Analytic Solutions in Nonlinear Massive Gravity

Kazuya Koyama, Gustavo Niz, and Gianmassimo Tasinato

Institute of Cosmology and Gravitation, University of Portsmouth, Dennis Sciama Building, Portsmouth, PO1 3FX, United Kingdom

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We study spherically symmetric solutions in a covariant massive gravity model, which is a candidate for a ghost-free nonlinear completion of the Fierz-Pauli theory. There is a branch of solutions that exhibits the Vainshtein mechanism, recovering general relativity below a Vainshtein radius given by $(r_g m^2)^{1/3}$, where m is the graviton mass and r_g is the Schwarzschild radius of a matter source. Another branch of exact solutions exists, corresponding to de Sitter–Schwarzschild spacetimes where the curvature scale of de Sitter space is proportional to the mass squared of the graviton.

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Introduction.—It is a fundamental question whether there exists a consistent covariant theory for massive gravity, where the graviton acquires a mass and leads to a large distance modification of general relativity (GR). The quest for massive gravity dates back to the work by Fierz and Pauli (FP) in 1939 [1]. They considered a mass term for linear gravitational perturbations, which explicitly breaks the gauge invariance of GR. As a result, there exist 5 degrees of freedom in the graviton, instead of the two found in GR. There have been intensive studies on what happens going beyond the linearized theory. In 1972, Boulware and Deser (BD) found that, at the nonlinear level, there appears a sixth mode in the graviton that becomes a ghost in the FP model [2]. This problem was reexamined using the effective theory approach [3], where additional (Stückelberg) fields were introduced to restore the gauge invariance, and whose scalar part represents the helicity-0 mode of the graviton. In the FP model, the scalar acquires nonlinear interaction terms that contain more than two time derivatives, signaling the existence of the ghost.

The Stückelberg approach also sheds light on the other puzzle in the FP gravity: if one linearizes the system, the solutions in the FP theory do not reduce to GR solutions in the massless limit. This is known as the van Dam, Veltman, Zakharov (vDVZ) discontinuity [4,5]. However, in this massless limit the scalar mode becomes strongly coupled and one cannot linearize the system. Therefore, due to strong coupling, the scalar interaction is shielded and GR can be recovered. This is known as the Vainshtein mechanism [6]. The strong coupling scale in the FP model is identified as $\Lambda_5 = (m^4 M_{\rm Pl})^{1/5}$, where $M_{\rm Pl}$ is the Planck scale and m is the graviton mass. This scale is tightly connected with the nonlinear interactions of the scalar mode that contain more than two time derivatives. In the decoupling limit, where $m \to 0$ and $M_{\rm Pl} \to \infty$, while the strong coupling scale Λ_5 is kept fixed, one obtains an effective theory for the scalar mode, where it is possible to study the consistency of the theory in more detail.

Until recently, it was believed that there is no consistent way to extend the FP model [7,8] to get a ghost-free model at all orders. A breakthrough came with a 5D braneworld model known as the Dvali-Gabadadze-Porrati (DGP) model [9]. In this model there appears a continuous tower of massive gravitons from a four-dimensional perspective. The nonlinear interactions of the helicity-0 mode of massive gravitons contain no more than two derivatives, which is crucial to avoid the BD ghost. Because of this fact, the strong coupling scale in this theory is given by $\Lambda_3 = (m^2 M_{\rm Pl})^{1/3}$ instead of Λ_5, where $m = r_c^{-1}$ and r_c is a crossover scale between 5D and 4D gravity [10,11]. Further studies have considered more general nonlinear interactions which contain no more than two derivatives. In 4D, only a finite number of terms satisfy this condition; these are dubbed Galileon terms because of a symmetry under field transformations of the form $\partial_\mu \pi \to \partial_\mu \pi + c_\mu$ [12]. Reference [13] constructed the extension of the FP theory that gives the Galileon terms in the decoupling limit, by choosing the correct parameters in the Lagrangian up to quintic order in perturbations. Reference [14] proposed a covariant nonlinear action that automatically ensures this property to all orders, which we will discuss below.

