus the fluctuation-dissipation theorem (FDT) defined by $G^K = (G^R - G^A)\coth(\omega/2T)$, where T is the temperature of the bosons, is violated. When $K = K_0$, FDT is recovered as $\coth(\omega/2T) \xrightarrow{T=0} \mathrm{sign}(\omega)$. Note that although the system is now out of equilibrium, each q mode is still infinitely long lived since $\delta = 0^+$.

S_0 can be used to evaluate the basic two-point correlation functions corresponding to the density fluctuation $C^K_{\phi\phi}$ and response $C^R_{\phi\phi}$, defined as $C^K_{\phi\phi}(r, t) = -i\,\mathrm{Re}[e^{-(\gamma^2/2)\langle[\phi_-(r,t)-\phi_+(0,0)]^2\rangle}]$ and $C^R_{\phi\phi}(r, t) = i\theta(t)\times \mathrm{Im}[e^{-(\gamma^2/2)\langle[\phi_-(r,t)-\phi_+(0,0)]^2\rangle}]$. Defining $K_{\rm eq} = \frac{\gamma^2}{4}K$, $K_{\rm neq} = \frac{\gamma^2}{8}K_0(K^2/K_0^2 + 1)$, we find,

$$e^{-(\gamma^2/2)\langle[\phi_-(r,t)-\phi_+(0,0)]^2\rangle} = e^{-(K_{\rm neq}/2)\ln([\alpha^2+(ut+r)^2]/\alpha^2)-(K_{\rm neq}/2)\ln([\alpha^2+(ut-r)^2]/\alpha^2)} \times e^{i[K_{\rm eq}\tan^{-1}[(ut+r)/\alpha]+K_{\rm eq}\tan^{-1}[(ut-r)/\alpha]]}, \tag{4}$$

where α is a short distance cutoff, and γ is an arbitrary coefficient which will later be related to the periodicity of an externally imposed lattice potential. In equilibrium $K_{\rm eq} = K_{\rm neq}$, and one recovers the usual power-law decay of Luttinger liquids. However out of equilibrium one finds power-law behavior with new decay exponents $K_{\rm neq}$. For the case of $K_0 = 1$ this power-law decay was obtained in [5], and can also be recovered using a GGE that accounts for the conservation of the occupation number of appropriate bosonic modes. Since $K_{\rm neq} > K_{\rm eq}$, the propagators always decay faster than in equilibrium.

The role played by the oscillating terms in Eq. (4) which differentiates between response and correlation functions has not been explored before, and will play an important role in the RG. Its importance can already be seen at this level by studying the FDT ratio defined by $\frac{C^K_{\phi\phi}(q,\omega)}{2\,\mathrm{Im}[C^R_{\phi\phi}(q,\omega)]}$. While for $K_{\rm eq} = K_{\rm neq}$ this ratio reduces to the equilibrium $T = 0$ result of $\mathrm{sign}(\omega)$, out of equilibrium it can be used to formally define a ω, $|q|$ dependent effective "temperature." In the limit $\omega \to 0$, $q \to 0$, the effective temperature $\bar{T}$ defined as

$$\frac{C^K_{\phi\phi}(q = 0, \omega = 0)}{2\,\mathrm{Im}[C^R_{\phi\phi}(q = 0, \omega \to 0)]} = \frac{2\bar{T}}{\omega} \tag{5}$$

and therefore assumed to be $\bar{T} > \omega$ is,

$$\bar{T} = \frac{K_{\rm neq} - 2}{2K_{\rm eq}}, \tag{6}$$

where the energy-scales are expressed in units of u/α, and length scales in units of α. The behavior of $C^K(q = 0.5, \omega)$ and $2\,\mathrm{Im}[C^R_{\phi\phi}(q = 0.5, \omega)]$ are plotted in Fig. 1 for the equilibrium case when $K_{\rm eq} = K_{\rm neq} = 2$ and the nonequilibrium case of $K_{\rm eq} = 2$, $K_{\rm neq} = 3$. Note that this temperature is dependent on the correlation function we use, contrary to the case of equilibrium for which each ratio between response and fluctuation defines the same temperature. Figure 1 shows that besides the appearance of an effective temperature, a striking effect is the appearance of a dissipation characterized by a nonzero slope of $\mathrm{Im}[C^R] \propto -i\eta\omega$. As we shall show below, the temperature and the dissipation already apparent at this stage will reappear in the RG analysis.

