Interferometric and Noise Signatures of Majorana Fermion Edge States in Transport Experiments

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Domain walls between superconducting and magnetic regions placed on top of a topological insulator support transport channels for Majorana fermions. We propose to study noise correlations in a Hanbury Brown–Twiss type interferometer and find three signatures of the Majorana nature of the channels. First, the average charge current in the outgoing leads vanishes. Furthermore, we predict an anomalously large shot noise in the output ports for a vanishing average current signal. Adding a quantum point contact to the setup, we find a surprising absence of partition noise which can be traced back to the Majorana nature of the carriers.

Elementary excitations (often called quasiparticles) of condensed-matter systems can show features that are not displayed by the bare particles that they are composed of. A striking example are quasiparticles that show neither fermionic nor bosonic statistics but an intermediate "anyonic" form [1]. Majorana fermions appearing at the core of half-vortices in p-wave superconductors have been predicted to exhibit anyonic statistics [2]. Theoretical proposals to observe their existence in tunneling experiments [3,4] were devised.

Recently, the possibility to realize Majorana-like quasiparticles on the surface of a three-dimensional topological insulator has attracted a lot of attention (see Ref. [5] and references therein). It has been shown that the domain wall of two superconducting regions support transport channels for Majorana fermions [6], and the interface of superconducting and magnetic regions give rise to transport channels for chiral Majorana fermions [7,8].

Up to now, these new excitations have not been observed experimentally, but a number of schemes to detect them has been put forward. These include interferometric structures in which electrons are converted to Majorana fermions and back [7–10], as well as scanning probe devices coupled to Majorana edge states that detect resonant Andreev reflection [11]. Also the measurement of the backaction of Majorana edge states to a coupled flux qubit could provide a hint of their existence [12].

In the proposals using interferometry [7,8], the authors considered a two-terminal Mach-Zehnder setup. A magnetic domain wall carrying chiral electronic excitations meets a superconducting region, where the incoming electron channel is split into two chiral Majorana fermion channels surrounding the superconductor. At the opposite side of the superconductor, the Majorana channels recombine to form an outgoing chiral electron channel. Depending on the phase change $2\pi\phi/\phi_0$ between the

two Majorana arms that is determined by their geometric length and the number of vortices threading the superconductor, an incoming electron is converted either to an outgoing electron or an outgoing hole. The *effective* flux ϕ threading the Mach-Zehnder interferometer includes the actual magnetic flux due to vortices, as well as the dynamical phase of the Majorana fermions; $\phi_0 = h/e$ is the flux quantum. The conductance G_{12}, where 1(2) stands for the incoming (outgoing) lead, is periodic in ϕ/ϕ_0: $G_{12} = (e^2/h)\cos(2\pi\phi/\phi_0)$, at zero bias and low temperatures. Negative conductances correspond to outgoing holes: charge conservation is ensured because the superconductor is grounded; i.e., this Mach-Zehnder interferometer is actually a three-terminal device. This form of the conductance shows the same periodicity as a normal (nonsuperconducting) interference experiment. Hence, there is a need for further signatures of Majorana physics beyond the Mach-Zehnder setup.

The structure we have in mind is a Hanbury Brown-Twiss (HBT) type interferometer built on the surface of a topological insulator. This setup is inspired by recent proposals [7–9,13] and is related to the two-particle Aharonov-Bohm effect [14]. We calculate the current cross-correlations in the two outgoing leads of this interferometer and predict the possibility to switch between negative and positive current cross-correlations by tuning the magnetic flux threading the superconductor. Positive cross-correlations are remarkable since noninteracting fermions will always show a negative sign [15]; see, however, [16–18]. The cross-correlations are predicted to be temperature-independent in a reasonable range of temperature and at low voltages. As in [9] we find that the cross-correlations vanish when only one source is active as the consequence of the transport through Majorana modes.

We then consider a setup that contains an additional quantum point contact (QPC), similarly as in [7].

Strikingly, the partition noise associated to the quantum point contact is predicted to vanish, which is an evidence of the neutrality, or equivalently, the Majorana nature, of the charge carriers.

