Equilibrium Circular Photogalvanic Effect in a Hybrid Superconductor-Semiconductor System

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A dc electric current can be induced in a hybrid semiconductor-superconductor system under illumination of it by a circularly polarized light with the frequency below the energy of semiconductor interband transitions. In conditions when the light beam is unable to create real electron-hole excitations, this phenomenon is reminiscent of the Meissner effect in the static magnetic field. Such an effect can be employed in systems combining cavity photons and superconducting quantum circuits.

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In noncentrosymmetric semiconductors a circularly polarized light can generate the dc electric current [1]. This so-called circular photogalvanic effect (CPE) is determined by the second-order response to the electric field of the incident light. It involves spin polarization of electrons excited from valence bands split by the spin-orbit interaction (SOI) into bands with the total angular moments $3/2$ and $1/2$. The polarized carriers, in their turn, give rise to the electric current, due to SOI associated with the lack of inversion symmetry. This phenomenon attracted much attention recently [2] in connection with perspective spintronic applications.

As expected, in the case of a dissipative transport in normal electron systems at stationary conditions, CPE takes place only if light illumination creates a thermodynamically nonequilibrium distribution of carriers in a semiconductor [3]. For this reason, one cannot expect this effect to occur in bulk semiconductors, if the frequency of the incident light is less than the energy gap between the valence and conduction bands and when indirect phonon or impurity assisted optical transitions are weak. On the other hand, in superconductors the flow of electrons might be created without exciting the system from its thermal equilibrium. Such a phenomenon becomes possible due to a macroscopic coherency of the condensate wave function, so that spacial variations of its phase determine the supercurrent. The well-known example is the Meissner effect, where the static magnetic field cannot produce electron-hole excitations. It, however, induces a superconducting electric current. The goal of this Letter is to show that CPE can be observed in a hybrid superconductor-semiconductor system even if the incident light does not drive it from the thermal equilibrium. The electric current is induced due to changes in the spectrum and wave function of the many-electron system, rather than from deviation of the electron's distribution function from its thermal equilibrium. Apart from the fundamental interest, this phenomenon has a practical value when it is employed in nanoscale optoelectronic devices, because it allows us to reduce dissipation losses compared to CPE in normal systems. It can also be an efficient tool to combine microwave electrodynamics in superconducting quantum circuits [4] with cavity optical fields coupled to atomic ensembles [5], like the recently observed [6] interaction of spin ensembles to superconductor resonators. This suggests evident connections to many topics of current interest, ranging from atomic and polaritonic Bose condensates to spintronics and quantum information processing.

The equilibrium CPE takes place in a hybrid semiconductor-superconductor system due to the proximity effect which induces Cooper pair correlations in the semiconductor. Various hybrid systems displaying the strong proximity effect have been recently fabricated [7]. In order to demonstrate the main features of CPE and make our quantitative analysis more transparent, a simple model will be considered here, where a noncentrosymmetric semiconductor film is in a planar contact with a superconductor having the singlet order parameter. The film contains an n-doped quantum well (QW) close to the metal surface (Fig. 1). Note, that such a sandwich system is now being considered as a key element of a topological superconductor [8].

Let us assume that the circularly polarized light with the frequency ω_i is incident onto the semiconductor surface.

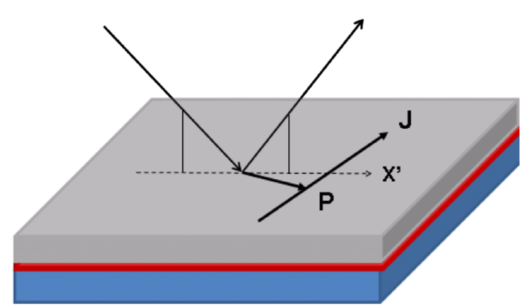

FIG. 1 (color online). A sketch of the system. An optically transparent semiconductor film (top) is in a contact with a superconductor (bottom). A thin layer between them depicts a doped quantum well. An incident electromagnetic wave induces in this well an electric current $\mathbf{J}$ perpendicular to the vector $\mathbf{P} \sim \mathbf{E} \times \mathbf{E}^*$, where $\mathbf{E}$ is the electromagnetic field. The light beam can be also incident from the metal side, if the metal film is thinner than the skin layer.