We now study how this nonequilibrium state is modified by a coupling between modes. Although in principle any form of nonlinear coupling, such as, for example, a ϕ^4 term can be used, we focus here on the case of a $\cos(\gamma\phi)$ perturbation. There are two reasons for such a choice: (i) if the phase ϕ represents real interacting bosons, the Hamiltonian cannot contain perturbations coupled directly

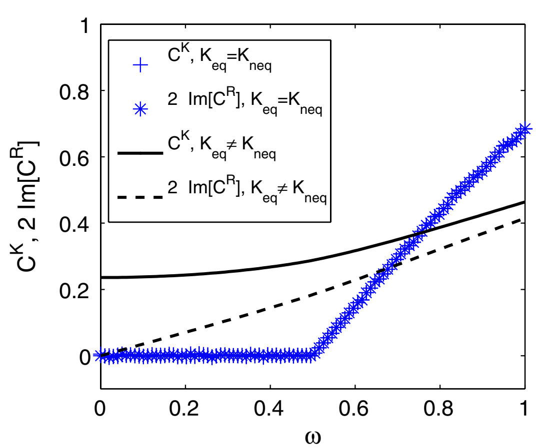

FIG. 1 (color online). $C^K(q = 0.5, \omega)$ and $2\,\mathrm{Im}[C^R(q = 0.5, \omega)]$ in equilibrium $K_{\rm eq} = K_{\rm neq} = 2.0$ and after a quench $K_{\rm eq} = 2$, $K_{\rm neq} = 3$.

to ϕ but only to derivative or periodic functions of ϕ; (ii) such a periodic term arises naturally when a periodic potential is added on the system. It is the source of the Mott transition in one dimension [13], and thus very natural to study in that context. As for the case of equilibrium we study this term by a renormalization group (RG) procedure, since the rest of the Keldysh action S_0 is quadratic. The Keldysh path integral is $Z_K = \int \mathcal{D}[\phi_{\rm cl}, \phi_q] e^{i(S_0+S_{\rm sg})}$ where

$$S_{\rm sg} = \frac{gu}{\alpha^2}\int dx \int dt[\cos(\gamma\phi_-) - \cos(\gamma\phi_+)]. \quad (7)$$

Writing such an action assumes that after the quench has long taken place, one switches on the cosine term infinitely slowly. Without the quench the system would thus relax to the ground state (at $T = 0$) *in the presence* of the cosine term. To which state the system will tend if the initial state is not in equilibrium is the very question we address here. In order to perform an RG analysis, we split the modes between slow and fast components $\phi_{{\rm cl},q}(xt) = \phi^<_{{\rm cl},q}(xt) + \phi^>_{{\rm cl},q}(xt)$ where

$$\begin{aligned} \phi^<_{{\rm cl},q}(xt) &= \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\Lambda'}^{\Lambda'}\frac{dq}{2\pi}e^{iqx-i\omega t}\phi_{q,{\rm cl}}(q,\omega) \\ \phi^>_{{\rm cl},q}(xt) &= \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{\Lambda>|q|>\Lambda'}\frac{dq}{2\pi}e^{iqx-i\omega t}\phi_{q,{\rm cl}}(q,\omega) \end{aligned} \quad (8)$$

and $\Lambda/\Lambda' = e^{d(\ln l)}$, and integrate out the fast modes. Care has to be taken to regularize the two-point functions appropriately, and no cutoff on time should be imposed. We choose here a standard momentum cutoff [13], the details of the computation will be given elsewhere.

