Exploring Symmetry Breaking at the Dicke Quantum Phase Transition

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We study symmetry breaking at the Dicke quantum phase transition by coupling a motional degree of freedom of a Bose-Einstein condensate to the field of an optical cavity. Using an optical heterodyne detection scheme, we observe symmetry breaking in real time and distinguish the two superradiant phases. We explore the process of symmetry breaking in the presence of a small symmetry-breaking field and study its dependence on the rate at which the critical point is crossed. Coherent switching between the two ordered phases is demonstrated.

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Spontaneous symmetry breaking at a phase transition is a fundamental concept in physics [1]. At zero temperature, it is caused by the appearance of two or more degenerate ground states in the Hamiltonian. As a result of fluctuations, a macroscopic system evolves into one particular ground state which does not possess the same symmetry as the Hamiltonian. Finding a clean testing ground to experimentally study the process of symmetry breaking is notoriously difficult, as external fluctuations and asymmetries have to be minimized or controlled. The protected environment of atomic quantum gas experiments and the increasing control over these systems offer new prospects to experimentally approach the concept of symmetry breaking. Recently, rapid quenches across a phase transition were studied in multicomponent Bose-Einstein condensates [2–4] and optical lattices [5,6]. Such a nonadiabatic quench causes a response of the system at correspondingly high energies. Therefore, a central characteristic of a phase transition, which is its diverging susceptibility to perturbations, remains partially hidden.

In this work, we study the symmetry-breaking process while slowly varying a control parameter several times across a zero-temperature phase transition. Compared to quenching, this allows us to explore the low energy spectrum of the system which probes its symmetry most sensitively. For very slow crossing speeds, we identify the presence of a residual symmetry-breaking field of varying strength. Larger values of this residual field can be correlated to the repeated observation of one particularly ordered state. For increasingly steeper ramps across the phase transition, the influence of the symmetry-breaking field almost vanishes.

We investigate the symmetry breaking in the motional degree of freedom of a Bose-Einstein condensate coupled to a single mode of an optical cavity. Our system realizes the Dicke model [7–9], which exhibits a second-order zero-temperature phase transition [10–13]. The broken symmetry is associated with the formation of one of two identical atomic density waves, which are shifted by half an optical wavelength [8,9,14,15]. Using an interferometric heterodyne technique, we monitor the symmetry-breaking process in real time while crossing the transition point. A similar technique has been used to test self-organization in a classical ensemble of laser-cooled atoms [15], where the symmetric phase is stabilized by thermal energy rather than kinetic energy [16].

The Dicke model [7] considers the interaction between N two-level atoms and the quantized field of a single-mode cavity, which is described by the Hamiltonian

$$\hat{H} = \hbar\omega_0 \hat{J}_z + \hbar\omega \hat{a}^\dagger \hat{a} + \frac{2\hbar\lambda}{\sqrt{N}}(\hat{a}^\dagger + \hat{a})\hat{J}_x. \qquad (1)$$

Here, $\hat{a}$ and $\hat{a}^\dagger$ denote the annihilation and creation operators, respectively, for the cavity mode at frequency ω, and $\hat{\mathbf{J}} = (\hat{J}_x, \hat{J}_y, \hat{J}_z)$ describes the atomic ensemble with transition frequency ω_0 in terms of a pseudospin of length $N/2$. The cavity light field couples with coupling strength λ to the collective atomic dipole $\hat{J}_x$. In the thermodynamic limit, the Dicke model exhibits a zero-temperature phase transition from a normal to a superradiant phase when the control parameter λ exceeds a critical value given by $\lambda_{\rm cr} = \sqrt{\omega\omega_0}/2$ [10–12]. Simultaneously, the parity symmetry of the Dicke Hamiltonian, given by the invariance under the transformation $(\hat{a}, \hat{J}_x) \rightarrow (-\hat{a}, -\hat{J}_x)$, is spontaneously broken [13]. While parity is conserved in the normal phase with $\langle\hat{a}\rangle = 0 = \langle\hat{J}_x\rangle$, two equivalent superradiant phases (denoted by even and odd) emerge for $\lambda > \lambda_{\rm cr}$, which are characterized by $\langle\hat{J}_x\rangle \lesseqgtr 0$ and $\langle\hat{a}\rangle \gtreqless 0$, respectively [Fig. 1(b)].

