Growth and Phase Velocity of Self-Modulated Beam-Driven Plasma Waves

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A long, relativistic particle beam propagating in an overdense plasma is subject to the self-modulation instability. This instability is analyzed and the growth rate is calculated, including the phase relation. The phase velocity of the wake is shown to be significantly less than the beam velocity. These results indicate that the energy gain of a plasma accelerator driven by a self-modulated beam will be severely limited by dephasing. In the long-beam, strongly coupled regime, dephasing is reached in a homogeneous plasma in less than four e foldings, independent of beam-plasma parameters.

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Plasma-based accelerators have attracted considerable attention owing to the ultrahigh field gradients sustainable in an electron plasma wave, enabling compact accelerators. The electric field amplitude of the electron plasma wave (space-charge oscillation) is on the order of $E_0 = cm_e\omega_p/e$, or $E_0[\mathrm{V/m}] \simeq 96\sqrt{n_0[\mathrm{cm}^{-3}]}$, where $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, n_0 is the ambient electron number density, m_e and e are the electron mass and charge, respectively, and c is the speed of light in vacuum. This field amplitude can be several orders of magnitude greater than conventional accelerators. Electron plasma waves with relativistic phase velocities may be excited by the nonlinear ponderomotive force of an intense laser [1] or the space-charge force of a charged particle beam, i.e., a plasma wakefield accelerator (PWFA) [2,3]. In 2006, high quality 1 GeV electron beams were produced using 40 TW laser pulses in cm-scale plasmas [4]. In 2007, a 42 GeV electron beam in a meter-long plasma was used to double the energy of a small fraction of electrons on the beam tail by the plasma wave excited by the beam head [5]. These experimental successes have resulted in further interest in the development of plasma-based acceleration as a basis for future linear colliders [6,7].

It has recently been proposed to drive a plasma accelerator with a highly relativistic proton beam, such as those available at CERN (European Organization for Nuclear Research) [8,9]. In general, exciting plasma waves requires a drive beam density profile with frequency components at the plasma frequency, i.e., a beam density longitudinal scale length on the order of the plasma wavelength $\lambda_p = 2\pi/k_p = 2\pi c/\omega_p$, or $\lambda_p[\mu\mathrm{m}] = 3.3\times 10^{10}/\sqrt{n[\mathrm{cm}^{-3}]}$. Compact, high-gradient accelerators require high plasma density, and therefore require short drive beams, e.g., $\lambda_p \sim 100\ \mu$m for $n_0 \sim 10^{17}\ \mathrm{cm}^{-3}$. Generating short proton beams (or proton beams with spatial structure at λ_p) is challenging, and it has been proposed to rely on a beam-plasma instability to modulate the beam at λ_p, driving a large amplitude plasma wave [10]. The self-modulation of the beam occurs through coupling of the transverse wakefield with the beam radius evolution. Periodic regions of focusing and defocusing modulate the beam density at λ_p, driving a larger plasma density modulation that further focuses the beam periodically. For beams long compared to λ_p, where self-modulation occurs, the instability is enabled by the drive beam dynamics, and therefore the wakefield properties will be strongly affected by the drive beam dynamics.

An important quantity characterizing the performance of a plasma accelerator is the phase velocity v_p of the plasma wave. For $v_p < c$, a highly relativistic electron will outrun the plasma wave and phase slip from the accelerating to the decelerating phase region of the plasma wave. This limits the electron energy gain to $\Delta W \sim \gamma_p^2(E_z/E_0)m_e c^2$ after acceleration over a dephasing length $L_d \sim \gamma_p^2\lambda_p$, where E_z is the electric field amplitude of the plasma wave and $\gamma_p = (1 - v_p^2/c^2)^{-1/2}$. For a plasma accelerator driven by a short ($< \lambda_p$) intense laser pulse, v_p can be relatively low ($\gamma_p \sim 10 - 100$) and dephasing can limit the energy gain [11]. For a PWFA driven by a short ($< \lambda_p$) highly relativistic beam, v_p can be sufficiently high so that dephasing is not an issue.