The first step of RG generates a correction to $S_{\rm sg}$ as well as corrections to S_0 which may be absorbed into a redefinition of K and u. However when $K \neq K_0$, additional terms of the form

$$\begin{aligned} \delta S = \int dR \int d(ut)\frac{1}{\pi K}\Big[&-2\frac{\eta}{u}\Big(\frac{\Lambda}{\Lambda'}\Big)\phi_q\partial_{ut}\phi_{\rm cl} \\ &+ i\frac{4T_{\rm eff}\eta}{u^2}\frac{K_0}{2K}\Big(1+\frac{K^2}{K_0^2}\Big)\Big(\frac{\Lambda}{\Lambda'}\Big)^2\phi_q^2\Big] \end{aligned} \quad (9)$$

are generated. These corrections can be summarized in the following RG equations,

$$\frac{dg}{d\ln l} = \Big[2 - \frac{\gamma^2}{8}K_0(1 + K^2/K_0^2)\Big]g, \quad (10)$$

$$\frac{dK^{-1}}{d\ln l} = \frac{\pi g^2}{4\alpha^4}\Big(\frac{\gamma^2}{2}\Big)^2\frac{K_0}{2}\Big(1+\frac{K^2}{K_0^2}\Big)I_K, \quad (11)$$

$$\frac{1}{Ku}\frac{du}{d\ln l} = \frac{\pi g^2}{4\alpha^4}\Big(\frac{\gamma^2}{2}\Big)^2\frac{K_0}{2}\Big(1+\frac{K^2}{K_0^2}\Big)I_u, \quad (12)$$

$$\frac{d\eta}{d\ln l} = \eta + \frac{\pi g^2 uK}{2\alpha^4}\Big(\frac{\gamma^2}{2}\Big)^2\frac{K_0}{2}\Big(1+\frac{K^2}{K_0^2}\Big)I_\eta, \quad (13)$$

$$\frac{d(T_{\rm eff}\eta)}{d\ln l} = 2T_{\rm eff}\eta + \frac{\pi g^2u^2K^2}{4\alpha^4}\Big(\frac{\gamma^2}{2}\Big)^2 I_T, \quad (14)$$

where $I_T = \int_{-\infty}^{\infty}dr\int_{-\infty}^{\infty}dt\,{\rm Re}[e^{-(\gamma^2/2)\langle[\phi_-(t,r)-\phi_+(0,0)]^2\rangle}F]$, $I_\eta = \int_{-\infty}^{\infty}dr\int_{-\infty}^{\infty}dt t\,{\rm Im}[e^{-(\gamma^2/2)\langle[\phi_-(t,r)-\phi_+(0,0)]^2\rangle}F]$, $I_{K/u} = \int_{-\infty}^{\infty}dr\int_0^{\infty}dt(r^2 \mp t^2){\rm Im}[e^{-(\gamma^2/2)\langle[\phi_-(t,r)-\phi_+(0,0)]^2\rangle}F]$ and $F = \frac{1}{2}(e^{i\Lambda(t+r)} + e^{i\Lambda(t-r)}) + \frac{i}{2}(\frac{K_{\rm eq}}{K_{\rm neq}} - 1)\{\sin[\Lambda(t+r)] + \sin[\Lambda(t-r)]\}$.