The vector potential of the total electromagnetic field in the QW region is represented by its Fourier components $\mathbf{A}$ and $\mathbf{A}^*$, corresponding to frequencies $\omega = \omega_i$ and $\omega = -\omega_i$, respectively. It will be assumed that the electromagnetic field causes only direct transitions between valence and conduction bands. Hence, phonon and impurity assisted indirect transitions, as well as a very small momentum transfer by photons, will be ignored. The corresponding matrix elements are given by $M_{\alpha v}(\omega_i) = (e/mc)\langle \mathbf{k}, \alpha|\mathbf{p}\cdot\mathbf{A}|\mathbf{k}, v\rangle$ and $M_{\alpha v}(-\omega_i) = (e/mc)\langle \mathbf{k}, \alpha|\mathbf{p}\cdot\mathbf{A}^*|\mathbf{k}, v\rangle$, where $\mathbf{k}$ denotes the two-dimensional wave vector of QW electrons and $\mathbf{p}$ is their momentum operator, α is the conduction electron spin variable, and v labels the valence bands. In 3D superconductors the latter are the heavy-hole and light-hole bands with the total angular moment $l = 3/2$ and its projections $\pm 3/2$ and $\pm 1/2$, respectively, plus the split-off band with $l = 1/2$, $l_z = \pm 1/2$. In QW the label v runs also through valence subband indexes, while only the lowest conduction subband is assumed to be occupied. In noncentrosymmetric semiconductors SOI splits the energies of bands with opposite angular moments. For CPE this splitting is crucial. In the case of an n-doped QW the conduction band splitting appears to be most important, although at strong resonance conditions the hole splitting can be equally important. Therefore, the latter will be ignored under the assumption that it is much less than the resonance detuning. At the same time, the conduction band splitting is determined by the SOI Hamiltonian $H_{\rm so} = \boldsymbol{\sigma}\cdot\mathbf{h}_\mathbf{k}$, where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of Pauli matrices and $\mathbf{h}_\mathbf{k} = -\mathbf{h}_{-\mathbf{k}}$ is the spin-orbit field. The latter will be assumed to have only x and y components. This takes place for the Rashba field, as well as for the Dresselhaus field in [001] oriented QW [9].

The semiconductor film contacts 3D metal through a high enough tunneling barrier, so that the broadening of QW states due to resonance with 3D continuum is much smaller than the metal superconducting gap. Therefore, in the leading approximation, the electric current in QW can be calculated by ignoring a leaking of electrons into the metal. The system is assumed to be clean enough, with the mean electron scattering rate in QW to be much smaller than the proximity induced energy gap and $h_\mathbf{k}$. Hence, the Green's functions are diagonal with respect to electron wave vectors. Considering a thermodynamic equilibrium with the temperature T, the stationary electric current density can be expressed in terms of the equilibrium Keldysh function as

$$\mathbf{J} = -\frac{ie}{4}\sum_{\mathbf{k},n}\int\frac{d\omega}{2\pi}\,\mathrm{Tr}[\mathbf{j}_{n\mathbf{k}}(G^r_{n\mathbf{k}}(\omega) - G^a_{n\mathbf{k}}(\omega))] \times \tanh\frac{\omega}{2k_BT}, \qquad (1)$$

where $G^{r(a)}_{n\mathbf{k}}(\omega)$ are the retarded and advanced Green's functions. The index n labels the energy bands including conduction $n = c$ and valence $n = v$ bands. Although the valence bands are far from the Fermi level, their inclusion into the current expression is necessary, because they contribute to important spectral corrections associated with the coupling of electrons to the electromagnetic field. The current operator of conduction electrons is given by

$$\mathbf{j}_\mathbf{k} = \frac{\partial\epsilon_{c\mathbf{k}}}{\partial\mathbf{k}} + \tau_3\frac{\partial(\boldsymbol{\sigma}\cdot\mathbf{h}_\mathbf{k})}{\partial\mathbf{k}}, \qquad (2)$$

while the valence-band currents are given by corresponding band velocities $\partial\epsilon_{v,\mathbf{k}}/\partial\mathbf{k}$, where $\epsilon_{n,\mathbf{k}}$ are the band energies measured with respect to the chemical potential. The above expressions are written in the Gor'kov-Nambu representation, where the new electron destruction operators $c_{\mathbf{k},n,\sigma,1} = c_{\mathbf{k},n,\sigma}$ and $c_{\mathbf{k},n,\sigma,2} = c^\dagger_{-\mathbf{k},\bar{n},\bar{\sigma}}$ are introduced, with Pauli matrices τ_1, τ_2, τ_3 acting in the space of indices 1 and 2. As mentioned in Ref. [10], this basis is more convenient for studying the spin-dependent transport in superconducting systems.