In this Letter we calculate the self-modulation of particle beams in plasma, including the properties of the excited plasma wave. In particular, we show that the phase velocity of the plasma wave excited by self-modulation is greatly reduced from the velocity of the drive beam. The phase velocity is determined by the growth of the instability and the beam-plasma dynamics. A similar effect occurs in self-modulated laser-driven plasma waves [12–14]. Analytic solutions for the growth rate and phase velocity in the long-beam regime are derived and compared to numerical solutions of the envelope equation for the particle beam. Owing to the low phase velocity of the plasma wave, the maximum energy gain in such a self-modulated beam-driven accelerator will be severely limited by dephasing.

The wake generated by a relativistic particle beam driver moving through an initially neutral plasma can be calculated using the cold plasma fluid and Maxwell equations. Here we consider a drive beam consisting of particles with charge $\mp e$ and mass M_b. In the linear wake regime, the normalized electron plasma density perturbation $\delta n/n_0 = (n - n_0)/n_0 < 1$ driven by a beam with density $n_b < n_0$ is

$$(\partial_\zeta^2 + k_p^2)\delta n/n_0 = \mp k_p^2 n_b/n_0, \tag{1}$$

where the $\mp$ corresponds to a negatively or positively charged particle beam. A highly relativistic beam is assumed with Lorentz factor $\gamma = (1 - \beta_b^2)^{-1/2} \gg 1$, and the quasistatic approximation is taken such that the plasma fluid quantities are functions of the comoving variable $\zeta = z - \beta_b t$. The beam-driven longitudinal electric field E_z and transverse fields E_r and B_θ are [15]

$$(\nabla_\perp^2 - k_p^2)E_z/E_0 = -k_p\partial_\zeta\delta_n/n_0, \tag{2}$$

$$(\nabla_\perp^2 - k_p^2)(E_r - B_\theta)/E_0 = -k_p\partial_r\delta_n/n_0. \tag{3}$$

The transverse beam-driven wakefield Eq. (3) is coupled to the envelope equation for the beam [16]

$$\frac{d^2R}{dz^2} - \frac{\epsilon_n^2}{4\gamma^2R^3} = \mp\frac{1}{\gamma R}\frac{m_e}{M_b}\langle k_p r(E_r - B_\theta)/E_0\rangle, \tag{4}$$

where $R = \langle r^2\rangle^{1/2}$ is the rms beam size, $\epsilon_n = \gamma[\langle r^2\rangle \times \langle(dr/dz)^2\rangle - \langle r dr/dz\rangle^2]^{1/2}/2$ is the normalized transverse emittance in cylindrical geometry, and the brackets indicate an average over the transverse beam distribution.

For simplicity, we consider a beam with a flat-top radial profile, $n_b = [n_{b0}r_{b0}^2/r_b^2]f(\zeta)\Theta(r_b - r)$, where n_{b0} is the initial peak density, f is the normalized longitudinal profile, Θ is the Heaviside function, $r_b(\zeta, z)$ is the beam radius, and $r_{b0} = r_b(\zeta, z = 0)$ is the initial beam radius. Using the solution to Eqs. (1) and (3) for a flat-top radial beam profile in Eq. (4) yields the envelope equation for the beam radius $r_b(\zeta, z) = \sqrt{2}R$ at any slice ζ

$$\frac{d^2r_b}{dz^2} - \frac{\epsilon_n^2}{\gamma^2r_b^3} = -\frac{4k_b^2r_{b0}^2I_2(k_pr_b)}{\gamma r_b}\int_\infty^\zeta d\zeta'\sin[k_p(\zeta - \zeta')] \times f(\zeta')K_1(k_pr_b(\zeta'))/r_b(\zeta'), \tag{5}$$

where $k_b^2 = 4\pi n_{b0}e^2/M_bc^2$ is plasma wave number of the beam. Here, I_m and K_m are the modified Bessel functions and we assumed the initial radius r_{b0} is independent of ζ. Equation (5) describes the coupled beam evolution and wakefield excitation.