In our experiment [8], we couple motional degrees of freedom of a Bose-Einstein condensate with a single cavity mode by using a transverse coupling laser [Fig. 1(a)]. Within a two-mode momentum expansion of the matter-wave field, the Hamiltonian dynamics of this system is described by the Dicke model [Eq. (1)] [8,9,17], where the effective atomic transition frequency is given by $\omega_0 = 2\omega_r$ with the recoil frequency $\omega_r = \hbar k^2/2m$, the atomic mass m, and the wavelength $\lambda_p = 2\pi/k$ of the coupling laser.

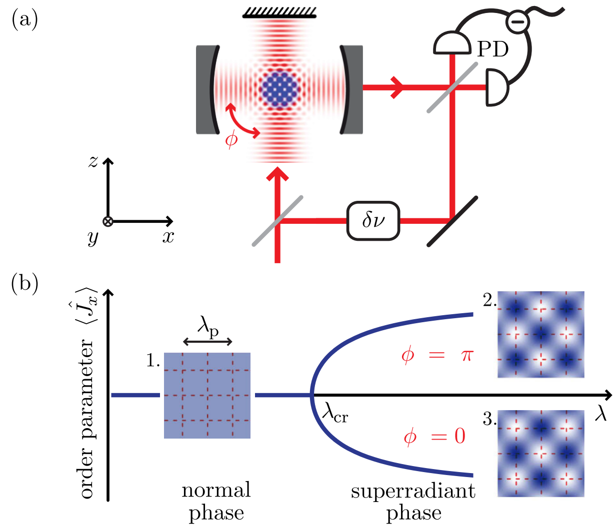

FIG. 1 (color online). (a) Experimental setup. A Bose-Einstein condensate is placed inside an optical cavity and driven by a far-detuned standing-wave laser field (wavelength λ_p) along the z axis. The phase and amplitude of the intracavity field are measured with a balanced heterodyne setup (PD, photodiodes). (b) Steady-state order parameter $\langle \hat{J}_x \rangle$ as a function of coupling strength λ, with corresponding atomic density distributions (1)–(3). The order parameter vanishes in the normal phase (1) and bifurcates at the critical point λ_{cr}, where a discrete $\lambda_p/2$-spatial symmetry is broken. The two emergent superradiant phases [(2) and (3)] can be distinguished via the relative time phase ϕ.

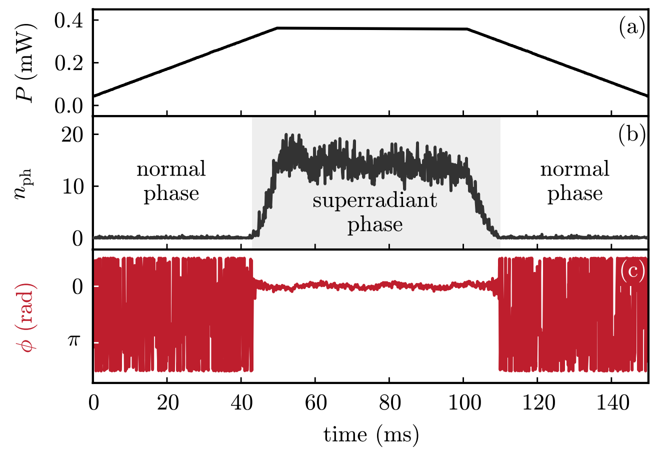

FIG. 2 (color online). Observation of symmetry breaking and steady-state superradiance. Shown are simultaneous traces of (a) the coupling laser power P, (b) the mean intracavity photon number n_{ph}, and (c) the relative time phase ϕ between the coupling laser and cavity field (both averaged over 150 μs). The coupling laser frequency is red-detuned by 31.3(2) MHz from the empty cavity resonance, and the atom number is $2.3(5) \times 10^5$. Residual atom loss causes a slight decrease of the cavity photon number in the superradiant phase.

The frequency and power of this laser control the effective mode frequency ω and the coupling strength λ, respectively [8]. Above a critical laser power, the discrete $\lambda_p/2$-spatial symmetry, defined by the optical mode structure $u(x, z) = \cos(kx)\cos(kz)$, is spontaneously broken, and the condensate exhibits either of two density waves [Fig. 1(b)]. Correspondingly, the atomic order parameter $\langle \hat{J}_x \rangle$, given by the population difference between the even $[u(x, z) > 0]$ and odd $[u(x, z) < 0]$ sublattices, exhibits a negative or positive macroscopic value, while the emergent coherent cavity field oscillates (for $\omega \gg \kappa$) either in ($\phi = 0$) or out of phase ($\phi = \pi$) with the coupling laser.