We propose to realize a Hanbury Brown-Twiss type interferometer consisting of a grounded superconductor surrounded by four magnetic domains, as shown in Fig. 1.

In the outer arms 1, 2, 3, and 4, Dirac electrons propagate, while in the center arms A, B, C, and D, at the edge of the superconductor, only Majorana chiral fermions exist at energies below the induced superconducting gap Δ. The incoming Dirac electrons are transformed into Majorana fermions which partially circle the superconductor and are converted back to Dirac electrons. This conversion process has already been studied in [7,8] and is expected to be perfectly symmetric: the Dirac electron is coherently split into the two arms with equal probabilities. For the reversed process, a single Majorana fermion is converted into a superposition of an electron and a hole with equal probabilities as well. An incoming electron can thus leave as a hole, in which case a Cooper pair will flow into the grounded superconductor. Figure 1 does not show the backflow currents between terminals 4 and 1 (2 and 3): since they are noiseless, they will not affect any of our conclusions below.

In the following, we would like to study the conductance and noise properties of the interferometer when varying the magnetic flux, i.e., the number of vortices threading the superconductor.

The Landauer-Büttiker formalism provides a straightforward analysis of this interferometer once we know its
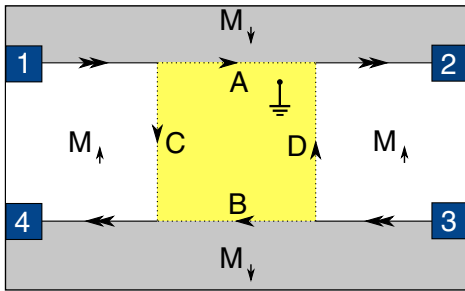

FIG. 1 (color online). Hanbury Brown–Twiss type interferometer. A grounded superconductor surrounded by four magnetic domains is placed on the surface of a three-dimensional topological insulator. The terminals 1, 2, 3, and 4 are connected to the outside circuit and biased at potentials $V_1, \ldots, V_4$ respectively. The magnetizations are chosen such that Dirac electron states at leads 1, 2, 3, and 4 exist and propagate in the direction of the double arrows. Electrons and holes can enter the interferometer at leads 1 and 3, Majorana fermions propagate along the arms A, B, C, and D in the direction of the single arrows and electrons and holes leave through leads 2 and 4. A magnetic flux in the form of n_v vortices threading the superconductor will control the phase difference between the arms of the interferometer.

scattering matrix [15]. The scattering properties of the Dirac to Majorana converter were established and discussed in [7,8]. At zero energy, the full scattering matrix is fixed by particle-hole symmetry. For small enough energies $E \ll (v_M/v_F)\Delta$ where v_M is the Majorana fermion velocity and v_F the electronic Fermi velocity at the surface of the bare topological insulator, the following still holds

$$\begin{pmatrix} a_2 \\ b_2 \\ a_4 \\ b_4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -\eta & \eta \\ 1 & -1 & \eta & -\eta \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \\ a_3 \\ b_3 \end{pmatrix}. \quad (1)$$

The interferometric phase factor $\eta = e^{i2\pi\phi/\phi_0} = (-1)^{n_v} e^{iE\delta L/\hbar v_M}$ has been concentrated to the arm B by a gauge choice. Here, n_v is the number of vortices threading the superconductor, $\delta L = L_A + L_B - L_C - L_D$ where L_i is the length of arm i of the interferometer. The operator a_i (b_i) is the annihilation operator of a Dirac electron (hole) in lead i. The scattering matrix shown in Eq. (1) is similar to the one obtained in [9], which, however, did not consider the possibility of vortices.

For topological reasons, one-particle quantities are not sensitive to the enclosed flux in this structure: because of the chiral nature of the Majorana states, no one-particle state will enclose the flux. One incoming electron or hole is scattered with equal probability to a hole or an electron at lead 2 or 4. The outgoing currents thus vanish on average. This vanishing conductance is a first hallmark of Majorana fermions: in a standard setup with Andreev processes this could occur only accidentally, and small perturbations would give rise to a nonzero conductance. This vanishing conductance could in principle be due to an interrupted circuit and has to be complemented by an additional measurement of, e.g., the current autocorrelation discussed below.