Consider a small perturbation about the long beam (where variation in the longitudinal beam profile may be neglected) equilibrium radius r_0, satisfying $\epsilon_n^2k_p = 4\gamma k_b^2r_0^3K_1(k_pr_0)I_2(k_pr_0)$. Assuming a small perturbation about this equilibrium, $r_b = r_0 + r_1$ with $|r_1/r_0| \ll 1$ and $r_{b0} = r_0$, Eq. (5) yields the evolution of the beam radius perturbation

$$\left(\frac{d^2}{d\hat{z}^2} + 4\kappa^2\right)r_1 = 2\nu\int_\infty^\zeta d\hat{\zeta}'\sin(\hat{\zeta} - \hat{\zeta}')r_1(\hat{\zeta}'), \tag{6}$$

with the constants $\nu = 4I_2(k_pr_0)K_2(k_pr_0)$ and

$$\kappa^2 = 2K_1(k_pr_0)\left[4\frac{I_2(k_pr_0)}{k_pr_0} + I_3(k_pr_0)\right], \tag{7}$$

and the normalized variables $\hat{\zeta} = k_p\zeta$ and $\hat{z} = k_bz/(2\gamma)^{1/2}$. In the limit of a narrow beam $k_pr_0 \ll 1$, $\nu \simeq 1 - (k_pr_0)^2/6$, and $\kappa^2 \simeq 1 + (k_pr_0)^2[C_\gamma - 1/4 + \ln(k_pr_0/2)]/2$, where $C_\gamma \simeq 0.577$ is the Euler-Mascheroni constant. Equation (6) may be analyzed in several regimes. The most relevant regime for plasma accelerators based on self-modulated drive beams is the strongly coupled (or long-beam, early-time) regime valid for $\hat{\zeta} \gg \hat{z}$.

Consider a slowly varying envelope, such that $r_1 = \hat{r}\exp(ik_p\zeta)/2 + \text{c.c.}$ with $|\partial_\zeta\hat{r}| \ll |k_p\hat{r}|$, and assume the strongly coupled regime where the growth length of the instability is short compared to $\gamma^{1/2}k_b^{-1}$, such that $|\partial_{\hat{z}}\hat{r}| \gg 2\kappa|\hat{r}|$. In this regime, after applying the linear plasma wave operator, Eq. (6) becomes

$$(\partial_{\hat{\zeta}}\partial_{\hat{z}}^2 + i\nu)\hat{r} = 0, \tag{8}$$

which describes the evolution of the slowly varying amplitude of the beam radius perturbation and may be solved using standard Laplace transform techniques. With the initial conditions $\hat{r}(z, \zeta = 0) = \delta r\Theta(z)$, $\hat{r}(z = 0, \zeta) = \delta r$, and $\partial_z\hat{r}(z = 0, \zeta) = 0$, the solution to Eq. (8) can be expressed as

$$\hat{r}/\delta r = \sum_{n=0}^\infty\frac{(i\nu|\hat{\zeta}|\hat{z}^2)^n}{n!(2n)!}. \tag{9}$$

The solution to Eq. (8) may also be evaluated asymptotically and has the form

$$r_1 = \delta r\frac{3^{1/4}}{(8\pi)^{1/2}}N^{-1/2}e^N\cos\left(\pi/12 - k_p\zeta - N/\sqrt{3}\right), \tag{10}$$

where the number of e foldings is

$$N = \frac{3^{3/2}}{4}\left(\nu\frac{n_{b0}m_e}{n_0M_b\gamma}k_p^3|\zeta|z^2\right)^{1/3}. \tag{11}$$

Note that the instability growth rate Eq. (11) [and the beam envelope equation, Eq. (5)] differ from that found in Ref. [10]. Behind the modulated beam the growth is given by Eq. (11) with $|\zeta| = L_b$, where L_b is the bunch length. Hence, for fixed r_{b0}, the growth scales as $N \propto (n_{b0}L_b)^{1/3} \propto Q_b^{1/3}$, where Q_b is the beam charge.

Figure 1 shows the beam radius modulation $r_b/r_0 = 1 + r_1$ versus $k_p\zeta$, after propagating $k_pz = 28\,000$ (red curve) and $k_pz = 25\,000$ (blue curve), obtained from numerical solution of Eq. (5) for a proton beam initially in equilibrium $r_{b0} = r_0$ with beam-plasma parameters $n_b/n_0 = 0.008$, $\gamma = 480$, and $k_pr_0 = 1$. The dashed curves are the envelope of the linear asymptotic solution