The next step is to specify important contributions to the electron self-energy due to the superconducting proximity effect and interaction with the electromagnetic field. The proximity effect is represented by a nondiagonal in the Nambu space contribution to the self-energy of QW electrons. In the case when parallel to the interface wave vectors of tunneling particles are conserved, this self-energy has the form

$$\Sigma_t^{r(a)}(\omega, \mathbf{k}) = \sum_{k_z}|T_{\mathbf{k},k_z}|^2G^{r(a)}_{\mathbf{k},k_z}(\omega), \qquad (3)$$

where $T_{\mathbf{k},k_z}$ is the tunneling matrix element, with k_z denoting the normal component of the wave vector in the metal side of the interface. Assuming the real order parameter Δ_0, the Green's function in (3) can be written as $G^{r(a)}_{\mathbf{k},k_z} = (\omega - \tau_3\epsilon_k - \sigma_z\tau_1\Delta_0 \pm i\delta)^{-1}$. For a wideband metal, where $T_{\mathbf{k},k_z}$ slowly varies near the Fermi level, the self-energy is expressed as

$$\Sigma_t^{r(a)}(\omega, \mathbf{k}) = -\Gamma_\mathbf{k}(0)\frac{\omega + \tau_1\sigma_z\Delta_0}{\sqrt{\Delta_0^2 - (\omega \pm i\delta)^2}}, \qquad (4)$$

where $\Gamma_\mathbf{k}(\epsilon) = \sum_{k_z}|T_{\mathbf{k},k_z}|^2\delta(\epsilon - \epsilon_k)$ determines a finite confinement lifetime of QW electrons due to resonance with the continuum of metal states. The proximity effect is represented by the second term in the numerator of Eq. (4). Since we assumed that $\Gamma_\mathbf{k} \ll \Delta_0$ and the temperature is low, one can neglect ω in comparison with Δ_0. In this case the gap Δ in the spectrum of QW electrons is given by $\Delta = \Gamma_\mathbf{k}(0)$ and it is reasonable to neglect a very weak dependence on $\mathbf{k}$, because the Fermi wave vector is much smaller in QW than in the metal.

The stationary effect of the electromagnetic field onto the electron current is determined by the second-order contribution to the conduction electron self-energy. Its 11 component in the Nambu space is

$$\Sigma_{11,\alpha\beta}(\omega,\mathbf{k}) = \sum_v [M_{\alpha v}(\omega_i)M^*_{\beta v}(\omega_i)G^0_{v\mathbf{k}}(\omega+\omega_i) + M_{\alpha v}(-\omega_i)M^*_{\beta v}(-\omega_i)G^0_{v\mathbf{k}}(\omega-\omega_i)], \tag{5}$$

where the unperturbed valence-band Green's functions are $G^0_{v\mathbf{k}}(\omega) = (\omega - \epsilon_{v\mathbf{k}})^{-1}$. From the above definition of Gor'kov-Nambu operators, Σ_{22} can be expressed as $\Sigma_{22,\alpha\beta}(\omega,\mathbf{k}) = -\Sigma_{11,\bar\beta\bar\alpha}(-\omega,-\mathbf{k})$. The valence-band wave functions that determine the matrix elements in Eq. (5) are linear combinations of the functions $|J\rangle|\sigma\rangle$, where $J = X$, Y, or Z denote $l = 1$ orbitals and σ is the spin index. Therefore, depending on the spin variables α and β in Eq. (5), the electromagnetic field enters as various combinations of $\mathbf{A}\times\mathbf{A}^*$, $\mathbf{A}\cdot\mathbf{A}^*$, and $A_zA_z^*$. The circular photogalvanic effect is associated with $\mathbf{P} = i\mathbf{A}\times\mathbf{A}^*$. In general, its contribution to Eq. (5) has the form [1] $C\mathbf{P}\cdot\boldsymbol{\sigma}$, where the factor C contains resonance denominators. The resonance detuning, however, was assumed to be much larger than Δ and $h_\mathbf{k}$. Since $\omega \sim \max(\Delta, h_\mathbf{k}, k_BT)$, a weak dependence of the self-energy on ω can be ignored and Σ becomes

$$\Sigma(\omega,\mathbf{k}) = \tau_3(\mathbf{P}\cdot\boldsymbol{\sigma})C_\mathbf{k}. \tag{6}$$

The electric current density will be calculated in the leading order with respect to $\Sigma(\omega,\mathbf{k})$. Hence, the corresponding correction to the Green's function of conduction electrons in Eq. (1) can be written as

$$\delta G^{r(a)}_\mathbf{k}(\omega) = G^{0r(a)}_\mathbf{k}(\omega)\Sigma(\omega,\mathbf{k})G^{0r(a)}_\mathbf{k}(\omega). \tag{7}$$

The unperturbed functions, in their turn, are given by

$$G^{0r(a)}_\mathbf{k}(\omega) = (\omega - \tau_3\epsilon_{c\mathbf{k}} - \boldsymbol{\sigma}\cdot\mathbf{h}_\mathbf{k} - \Sigma^{r(a)}_t(\omega,\mathbf{k}) \pm i\delta)^{-1}, \tag{8}$$

where, as was mentioned above, $\Sigma^{r(a)}_t(\omega,\mathbf{k}) \simeq -\tau_1\sigma_z\Delta$.