A remaining crucial question is whether this property, holding in the decoupling limit, is sufficient to ensure the absence of the BD ghost or not. In Ref. [14], it was shown that there is no BD ghost in the decoupling limit to all orders in perturbation theory, but only up to and including quartic order away from this limit. However, it is very hard to show the absence of the BD ghost at all orders if one starts from Minkowski and studies nonlinear interactions perturbatively. Therefore, it is important to obtain nonperturbative background solutions in this theory and study fluctuations around them. Moreover, it is interesting to find solutions in this covariant nonlinear theory that can describe features of the observed Universe. These are the topics of the present work.

Covariant nonlinear massive gravity.—We first construct the action for the generalized FP model based on Refs. [13,14]. We define the tensor $H_{\mu\nu}$ as a covariantization of the metric perturbations:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \equiv H_{\mu\nu} + \eta_{\alpha\beta}\partial_\mu\phi^\alpha\partial_\nu\phi^\beta. \quad (1)$$

The Stückelberg fields $\phi^\alpha = (x^\alpha - \pi^\alpha)$ transform as scalars, while $\eta_{\alpha\beta}$ corresponds to a nondynamical background metric that is needed to define the potential, which is assumed to be the Minkowski metric. The covariant tensor $H_{\mu\nu}$ can then be expanded as

$$\begin{aligned} H_{\mu\nu} &= h_{\mu\nu} + \eta_{\beta\nu}\partial_\mu\pi^\beta + \eta_{\alpha\mu}\partial_\nu\pi^\alpha - \eta_{\alpha\beta}\partial_\mu\pi^\alpha\partial_\nu\pi^\beta, \\ &\equiv h_{\mu\nu} - \mathcal{Q}_{\mu\nu}, \end{aligned} \quad (2)$$

and under the coordinate transformation $x^\mu \to x^\mu + \xi^\mu$, π^μ transforms as

$$\pi^\mu \to \pi^\mu + \xi^\mu. \quad (3)$$

Before proceeding with the massive gravity theory, indices are raised and lowered, from now on, with the dynamical metric $g_{\mu\nu}$; for example, $H^\mu_\nu = g^{\mu\rho}H_{\rho\nu}$.

We define a new tensor $\mathcal{K}^\nu_\mu$ as

$$\mathcal{K}^{\ \nu}_{\mu} \equiv \delta^\nu_\mu - \left(\sqrt{g^{-1}[g-H]}\right)^\nu_\mu, \quad (4)$$

where the square root is formally understood as $\sqrt{A}^\alpha_\mu\sqrt{A}^\nu_\alpha = A^\nu_\mu$. This allows us to represent the complete potential for gravitational interactions as

$$\begin{aligned} \mathcal{L} &= \frac{M^2_{\rm Pl}}{2}\sqrt{-g}(R - m^2\mathcal{U}), \\ \mathcal{U} &= [\mathrm{tr}(\mathcal{K}^2) - (\mathrm{tr}\mathcal{K})^2]. \end{aligned} \quad (5)$$

By expanding the potential in $H_{\mu\nu}$, we get an infinite sum of interaction terms for $H_{\mu\nu}$, with the FP term at the lowest order.