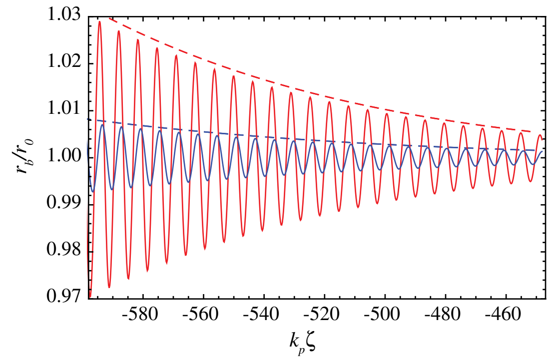

FIG. 1 (color online). Beam radius modulation r_b/r_0 vs $k_p\zeta$ with beam-plasma parameters $n_b/n_0 = 0.008$ (proton beam), $\gamma = 480$, and $k_p r_0 = 1$ (and $r_{b0} = r_0$), obtained from numerical solution of Eq. (5), at $k_p z = 25\,000$ (blue curve) and $k_p z = 28\,000$ (red curve). Dashed curves are the envelope of the asymptotic linear solution Eq. (10).

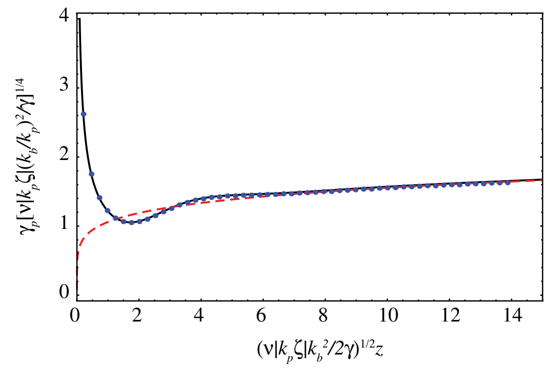

FIG. 2 (color online). Normalized phase velocity of accelerating wakefield $\gamma_p[\nu(k_b/k_p)^2|\hat{\zeta}|/\gamma]^{1/4}$ vs normalized propagation distance $(\nu|\hat{\zeta}|)^{1/2}\hat{z}$ in the long-beam regime: solid (black) curve is the series solution Eq. (12), dashed (red) curve is the asymptotic solution Eq. (14), and dots (blue) are from the numerical solution of the envelope equation Eq. (5).

Eq. (10). Figure 1 shows the growth versus distance behind the head of the beam (at $k_p\zeta = 0$) and versus propagation distance. Also shown is the shift in phase of the modulation versus propagation distance, resulting in a reduced phase velocity. Physically the peaks of the focusing force (transverse wakefield) lag behind (shifted in phase by $2\pi/3$) the peaks of the beam density. This phase shift results in the peaks of the beam density modulation moving back toward the peaks in the focusing force, reducing the phase velocity of the modulation with respect to the beam velocity.

The above solution Eq. (9) assumed $|k_p\hat{r}| \gg |\partial_\zeta\hat{r}|$, or $1 \gg |k_p^{-1}(\partial_\zeta N)|$. This condition may be approximately expressed as $\hat{\zeta} \gg \hat{z}$, which will be satisfied for long beams sufficiently early in the beam propagation. It was also assumed that $|\partial_{\hat{z}}\hat{r}| \gg 2\kappa|\hat{r}|$, which is satisfied provided $\hat{\zeta} \gg \hat{z}$ is satisfied. The above analysis is also based on linear theory, and nonlinear effects (i.e., when $r_1 \sim r_0$ or $E_z \sim E_0$) may saturate the instability.

The beam radius perturbation $r_1 = \hat{r}\exp(ik_p\zeta)/2 + \text{c.c.}$ modulates beam density $n_b \simeq n_b(r_0)(1 - 2r_1/r_0) + n_{b0}r_1\delta(r_0 - r)$. This beam density modulation drives a modulation in the electron plasma density $\hat{n}\exp(ik_p\zeta)/2 + \text{c.c.}$, via Eq. (1), i.e., $\partial_\zeta\hat{n} \simeq \mp ik_p n_{b0}\times[\Theta(r_0 - r) - \delta(r_0 - r)r_0/2](\hat{r}/r_0)$. The plasma density modulation drives the accelerating wakefield $E_z/E_0 = \hat{E}_z\exp(ik_p\zeta)/2 + \text{c.c.}$, via Eq. (2), i.e., $(\nabla_\perp^2 - k_p^2)\hat{E}_z = -ik_p^2\hat{n}/n_0$. For the same initial conditions as above, the solution for the accelerating wakefield in the long-beam regime is

$$\hat{E}_z = \mp H_R(r, r_0)\frac{n_{b0}}{n_0}\frac{\delta r}{r_0}|\hat{\zeta}|\sum_{n=0}^{\infty}\frac{(i\nu|\hat{\zeta}|\hat{z}^2)^n}{(n+1)!(2n)!}, \qquad (12)$$