As described previously [8], we prepare Bose-Einstein condensates of typically 2×10^5 ^{87}Rb atoms in a crossed-beam dipole trap centered inside an ultrahigh-finesse optical Fabry-Perot cavity, which has a length of 176 μm. The transverse coupling laser at wavelength $\lambda_p = 784.5$ nm is red-detuned by typically ten cavity linewidths $2\kappa = 2\pi \times 2.5$ MHz from a TEM_{00} cavity mode, realizing the dispersive regime $\omega \gg \omega_0$ of the Dicke model. We monitor the amplitude and phase of the intracavity field in real time by using a balanced heterodyne detection scheme [Fig. 1(a)]. Because of slow residual drifts of the differential path length of our heterodyne setup, which translate into drifts of the detected phase signal of about 0.1 π/s, we cannot relate the phase signals between consecutive experimental runs separated by 60 s.

To observe symmetry breaking, we gradually increase the coupling laser power across the critical point [Fig. 2(a)]. The transition from the normal to the superradiant phase is marked by a sharp increase of the mean intracavity photon number [Fig. 2(b)]. Simultaneously, the time phase ϕ between the two light fields locks to a constant value, implying that the symmetry of the system has been broken [Fig. 2(c)]. The observation of a constant time phase above threshold confirms that the system reaches a steady-state superradiant phase in which the induced cavity field oscillates at the coupling laser frequency. When lowering the laser power to zero again, the system recovers its initial symmetry and a pure Bose-Einstein condensate is retrieved, as was inferred from absorption imaging after free ballistic expansion.

To identify the two different superradiant states [Fig. 1(b)], we cross the phase transition multiple times within one experimental run [Fig. 3(a)]. Above threshold, the corresponding phase signal takes always one of two constant values. From multiple traces of this type, we extract a time-phase difference of $1.00(2) \times \pi$ between the two superradiant phases, where the statistical error can be attributed to residual phase drifts of our detection system.

If the system were perfectly symmetric, the two ordered phases would be realized with equal probabilities, when repeatedly crossing the phase transition. However, the presence of any symmetry-breaking field will always drive the system into the same particularly ordered state when

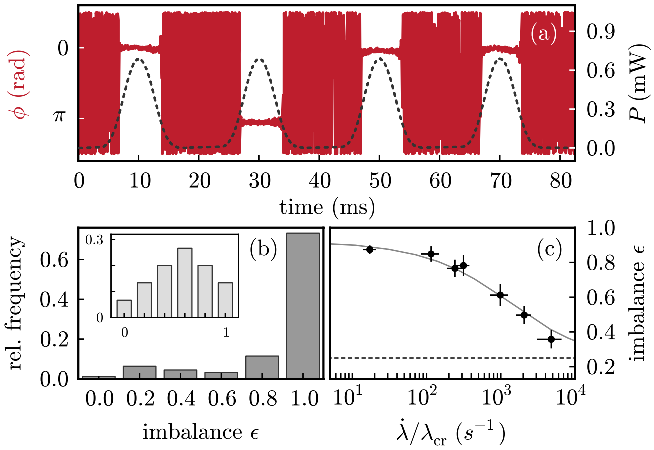

FIG. 3 (color online). (a) Cavity time phase (red, averaged over 30 μs) for a single run and the corresponding time sequence of the coupling laser power P (dashed line). (b) Probability distribution of the imbalance ϵ (see the text) for 156 runs, where the phase transition was crossed at a rate of $\dot{\lambda}/\lambda_{cr} = 18(3)\ s^{-1}$. The inset displays the distribution of postselected data (see the text). (c) Mean imbalance (dots) as a function of the rate $\dot{\lambda}/\lambda_{cr}$ at which the transition was crossed (extracted from 356 runs in total) and the theoretical model (solid line). The error bars indicate the standard error of the mean of ϵ and systematic changes of λ_{cr} during probing.

adiabatically crossing the critical point. We experimentally quantify the even-odd imbalance by performing 156 experimental runs [similar to Fig. 3(a)], in each of which the system enters the superradiant phase 10 times within 1 s. A measure for the even-odd imbalance is given by the parameter $\epsilon = (m_1 - m_2)/10$, where $m_2 \leq m_1$ denote the number of occurrences of the two superradiant configurations in individual traces. In 73% of the traces, the system realized 10 times the same time phase, corresponding to the maximum imbalance of $\epsilon = 1$ [Fig. 3(b)]. However, 12% of the runs exhibited an imbalance below 0.5, which is not compatible with a constant even-odd asymmetry.