In order to study exact solutions associated with the previous Lagrangian, it is convenient to express $\mathcal{K}$ in terms of matrices, namely,

$$\mathcal{K} = \mathbb{I} - \sqrt{g^{-1}[\eta + \mathcal{Q}]}, \quad (6)$$

where $\mathbb{I}$ denotes the identity matrix, and we have used $H_{\mu\nu} = g_{\mu\nu} - (\eta_{\mu\nu} + \mathcal{Q}_{\mu\nu})$. The potential in four dimensions then reads

$$\begin{aligned} \mathcal{U} = \mathrm{tr}g^{-1}[\eta + \mathcal{Q}] - 12 + \mathrm{tr}\sqrt{g^{-1}[\eta + \mathcal{Q}]} \\ \times\left(6 - \mathrm{tr}\sqrt{g^{-1}[\eta + \mathcal{Q}]}\right). \end{aligned} \quad (7)$$

In general, the task is to calculate the trace of $\sqrt{g^{-1}[\eta + \mathcal{Q}]}$. Given that $g^{-1}[\eta + \mathcal{Q}]$ is a square matrix, the Schur decomposition theorem ensures that it can be expressed as

$$g^{-1}[\eta + \mathcal{Q}] = \mathcal{T} D \mathcal{T}^{-1}, \quad (8)$$

for some unitary matrix $\mathcal{T}$ and an upper triangular matrix D. The diagonal entries of D are the eigenvalues of $g^{-1}[\eta + \mathcal{Q}]$, and we call these eigenvalues $\lambda_1, \ldots, \lambda_4$. Then, since $\sqrt{g^{-1}[\eta + \mathcal{Q}]} = \mathcal{T}\sqrt{D}\mathcal{T}^{-1}$, one can express the traces in the formulas above, in terms of eigenvalues, as

$$\mathrm{tr}\, g^{-1}[\eta + \mathcal{Q}] = \sum_i \lambda_i, \quad \mathrm{tr}\sqrt{g^{-1}[\eta + \mathcal{Q}]} = \sum_i \sqrt{\lambda_i}. \quad (9)$$

Plugging these expressions into the potential, Eq. (7), we find the following expression for $\mathcal{U}$:

$$\mathcal{U} = \sum_i \lambda_i + \left(\sum_j \sqrt{\lambda_j}\right)\left(6 - \sum_i \sqrt{\lambda_i}\right) - 12. \quad (10)$$

Spherically symmetric configurations.—We now focus on analyzing properties of spherically symmetric configurations in this setup. We start by considering static configurations in the unitary gauge, $\pi^\mu = 0$ (see Ref. [15] for spherical symmetric solutions in the FP theory). The most general form of the metric respecting spherical symmetry is

$$ds^2 = -C(r)dt^2 + A(r)dr^2 + 2D(r)dtdr + B(r)d\Omega^2, \quad (11)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. We choose to write the nondynamical flat metric as $ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$. Notice that in GR one can set $B(r) = r^2$ by a coordinate transformation, but this is not possible here, since we have already fixed the gauge completely. In order to simplify the analysis, it is convenient to define the combination $\Delta(r) = A(r)C(r) + D^2(r)$. We plug the previous metric into the Einstein equations

$$G_{\mu\nu} = T^{\mathcal{U}}_{\mu\nu}, \quad (12)$$

where the energy momentum tensor from the potential $\mathcal{U}$ of Eq. (10) is defined as $T^{\mathcal{U}}_{\mu\nu} = \frac{m^2}{\sqrt{-g}}\frac{\delta\sqrt{-g}\mathcal{U}}{\delta g^{\mu\nu}}$. The Einstein tensor $G_{\mu\nu}$ satisfies the identity $D(r)G_{tt} + C(r)G_{tr} = 0$, which implies the algebraic constraint

$$\begin{aligned} 0 &= D(r)T^{\mathcal{U}}_{tt} + C(r)T^{\mathcal{U}}_{tr} \\ &= m^2\frac{D(r)[2r - 3\sqrt{B(r)}]\sqrt{\Delta(r)}}{\sqrt{B(r)}[A(r) + C(r) + 2\sqrt{\Delta(r)}]^{1/2}}. \end{aligned} \quad (13)$$

The previous condition can be satisfied in two ways, which lead to two different branches of solutions. We can either set $D(r) = 0$, and focus on diagonal metrics, or alternatively, set $B(r) = 4r^2/9$. The fact that there are two branches of solutions indicates that, unlike in GR where Birkhoff's theorem holds, there is no uniqueness theorem for spherically symmetric solutions in this theory. We analyze the two branches in turn.