where $H_R(r, r_0) = 1 - k_p r_0 K_1(k_p r_0)I_0(k_p r) - I_0(k_p r)\times K_0(k_p r_0)r_0^2/2$ for $r \leq r_0$ and $k_p r_0 I_1(k_p r_0)K_0(k_p r) - I_0(k_p r_0)K_0(k_p r)r_0^2/2$ for $r > r_0$. In the asymptotic limit, the accelerating wakefield has the form $E_z/E_z(z = 0) \simeq 3^{7/4}(32\pi)^{-1/2}N^{-3/2}\exp(N)\cos(\psi)$, where the number of e foldings of growth of the accelerating wake is given by Eq. (11) and the phase is

$$\psi = \frac{\pi}{4} - k_p\zeta - \frac{3}{4}\left(\nu\frac{k_b^2 k_p}{\gamma}|\zeta|z^2\right)^{1/3}. \qquad (13)$$

The phase velocity of the accelerating wake is $\beta_p = -\partial_t\psi/\partial_z\psi = \partial_\zeta\psi/(\partial_\zeta + \partial_z)\psi \simeq 1 - \partial_z\psi/\partial_\zeta\psi$. Using the phase Eq. (13), the phase velocity is $\beta_p \simeq 1 + k_p^{-1}\partial_z\psi = 1 - (2/3^{3/2})(N/k_p z)$. The phase velocity of the self-modulated beam-driven wakefield is less than the beam velocity $\beta_b \simeq 1$, varies along the beam ζ and during propagation z. Asymptotically, the Lorentz factor of the phase velocity is

$$\gamma_p = \left(\frac{\gamma n_0 M_b}{\nu n_{b0} m_e}\frac{z}{|\zeta|}\right)^{1/6}. \qquad (14)$$

Note that, behind the modulated beam the phase velocity is given by Eq. (14) with $|\zeta| = L_b$.

Figure 2 shows the normalized Lorentz factor of the phase velocity of the accelerating wakefield $\gamma_p[\nu(k_b/k_p)^2|\hat{\zeta}|/\gamma]^{1/4}$ versus normalized propagation distance $(\nu|\hat{\zeta}|)^{1/2}\hat{z}$. The solid curve in Fig. 2 is obtained from the series solution Eq. (12), $\beta_p = 1 - |k_p^{-1}\partial_z[\arctan(\Im\hat{E}_z/\Re\hat{E}_z)]|$, the dashed curve is the asymptotic solution Eq. (14), and the dots are from the numerical solution (with the proton beam parameters $\gamma = 480$, $n_{b0}/n_0 = 0.008$, $k_p r_0 = 1$, and $k_p L_b = 715$) of the envelope equation Eq. (5). Figure 2 indicates that there is a minimum phase velocity. The minimum phase velocity can be estimated by using Eq. (12). The minimum phase velocity occurs at $(\nu|\hat{\zeta}|)^{1/2}\hat{z} \simeq 1.72$, with

$$\gamma_{\min} \simeq 1.05\left(\frac{\gamma n_0 M_b}{\nu n_{b0} m_e k_p |\zeta|}\right)^{1/4}. \quad (15)$$

As shown in Fig. 2, after reaching $\gamma_{\min}$, the phase velocity grows slowly as the beam propagates $\gamma_p \propto z^{1/6}$ [cf. Equation (14)]. For example, consider a wake driven by a 450 GeV proton beam ($\gamma = 480$), with $r_0 = 180\ \mu$m, $L_b = 12$ cm, and 10^{11} particles (i.e., near the parameters of the CERN Super Proton Synchrotron). Operating at $n_0 = 10^{15}$ cm^{-3}, corresponds to $n_{b0}/n_0 = 0.008$, $E_0 = 3$ GV/m, $k_p r_0 = 1.0$, $k_p L_b = 715$, and $\nu = 0.88$. For this example, $\gamma_{\min} \simeq 21$ behind the drive beam after $z \simeq 17$ cm (i.e., $\approx 160\lambda_p$) of propagation.