We attribute our observations to the finite spatial extension of the atomic cloud. This can result, even for zero coupling λ, in a small, but finite population difference between the even and odd sublattices, determined by the spatial overlap $\mathcal{O}$ between the atomic column density $n(x, z)$ (normalized to N) and the optical mode profile $u(x, z)$. This asymmetry enters the two-mode description [Eq. (1)] via the symmetry-breaking term $2\hbar\lambda\mathcal{O}(\hat{a}^\dagger + \hat{a})/\sqrt{N}$ and renormalizes the order parameter $\langle \hat{J}_x \rangle$ by the additive constant $\mathcal{O}$. The resulting coherent cavity field below threshold drives the system dominantly into either of the two superradiant phases, depending on the sign of $\mathcal{O}$. In the experiment, the resulting even-odd imbalance is likely to change between experimental runs, as the overlap integral $\mathcal{O}$ depends λ_p-periodically on the relative position between the mode structure $u(x, z)$ and the center of the trapped atomic cloud, with amplitude $\mathcal{O}_0$. We can exclude a drift of the relative trap position by more than half a wavelength λ_p on the time scale given by our probing time of 1 s, as it would lead to equal probabilities of the two phases, pretending spontaneous symmetry breaking.

The openness of the system provides us with direct experimental access to the symmetry-breaking field proportional to $\mathcal{O}$. Indeed, we detect a small coherent cavity field ($n_{ph} < 0.02$) in the normal phase whose magnitude varies between experimental runs. In all runs exhibiting an imbalance of $\epsilon = 1$ [Fig. 3(b)], the relative time phases of the cavity field detected below and above threshold are equal. Furthermore, the even-odd imbalance increases significantly with the light level observed below threshold. Postselection of those 10% of the runs with the smallest light level yields a much smaller mean imbalance [Fig. 3(b), inset].

In general, the influence of a symmetry-breaking field becomes negligible, if the mean value of the order parameter, induced by this field, is smaller than the quantum or thermal fluctuations present in the system. From a mean-field calculation performed in the Thomas-Fermi limit for $N = 2 \times 10^5$ harmonically trapped atoms, we estimate a maximum order parameter of $\mathcal{O}_0 = 40$ for zero coupling strength, corresponding to an even-odd population difference of 40 atoms. This value is much smaller than the uncertainty $\Delta J_x = \sqrt{N}/2 = 224$, given by vacuum fluctuations of the excited momentum mode. Therefore, one expects in the extreme case of a sudden quench of the coupling strength beyond λ_{cr} that the apparent symmetry is spontaneously broken, resulting in nearly equal probabilities of the two superradiant phases.

In the experiment, we determined the even-odd imbalance ϵ for increasingly larger rates $\dot{\lambda}/\lambda_{cr}$ at which the critical point was crossed, i.e., in an increasingly nonadiabatic situation [Fig. 3(c)]. As the transition is crossed faster, the mean imbalance between the two superradiant phases decreases significantly and approaches the value $\epsilon \approx 0.25$ corresponding to the balanced situation [Fig. 3(c), dashed line]. This indicates that the effect of the symmetry-breaking term can be overcome by nonadiabatically crossing the phase transition.