On the other hand, when both sources are active we expect to see a manifestation of an interesting two-particle Aharonov-Bohm effect [14] for Majorana fermions. As an example consider two incoming electrons in leads 1 and 3

$$a_1^\dagger a_3^\dagger = -(a_2^\dagger b_2^\dagger + \eta a_4^\dagger b_4^\dagger)/2 - (\eta + 1)(a_2^\dagger a_4^\dagger - b_2^\dagger b_4^\dagger)/4$$
$$+ (-\eta + 1)(b_2^\dagger a_4^\dagger - a_2^\dagger b_4^\dagger)/4. \quad (2)$$

The current cross-correlations between leads 2 and 4 are thus expected to be sensitive to the parity of the number n_v of enclosed vortices through the phase parameter η. In particular, as shown later, it is possible to switch between positive and negative cross-correlations by tuning the magnetic field threading the superconductor. As a side remark, note that post-selecting events with one fermion per lead for $\eta = \pm 1$ yields maximally entangled pairs in particle-hole space, and we can equivalently speak of antibunching [13].

Assuming the reservoirs connected to the incoming leads are specified by the electron and hole distribution functions $n_{i,e}(E) = [\exp((E - eV_i)/k_BT) + 1]^{-1} = 1 - n_{i,h}(-E)$ for lead i, we can give explicit expressions for the current-current correlations. The current cross-correlation between leads 2 and 4 $S_{24} = \frac{1}{2}\langle\langle I_2, I_4\rangle\rangle$ are of special interest

$$S_{24} = -\frac{e^2}{h}\int_0^\infty dE\, \Re(\eta(E))(n_{1,e} - n_{1,h})(n_{3,e} - n_{3,h}),$$
(3)

which is sensitive to the magnetic flux through the real part of η, $\Re(\eta) = (-1)^{n_v}\cos E\delta L/\hbar v_M$. At equilibrium $S_{24} = 0$; i.e., there is no thermal noise in this quantity (electrons and holes compensate each other). This temperature independence is expected to hold as long as $k_BT \ll (v_M/v_F)\Delta$. For voltages $V_2 = V_4 = 0$, $V_1 = V_3 = V$ (with respect to the potential of the superconductor), temperatures such that $k_BT \ll eV$ and an approximately symmetric interferometer, $\delta L \ll \hbar v_M/eV$,

$$S_{24} = (-1)^{n_v+1}\frac{e^2}{h}\int_0^\infty dE(n_e + n_h) = (-1)^{n_v+1}\frac{e^2}{h}e|V|.$$
(4)

Thus, the sign of the cross-correlation is given by the parity of the number of vortices. The possibility to achieve positive cross-correlations for fermions is attributed here to electron-hole conversions.

We now look at the current autocorrelations in the outgoing leads. While the outgoing current is zero on average, it is carried by electrons and (the same number of) holes. Current fluctuations are thus expected to be relevant. Indeed,

$$S_{22} = \frac{e^2}{h}\int_0^\infty dE[n_{1,e} + n_{1,h} + n_{3,e} + n_{3,h}$$
$$- (n_{1,e} + n_{1,h})(n_{3,e} + n_{3,h})].$$
(5)

At zero bias, this reduces to the usual Johnson-Nyquist noise $S_{22} = \frac{4e^2}{h}k_BT$, while for voltages $V_2 = V_4 = 0$, $V_1 = V_3 = V$ and $k_BT \ll eV$, we obtain the shot noise result $S_{22} = \frac{e^3}{h}|V|$, which is 4 times larger than the maximal expected shot noise due to a beam splitter of chiral electrons. This remarkable result can be explained by noting that in each scattering event both outgoing electrons and holes contribute to the charge fluctuations, while giving a zero average current as a consequence of the perfect electron-hole symmetry imposed by the Majorana conversion.