With the phase velocity of the self-modulated wake determined, the dephasing length may be calculated. For a linear wake, the dephasing length is the propagation distance required for an ultrarelativistic particle $\beta_b \simeq 1$ to slip $\lambda_p/4$ (or a wake phase of $\pi/2$) with respect to the plasma wave. Assuming the phase velocity is well-approximated by the asymptotic solution in the strongly-coupled regime Eq. (14), the dephasing length is $L_d = (2\pi/3)^{3/2}(\nu k_b^2 k_p |\zeta_i|/\gamma)^{-1/2}$. Including the early-time response via Eq. (12), the dephasing length is

$$L_d \simeq 4.9(\nu k_b^2 k_p |\zeta_i|/\gamma)^{-1/2}, \quad (16)$$

where ζ_i is the injection position of the witness bunch (e.g., initially at a peak of the accelerating field). For a witness bunch injected behind the drive beam, $|\zeta_i| = L_b$. This reduced dephasing length will greatly limit the energy gain of a witness electron beam trailing the drive bunch. For example the number of e foldings of the self-modulation instability that have occurred at the dephasing length Eq. (16) is $N(z = L_d, \zeta_i) \simeq 3.8$. Note that the number of e foldings at a dephasing length $N(z = L_d)$ is independent of injection location and the beam-plasma parameters. After a dephasing length, the witness beam will move into a decelerating phase, and then into a defocusing phase of the plasma wave (which will scatter the beam transversely).

The number of e foldings required to reach an interesting accelerating gradient will be determined by the instability seed. Assuming the beam is Gaussian, the seed generated by the wake of the long-beam envelope is $\delta n_{\rm seed}/n_0 \sim (k_p \sigma_z)\exp[-(k_p\sigma_z)^2/2] n_{b0}/n_0$, which vanishes for $k_p \sigma_z \gg 1$, where σ_z is the rms bunch length. One possibility to seed the modulation is using a leading intense short-pulse laser or short electron bunch that drives a wake. Consider a seed $E_{z,\rm seed}/E_0 \simeq 10^{-2}$ (i.e., $E_{z,\rm seed} \simeq 30$ MV/m at $n_0 = 10^{15}$ cm^{-3}) driven by a resonant laser pulse (requiring a 93 TW, 130 J, 0.8 μm wavelength pulse with spot size $k_p r_L = 2$ and laser strength parameter $a_0 = 0.165$). The peak electric field after a dephasing length is $E_z \approx 106$ MV/m, and the energy gain of a trailing electron beam after a dephasing length (≈ 34 cm) is ≈ 10 MeV, assuming a wakefield driven by a self-modulated 450 GeV proton beam with $n_{b0}/n_0 = 0.008$, $k_p r_0 = 1$, $k_p L_b = 715$, and $n_0 = 10^{15}$ cm^{-3}.

Improved efficiency may be possible by tapering the plasma density, i.e., increasing the background plasma density to reduce the plasma wavelength, thereby increasing the phase velocity [17], although variation of the plasma density may affect the instability growth. Alternatively the accelerator may use a staged approach, where a long plasma region self-modulates the drive beam until saturation of the instability, followed by a second stage where a witness bunch would be injected following the modulated drive beam. Such a two-staged approach could potentially be limited by the hose (or transverse two-stream) instability [18], which grows in the long-beam limit with a comparable growth rate $\sim N$. This implies that to drive large amplitude accelerating fields via the self-modulation instability without hosing requires strongly seeding the self-modulation instability.

The long-beam, early-time regime described above will be valid for $\hat{z} \ll \hat{\zeta}$. After sufficiently long propagation distances, or for sufficiently short beams, the instability may enter a weakly-coupled regime where the instability growth length is long compared to $\gamma^{1/2}/k_b$. The instability will transition to the weakly-coupled regime after a propagation distance approximately $\hat{z} \sim \hat{\zeta}$, or, using Eq. (11), after approximately $N \sim k_p \zeta$. For long beams $k_p \zeta \gg 1$, nonlinear effects will typically appear before the instability enters the weakly coupled regime.

In this Letter we have calculated the beam self-modulation instability growth rate, in the long-beam regime, including the phase dependence. The phase velocity of the accelerating wakefield was calculated and shown to be significantly less than the drive beam velocity. The dephasing length was calculated, and a witness beam will reach dephasing in less than four e foldings, independent of beam-plasma parameters. This indicates that the energy gain in a plasma accelerator driven by a self-modulated PWFA in a homogeneous plasma will be limited by dephasing.

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