Our observations [Fig. 3(c)] are in quantitative agreement with a simple model based on the adiabaticity condition known from the Kibble-Zurek theory [18,19]. We divide the evolution of the system during the increase of the transverse laser power into a quasiadiabatic regime, where the system follows the change of the control parameter, and an impulse regime, where the system is effectively frozen. After crossing the critical point, fluctuations of the order parameter, which are present at the instance of freezing, become instable and are amplified. The coupling strength which separates the two regimes is determined by Zurek's equation [18] $|\dot{\zeta}/\zeta| = \Delta/\hbar$, with

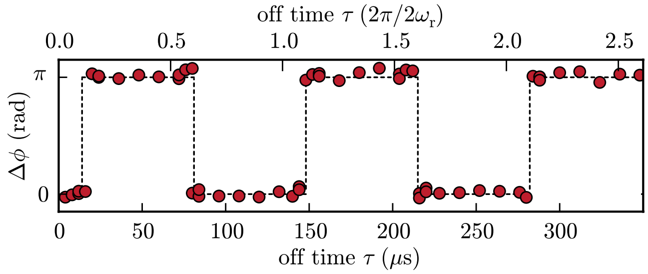

FIG. 4 (color online). Coherent switching between the two ordered phases. After adiabatically preparing the system in one of the two superradiant phases, the coupling field is turned off for a time τ. Displayed is the steady-state cavity time phase $\Delta\phi$ (averaged over 0.5 ms) after turning on the coupling field, referenced to the value recorded before the turning-off. Each data point corresponds to a single measurement. The dashed line shows the time evolution as expected from the two-mode model.

$\zeta = (\lambda_{\rm cr} - \lambda)/\lambda_{\rm cr}$, and the energy gap between the ground and first excited state given by $\Delta = \hbar\omega_0\sqrt{1 - \lambda^2/\lambda_{\rm cr}^2}$ for $\omega \gg \omega_0$ [13,17].

We deduce the probability with which the system chooses the even phase, $p_{\rm even} = \int_0^\infty p(\Theta)d\Theta$, from the probability distribution $p(\Theta)$ at the instance of freezing, where $\hat{\Theta}$ denotes the shifted dipole operator $\hat{\Theta} = \hat{J}_x + \mathcal{O}$. In the thermodynamic limit, the distribution $p(\Theta)$ becomes Gaussian with a mean value $\langle\hat{\Theta}\rangle = \langle\hat{J}_x\rangle + \mathcal{O}$ and a width determined by the quantum fluctuations of the order parameter ΔJ_x. These values are determined from the linear quantum Langevin equations based on the Dicke model [17] including the symmetry-breaking term. Besides the decay of the cavity field, we also take into account dissipation of the excited momentum state at a rate $\gamma = 2\pi \times 0.6$ kHz. This value was deduced from independent measurements of the cavity output field below threshold [20].

From the steady-state solution of the quantum Langevin equations, we find that the mean order parameter $\langle\hat{\Theta}\rangle$ grows faster in λ than its fluctuations. If $\mathcal{O} > 12$, the order parameter exceeds its uncertainty already below critical coupling. Thermal fluctuations are neglected in this analysis. For our typical condensate temperatures of about 100 nK, quantum fluctuations dominate as long as $\zeta > 0.005$. We account for shot-to-shot fluctuations of the overlap $\mathcal{O}$ by suitably averaging over the position of the harmonic trap. The solid line in Fig. 3(c) shows a least-squares fit of our model to the data with the single free parameter $\mathcal{O}_0$. We obtain a value of $\mathcal{O}_0 = 77$, which is in reasonable agreement with the theoretically expected value of $\mathcal{O}_0 = 40$. This verifies the predominance of the considered symmetry-breaking field over other possible noise terms.

Finally, we experimentally demonstrate coherent switching between the two ordered states. To this end, we suddenly turn off the coupling laser field after adiabatically preparing the system in one of the two superradiant phases. The atoms are then allowed to freely evolve according to their momentum state occupation, giving rise to standing-wave oscillations of the atomic density distribution. In the two-mode description, this corresponds to harmonic oscillations of the order parameter $\langle\hat{J}_x\rangle$ at frequency $2\omega_r$. We probe this time evolution by turning on the coupling laser after a variable off time τ, thereby deterministically retrapping the atoms either in the initial or in the opposite superradiant state. As expected, we observe regular π jumps in the difference $\Delta\phi$ between the steady-state phase signals measured before and after the free evolution, with a frequency of $2\omega_r$ (Fig. 4, dashed line). The inertia of the atoms traveling at finite momentum causes the π jumps in Fig. 4 to occur before those times at which the order parameter has evolved by an odd number of quarter periods.

In conclusion, we have experimentally monitored symmetry breaking in the Dicke quantum phase transition and identified the interplay between a residual symmetry-breaking field, fluctuations, and the crossing speed.

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