The Green's functions of valence electrons in Eq. (1) are calculated in a similar way. The electromagnetic field contributes to their self-energy through the same matrix elements as in Eq. (5), with the intermediate states coming from the conduction band. It is easy to show that

$$\sum_v \mathrm{Tr}[\mathbf{j}_{v\mathbf{k}}G_{v\mathbf{k}}(\omega)] = \mathrm{Tr}\left[G^0_\mathbf{k}(\omega)\frac{\partial\Sigma(\omega,\mathbf{k})}{\partial\mathbf{k}}\right]. \tag{9}$$

Let us choose the x axis parallel to the xy projection of $\mathbf{P}$. For simplicity, the conduction and valence bands are assumed to be isotropic and SOI is taken in the form of the Rashba interaction, where $h_x = \gamma k_y$, $h_y = -\gamma k_x$. Substituting Eqs. (7) and (9) into Eq. (1), after some algebra one arrives to $J_x = 0$ and

$$J_y = \frac{eN_F\gamma C_{k_F}P_x}{2}\int d\epsilon\left[\frac{\Delta^2}{E(\epsilon^2-h^2)}\tanh\frac{E}{k_BT} - \frac{b}{2k_BT}\left(\cosh^{-2}\frac{E}{k_BT} - \cosh^{-2}\frac{\epsilon}{k_BT}\right)\right], \tag{10}$$

where $E = \sqrt{\epsilon^2+\Delta^2}$, $b = (1 + d\ln C_k/d\ln k)_{k=k_F}$, and $h = \gamma k_F$. As expected, the equilibrium CPE turns to 0 in the normal state [3]. One can immediately see it from Eq. (10) at $\Delta \to 0$. In the opposite limit of the large superconducting gap $\Delta \gg k_BT$ the current reaches its maximum magnitude. In this case $\cosh^{-2}(E/k_BT)$ in the integrand is exponentially small and in the leading approximation J_y is given by

$$J_y = \frac{eN_F\gamma C_{k_F}P_x}{2}\left(\frac{\zeta^2-1}{\zeta}\ln\left|\frac{1-\zeta}{1+\zeta}\right| + 2b\right), \tag{11}$$

where $\zeta = \sqrt{1+(\Delta^2/h^2)}$. We note that the expression in brackets smoothly varies between $2b-2$, at $h \ll \Delta$ and $2b$, at $h \gg \Delta$. Therefore, if the light frequency is not too close to the interband transition energy, one gets $b \sim 1$ and this expression is of the order of unity.

It is important to note that CPE is determined by the dependence of the electron-photon second-order scattering on the electron spins. The corresponding self-energy $\Sigma(\omega,\mathbf{k})$ results in the spin orientation of electrons in the direction parallel to $\mathbf{P}$ and, being combined with SOI, gives rise to the electric current. A basic distinction of the equilibrium CPE in superconductors is that at low temperatures there are no single-particle spins to be polarized. Their role, however, is played by triplet Cooper pairs, which admix to singlets due to SOI. This situation resembles the effect of the Zeeman interaction, which together with SOI also leads to the electric current in superconducting systems [11].

Some comments are needed regarding the vector $\mathbf{P}$ in Eqs. (6) and (10). In the considered geometry (Fig. 1), the electromagnetic field near the QW is a sum of the incident and reflected waves. Therefore, in the presence of highly reflecting metal, this vector is strongly modified with respect to its plane-wave value. Let the incident and reflected light beams in the $x'z$ plane. Near QW the circularly polarized incident wave has the electric field components $E^i_z = -\sin\theta E_0$, $E^i_{x'} = \cos\theta E_0$, and $E^i_{y'} = iE_0$, where θ is the incident angle at the metal-semiconductor interface. In general, the total electric field $\mathbf{E}$ near the interface may be expressed in terms of the surface impedance $Z(\omega_i)$. From these well-known expressions [12], the in-plane components of the vector $\mathbf{P} = i(c^2/\omega_i^2)(\mathbf{E}\times\mathbf{E}^*)$ may be easy calculated:

$$P_{x'} = |E_0|^2\frac{8c^2}{\omega_i^2}\sin\theta\cos^2\theta\,\mathrm{Re}\left(\frac{Z}{Z^*+\cos\theta}\right),$$
$$P_{y'} = -|E_0|^2\frac{8c^2}{\omega_i^2}\frac{\sin\theta\cos^2\theta}{|Z+\cos\theta|^2}\,\mathrm{Im}Z, \tag{12}$$

where the terms $Z\cos\theta$ have been neglected, taking into account that the impedance of a highly reflective and thick enough metal film is small. In the range of not too small frequencies, Z is represented by its Fresnel expression $Z(\omega_i) = \sqrt{-i\omega_i \epsilon_s(\omega_i)/4\pi\sigma(\omega_i)}$, where $\sigma(\omega_i)$ and $\epsilon_s(\omega_i)$ are the metal conductivity and dielectric function of the semiconductor film, respectively. At $\omega_i \gg \Delta_0$ the former is almost the same as that of the normal metal. Both $\epsilon_s(\omega_i)$ and $\sigma(\omega_i)$ depend on the frequency and many other factors. Therefore, it is useful to consider typical situations. In the case of a highly reflective metal and $\epsilon_s(\omega_i) > 0$ we have $\mathrm{Re}(Z) \ll \mathrm{Im}(Z)$. Hence, Eq. (12) gives the vector $\mathbf{P}$ perpendicular to the $x'z$ plane. Since the x axis in Eq. (10) has been chosen parallel to $\mathbf{P}$, this means that the electric current will be directed parallel to the x' axis. At the same time, for relatively low reflective metals the real and imaginary parts of Z will be of the same order of magnitude. Therefore, the vector $\mathbf{P}$ and, hence, the electric current will be directed arbitrarily in the $x'y'$ plane, their directions varying with the light frequency. It should be noted that a setup different from Fig. 1 might be chosen. For example, the light beam can be incident from the superconductor side, penetrating through a semitransparent film whose thickness is less than the skin-layer depth. In this case the vector $\mathbf{P}$ will have components quite different from Eq. (12).

For an order-of-magnitude evaluation of CPE, the factor C may be approximated by its bulk expression [1], that is valid far enough from the resonance:

$$C_k = \frac{2e^2 p_{cv}^2}{3m^2c^2}\omega_1\left(\frac{1}{E_g^2 - \omega_1^2} - \frac{1}{(E_g + \Delta_{\mathrm{so}})^2 - \omega_1^2}\right), \quad (13)$$

where E_g is the semiconductor energy gap and Δ_{so} denotes the split-off energy. Taking $\omega_1 \sim E_g \sim \Delta_{\mathrm{so}} \sim 0.4$ eV (InAs), we get $C \sim (e^2/mc^2)(p_{cv}^2/mE_g) \sim (e^2/mc^2) \times (m/m^*)$, where m^* is the effective electron mass. The Rashba parameter γ can vary depending on QW characteristics. For InAs it can be more than 10^{-11} eV m [9]. We take $\gamma = 10^{-12}$ eV m, fixing thus h around 1 meV at $k_F = 10^6$ cm^{-1}, that is much less than the Fermi energy of QW electrons. At $P_x \sim c^2|E_i|^2/\omega_i^2$ and the moderate electric field strength $E_i = 10^4$ V/cm, Eq. (11) gives $J_y \sim 1$ μA/cm. This value will increase considerably at resonance conditions, by the factor $E_g/\delta_{\mathrm{res}}$, where δ_{res} is a detuning from the resonance. The above theory restricts δ_{res} by a much larger value than the electron and hole spin-orbit splittings. In the safe range of relatively large detunings $\sim$10 meV, the resonance enhancement factor is about 40 for InAs based QW.

A really dramatic enhancement can be reached, if the incident light intensity is periodically modulated with a microwave frequency that is close to an eigenfrequency of a quantum circuit incorporating the considered superconducting system. In principle, such a modulation could be provided by the Rabi splitting of an optical cavity mode strongly coupled to atomic ensembles having spin resolved electron transitions. In this way CPE gives rise to a nonlinear interaction of optical cavity fields + associated atomic ensembles with quantum circuits.

In conclusion, the electric current induced by the circular photogalvanic effect has been calculated in the case of a superconducting system. It has been shown that, unlike CPE in a normal system, this effect becomes possible without driving electrons out of the thermodynamic equilibrium and without mediation of the impurity or phonon scattering, as well as intersubband optical transitions. This phenomenon has been considered for a noncentrosymmetric semiconductor QW, where superconductivity is induced by the proximity effect and the spin-orbit interaction is represented by the Rashba term. In the considered setup the magnitude and direction of the electric current can be varied by changing the incident light frequency and polarization.

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