Diagonal-metric branch: Asymptotically flat solutions.—The case of diagonal metrics, $D(r) = 0$ in Eq. (11), leads to equations which in general cannot be solved analytically. Thus, we will analyze them perturbatively, showing that they lead to asymptotically flat solutions (see Ref. [16]

for a more detailed discussion on this branch of solutions). We expand the functions A, C, and B as

$$A(r) = \frac{1}{1+f}, \qquad C(r) = (1+n)^2, \qquad B(r) = \frac{r^2}{(1+h)^2}, \tag{14}$$

and truncate the field equations to first order in n, f, and h. It is more convenient to introduce a new radial coordinate $\rho = \sqrt{B(r)}$, so that the linearized metric is expressed as

$$ds^2 = -(1+2n)dt^2 + (1-\tilde{f})d\rho^2 + \rho^2 d\Omega^2, \tag{15}$$

where $\tilde{f} = f - 2h - 2\rho h'$, and a prime denotes a derivative with respect to ρ. In this new coordinate, the solutions for n and $\tilde{f}$ are then given by

$$2n = -\frac{8GM}{3\rho} e^{-m\rho}, \quad \tilde{f} = -\frac{4GM}{3\rho}(1+m\rho)e^{-m\rho}, \tag{16}$$

where we fix the integration constant so that M is the mass of a point particle at the origin and $8\pi G = M_{\rm Pl}^{-2}$. See the right-hand plot of Fig. 1 for the general behavior of these solutions. Notice that these configurations, as anticipated, are asymptotically flat and exhibit the vDVZ discontinuity; i.e., they do not agree with the GR solutions ($2n_{\rm GR} = \tilde{f}_{\rm GR} = -2GM/\rho$) in the limit $m \to 0$. However, in order to understand what really happens in this limit, one should take into account the nonlinear behavior of h. Let us consider scales below the Compton wavelength $m\rho \ll 1$, and at the same time ignore higher order terms in GM. Under these approximations, the equations of motion can still be truncated to linear order in $\tilde{f}$ and n, but since h is not necessarily small, we keep all nonlinear terms in h. Then we obtain the following equations:

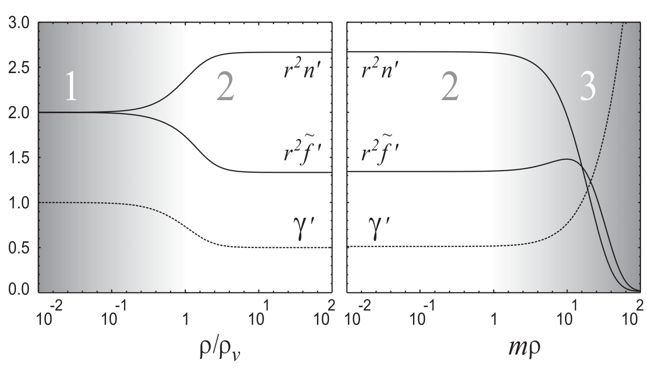

FIG. 1. Numerical solution for $\partial_r \tilde{f} = \tilde{f}'$, $\partial_r n = n'$, and the quotient $\gamma' \equiv \tilde{f}'/2n'$ around the Vainshtein radius ρ_v (left) and the Compton wavelength $\rho \sim 1/m$ (right). Region 1 (2) shows how GR solutions are (not) recovered inside (outside) the Vainshtein radius ρ_V. Region 3 shows the asymptotic decay of the linear solutions [Eq. (16)]. Here, $GM = 1$.