Equation (10) reflects the new scaling dimension of the operator $e^{i\gamma\phi}$ due to the change in the decay exponent from $K_{\rm eq}$ to $K_{\rm neq}$. It still defines two regimes, one in which the cosine is irrelevant, and one for which the perturbative RG would lead to strong coupling for the cosine term. In equilibrium this would reflect the Berezinski-Kosterlitz-Thouless transition corresponding to the Mott transition ($K = 2$ and infinitesimal g). Equation (11) is the usual scaling of K, which is reduced by the presence of the cosine term. This equation is also slightly modified compared to the equilibrium one when $K_{\rm eq} \neq K_{\rm neq}$. Equation (12) is a renormalization of the velocity. It appears here because we took a pure momentum cutoff which thus does not respect the Lorentz invariance. It would appear also in equilibrium with the same cutoff structure. These three equations would thus lead to two separate phases, one in which the cosine is irrelevant, and a strong coupling regime whose physics would be beyond the reach of the perturbative RG. In order to stay in the regime for which the RG is reliable even asymptotically we concentrate here on the case $K_{\rm eq}$, $K_{\rm neq} > 2$ for which the cosine term is irrelevant according to Eq. (10). Other regimes of the phase diagram will be discussed elsewhere. In this regime one could naively expect to recover the same physics as without the cosine [namely the athermal state corresponding to Eq. (2)]. However, the two remaining Eqs. (13) and (14) introduce qualitatively new physics and lead to quite a different state.

Equation (13) shows that contrary to the case of an equilibrium quantum system, for which the friction coefficient remains always infinitesimal ($\eta = 0^+$), even at finite temperature, here because of the combination of the cosine term and the initial out of equilibrium action, a *finite* friction is generated. If one starts from the equilibrium situation $K = K_0$ then of course $I_{T,\eta} = 0$ and one recovers the conventional results. The finite friction causes a crossover of the mode dispersion at low energies from a pure quantum behavior, dominated by $(\partial_t\phi)^2 \to \omega^2\phi^2$, to a more classical one $\eta\partial_t\phi \to i\eta\omega\phi$, and the correlation functions will reflect this. Interestingly, the physics of dissipation can also be recovered in a quench involving fermions [15]. Similar to the case studied here, an initial quench on a system of noninteracting fermions can cause it to reach a nonequilibrium steady state characterized by a highly broadened distribution function. As a result, switching on infinitesimal interactions (which can be treated within the random phase approximation) can cause efficient scattering and an enhanced particle-hole continuum which leads to a damping of collective modes, at least for attractive interactions between fermions.

In addition to the generation of the friction, Eq. (14) shows that a constant term (in the limit $\omega \to 0$) is added to the Keldysh part of the action. In this limit and in equilibrium, this term would simply be $\propto \omega\eta \coth\frac{\omega}{2T} \xrightarrow{\omega=0} 2T\eta$. Thus the constant term together with the appearance of a *finite* friction can be interpreted as a finite temperature, at least for small enough frequencies. One thus recovers at small frequencies the action (so called Martin-Siggia-Rose action) of a classical system with a finite friction and a thermal noise. Note that because we have shown that the full action renormalizes to a *quadratic* one, and that the Keldysh term tends to a constant, the temperature as defined above is indeed the one that will appear in *all* ϕ correlations, at least asymptotically for low frequencies, contrary to the case of (2). Therefore the nonlinear coupling of the modes leads to a thermalization of the system. The full frequency dependence of such a noise is however quite complicated leading to an interesting crossover, depending on the frequency scale, between the athermal distribution and the classical, finite temperature one at low frequencies. In particular the RG flow itself has been derived from the quantum athermal correlations. The corrections generated by the RG will thus change significantly at a scale for which $\omega^2 = \eta(\omega)\omega$. Since at this scale the system enters in a more classical regime with exponentially decaying correlation functions (see below), this regime will not change the fact that the cosine is irrelevant, and will simply slightly modify the final values of the friction and temperature.