We would now like to discuss a second possibility to obtain a signature of Majorana fermions by adding a QPC to the previous setup; see Fig. 2(a). A novel feature will appear in the noise properties, which we want to study in the same spirit as in the previous section.
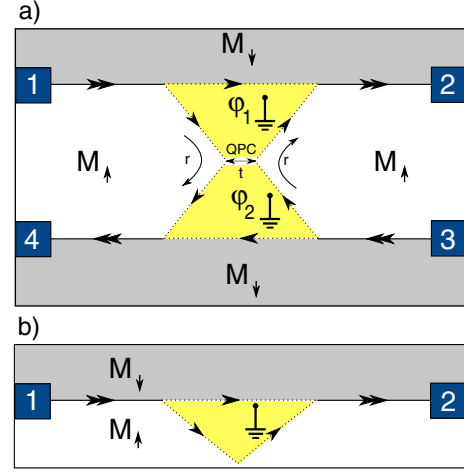

FIG. 2 (color online). (a) Modified Hanbury Brown–Twiss interferometer. Majorana excitations will propagate along the boundaries of the two triangular superconducting structures with phases φ_1, φ_2. An additional short gapped channel appears at the domain wall between the two superconducting regions, forming a quantum point contact characterized by reflection and transmission amplitudes r, t. The setup is similar to the one proposed in Ref. [7]. (b) In the fully transmissive case, $t = 1$, $r = 0$, the device splits into two separate Mach-Zehnder interferometers, one of which is shown here.

As explained in [7], the transmission and reflection amplitudes t, r of the QPC can be strongly tuned by altering the geometry of the QPC itself, or by changing the phase difference φ between the two superconducting parts. A narrow constriction would be dominated by direct tunneling and thus hardly sensitive to the phase difference. Therefore, the geometry we want to consider is closer to a line junction supporting a nonchiral Majorana channel on its own. By changing $\varphi = \varphi_1 - \varphi_2$ from $\varphi = 0$ to $\varphi = \pi$, the channel appearing at the interface of the two superconductors can be tuned from closed ($t \ll 1$) to fully open ($t \lesssim 1$) at zero energy. For intermediate values of the phase, the channel is gapped and the transmission amplitude strongly depends on energy.

We would first like to look at the limiting cases. For $t = 1$, $r = 0$, the upper and lower channels are not connected by the QPC. As a consequence, the setup effectively reduces to two independent copies of a Mach-Zehnder interferometer between terminals 1 and 2 (3 and 4) [see Fig. 2(b)]. The full current-current correlation matrix S_{MZ+MZ} for the outgoing leads is easy to obtain in that case: the cross-correlations vanish since they are not connected in any way, and the autocorrelations are given in Table I. For $t = 0$, the setup is equivalent to the HBT interferometer of the previous section, whose correlation matrix S_{HBT} is given by Eqs. (3) and (5). At intermediate values of t, we use the same formalism as for the HBT setup, taking the QPC into account in the scattering matrix:

TABLE I. Summary of conductance and noise properties of normal electron (as in Ref. [14] for the HBT setup) and Majorana interferometers at zero temperature. In the Mach-Zehnder (MZ) interferometer, 1(2) labels the incoming (outgoing) lead. In the Hanbury Brown–Twiss (HBT) interferometer, 1 and 3 (2 and 4) refer to the incoming (outgoing) leads.

	Normal	Majorana
$G_{12}^{\mathrm{MZ}}[e^2/h]$	$\frac{1}{2}[1 + \cos(2\pi\phi/\phi_0)]$	$\cos(2\pi\phi/\phi_0)$
$S_{22}^{\mathrm{MZ}}[e^3V/h]$	$\frac{1}{8}[1 - \cos(4\pi\phi/\phi_0)]$	$\frac{1}{2}[1 - \cos(4\pi\phi/\phi_0)]$
$G_{12}^{\mathrm{HBT}}[e^2/h]$	$1/4$	0
$S_{22}^{\mathrm{HBT}}[e^3V/h]$	$1/4$	1
$S_{24}^{\mathrm{HBT}}[e^3V/h]$	$-\frac{1}{4}[1 + \cos(2\pi\phi/\phi_0)]$	$-\cos(2\pi\phi/\phi_0)$