Figure 2 shows the solution for the renormalized η for two different g and $K_0 = 3$. The nonmonotonic behavior arises because the larger is K the more rapidly g renormalizes to zero leading to a smaller renormalized η. While at the other end, when $K = K_0$, $\eta = 0$. These two behaviors have to go through a maxima. Quite naturally the friction is proportional to g^2. This is however not the case for the temperature $T_{\rm eff}$ which for small g is found to reach the following value independent of g (where we have set $F = 1$ in the RG equations)

$$T^*_{\rm eff} = \frac{K^*_{\rm neq} - 2}{2K^*_{\rm neq}} \quad (15)$$

While as $K_{\rm neq} \to K_{\rm eq}$, η, $\eta T_{\rm eff} \to 0$, $T_{\rm eff}$ is nonzero. This is because the order of limits $\omega \to 0$, $K_{\rm eq} \to K_{\rm neq}$ do not commute. Further $T^*_{\rm eff} K^*_{\rm neq}/K^*_{\rm eq} \simeq \bar{T}$, and hence is consistent with the noninteracting estimate for the temperature (6).

Let us finally compute the correlations at the thermal fixed point where the action is (dropping ω^2 terms in comparison to $\omega\eta$)

$$S^* = \sum_{q,\omega} (\phi^*_{\rm cl} \quad \phi^*_q) \frac{1}{\pi K^* u} \times \begin{pmatrix} 0 & -i\eta^*\omega - u^2q^2 \\ i\eta^*\omega - u^2q^2 & 4iT^*_{\rm eff}\eta^*\frac{K_0}{2K^*}\left(1 + \frac{K^{*2}}{K_0^2}\right) \end{pmatrix} \begin{pmatrix} \phi_{\rm cl} \\ \phi_q \end{pmatrix} \quad (16)$$

The above implies that equal-time two-point correlation functions decay exponentially in position,

$$\langle e^{i\phi_{\rm cl}(x)} e^{-i\phi_{\rm cl}(y)} \rangle \simeq e^{-(K^*_{\rm neq}/K^*_{\rm eq})T^*_{\rm eff}(\pi K^*/u)|x-y|} \quad (17)$$

while the dissipation affects unequal-time correlation functions $G^R(q,t) = -\theta(t)(\pi K^* u/\eta^*)e^{-u^2q^2t/\eta^*}$, $G^K(q,t) = -(\frac{2\pi i K^*}{uq^2})(\frac{T^*_{\rm eff}K^*_{\rm neq}}{K^*_{\rm eq}})e^{-u^2q^2|t|/\eta^*}$. Thus in an experiment involving a one dimensional Bose gas in a periodic potential [1], a probe of the density-density response function which directly correspond to the correlators $\langle e^{i\gamma\phi}e^{-i\gamma\phi}\rangle$, should reveal the dissipative and thermal effects we predict.

In summary, by studying the particular example of a quenched Luttinger liquid in the presence of a lattice, we have found a mechanism, that we believe is generic by which a nonequilibrium system in the presence of mode coupling will both thermalize and acquire a finite friction or lifetime for the modes. It is important to note that these effects are related to the presence of a continuum of excitations in the system by which local degrees of freedom can relax and exchange energy. By this argument it is possible that thermalization might not occur in the Mott insulator phase, and the fact that $T_{\rm eff}$ vanishes near the critical point, might be a prelude to this physics. An investigation of this issue and also how the results depend upon the rapidity with which the cosine potential is switched on, are important open questions left for future studies.

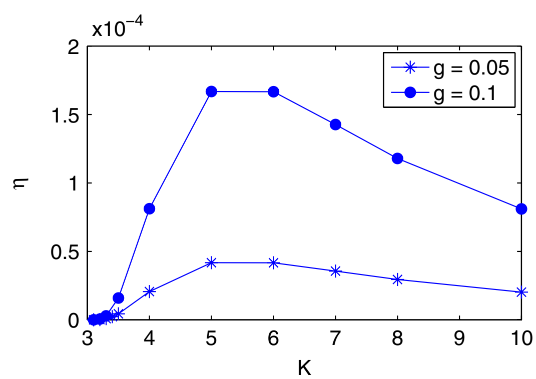

FIG. 2 (color online). Strength of the dissipation η for $K_0 = 3$, $\gamma = 2$ and $g = 0.05$ and $g = 0.1$.

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