$$\begin{pmatrix} a_2 \\ b_2 \\ a_4 \\ b_4 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \eta_1 - t & \eta_1 + t & r & r \\ \eta_1 + t & \eta_1 - t & -r & -r \\ r & -r & -\eta_2 + t & \eta_2 + t \\ r & -r & \eta_2 + t & -\eta_2 + t \end{pmatrix}\begin{pmatrix} a_1 \\ b_1 \\ a_3 \\ b_3 \end{pmatrix}. \tag{6}$$

Here, $\eta_{1(2)}$ is the interferometric phase factor for Majorana fermions around the upper (lower) superconductor.

In this case, the average currents do not identically vanish. In fact the conductances $G_{12} = G_{34} = \frac{e^2}{h}|t|$ are proportional to the transmission *amplitude*. This allows to experimentally access the QPC properties. The two remaining conductances G_{14} and G_{32} still vanish.

We now focus on the quantities of interest, namely, the current-current correlations S_{22}, S_{44}, and S_{24}. Proceeding through the usual steps, the resulting 2×2 correlation matrix, for the two outgoing leads, at fixed energy can be decomposed as

$$S = R \times S_{\mathrm{HBT}} + T \times S_{\mathrm{MZ+MZ}}, \tag{7}$$

where $R = |r|^2$ and $T = |t|^2$ are the reflection and transmission probabilities of the QPC. The QPC effectively interpolates between the two limiting cases: surprisingly, there are no mixed terms proportional to RT; in other words, while there are the (auto and cross-correlation) noise terms related to the HBT and MZ interferometer present in the structure, there is no partition noise. This is one of the main results of our Letter and is deeply rooted in the Majorana nature of the excitations transported along the boundaries of the superconductor.

In the following we give an intuitive explanation of this remarkable feature of Eq. (7). Partition noise in the context of an electronic beam splitter is due to the transport of charge in discrete units. An incoming electron is coherently split into, e.g., two channels, and in a current measurement the electron will contribute to the current in one, and only one, outgoing channel. The splitting thereby induces current fluctuations proportional to the charge of the electron. Majorana fermions, on the other hand, fail to

generate electric current fluctuations since they are neutral. We thus believe that the absence of electronic partition noise predicted by Eq. (7) is a signature of channels supporting Majorana fermions. Importantly, this absence occurs while the QPC is proven to actually scatter the fermions because of the dependence on R and T.

Our results for the zero-temperature conductance and noise properties of normal electron and Majorana interferometers in a two-terminal (Mach-Zehnder) and four-terminal (Hanbury Brown–Twiss) setup are summed up in Table I.

In conclusion, we have analyzed a Hanbury Brown-Twiss type interferometer for Majorana fermions. We have calculated its conductance and noise properties. The sign of the cross-correlations of the outgoing currents of the interferometer is predicted to be positive if the parity of the number of vortices threading the superconductor is odd. Our main results are three signatures for the Majorana nature of the transport channels defined by domain walls between superconducting and magnetic regions placed on the surface of a three-dimensional topological insulator. On the one hand, the average charge current in the outgoing leads vanishes since there are symmetric probabilities for outgoing electrons or holes; see the discussion before Eq. (2). This vanishing conductance needs to be complemented by a check that the structure is functional, which is provided by the finite current autocorrelation. On the other hand, we find a finite zero-temperature shot noise at the output port of the interferometer even for a vanishing average current reflecting the finite fluctuations of the Majorana particle around charge neutrality. Finally, our calculations predict the absence of electronic partition noise in a quantum point contact, whereas the parameter dependence of the scattering matrix proves that the point contact actually scatters the fermions. These signatures will be an important help in verifying the existence of Majorana excitations in interferometric structures at the surface of topological insulators.

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