$$\begin{aligned} 2\rho n' &= \frac{2GM}{\rho} - (m\rho)^2 h, \\ \tilde{f} &= -2\frac{GM}{\rho} - (m\rho)^2(h - h^2), \\ \frac{GM}{\rho} &= -(m\rho)^2\left(\frac{3}{2}h - 3h^2 + h^3\right). \end{aligned} \tag{17}$$

We should stress that these are exact equations in the limit $m\rho \ll 1$, $GM/\rho \ll 1$; i.e., there are no higher order corrections in h. For large radial ρ values, one can linearize the equations in h, recovering the solution in Eq. (16), to first order in $m\rho$. On the other hand, the Vainshtein mechanism applies, and below the so-called Vainshtein radius, $\rho_V = (GMm^{-2})^{1/3}$, h becomes larger than 1 and the nonlinear terms in h become important, recovering GR close to a matter source. Actually, for $\rho \ll \rho_V$ the solution for h is simply given by $|h| = \rho_V/\rho \gg 1$. The latter solution for h and Eq. (17) imply

$$\begin{aligned} 2\rho n' &= \frac{2GM}{\rho}\left[1 + \frac{1}{2}\left(\frac{\rho}{\rho_V}\right)^2\right], \\ \tilde{f} &= -\frac{2GM}{\rho}\left[1 - \frac{1}{2}\left(\frac{\rho}{\rho_V}\right)\right]. \end{aligned} \tag{18}$$

Therefore, corrections to the GR solutions are indeed small for $\rho < \rho_V$, as shown in the left-hand plot of Fig. 1.

The Vainshtein mechanism becomes also transparent in a nonunitary gauge. Indeed, by performing the coordinate transformation $\rho = \sqrt{B(r)}$, we excite the ρ component of the Stückelberg field [see Eq. (3)], $\pi^\rho = -\rho h$. Thus the strong coupling nature of h is encoded in π^ρ in this coordinate. It is possible to construct an effective theory for this Stückelberg field in the so-called decoupling limit [13]: first we introduce a scalar so that $\pi_\mu = \partial_\mu \pi/\Lambda_3^3$, where $\Lambda_3^3 = m^2 M_{\rm Pl}$. Then the covariantization of metric perturbations $H_{\mu\nu}$ is written as

$$H_{\mu\nu} = h_{\mu\nu} + \frac{2}{M_{\rm Pl} m^2}\Pi_{\mu\nu} - \frac{1}{M_{\rm Pl}^2 m^4}\Pi^2_{\mu\nu}, \tag{19}$$

where $\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$ and $\Pi^2_{\mu\nu} = \Pi_{\mu\alpha}\Pi^\alpha_\nu$. Formally, the decoupling limit is achieved by taking $m \to 0$ and $M_{\rm Pl} \to \infty$, but keeping Λ_3 fixed. By substituting Eq. (19) into the action, one can show that the kinetic terms of π become total derivatives and a mixing appears between $h_{\mu\nu}$ and π, which can be diagonalized using the definition

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + \frac{\pi}{M_{\rm Pl}}\eta_{\mu\nu} - \frac{1}{\Lambda_3^3 M_{\rm Pl}}\partial_\mu\pi\partial_\nu\pi. \tag{20}$$

The Lagrangian is then written as

$$\mathcal{L} = \mathcal{L}_{\rm GR}(\hat{h}_{\mu\nu}) + \frac{3}{2}\pi\Box\pi - \frac{3}{2\Lambda_3^3}(\partial\pi)^2\Box\pi + \frac{1}{2\Lambda_3^6}(\partial\pi)^2([\Pi^2] - [\Pi]^2) + \frac{5}{2\Lambda_3^9}(\partial\pi)^2([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]), \quad (21)$$

where $[\Pi] = \Pi^{\mu}_{\mu}$, $[\Pi^2] = \Pi^{\mu\nu}\Pi_{\mu\nu}$, $[\Pi^3] = \Pi^{\mu\nu}\Pi_{\nu\alpha}\Pi^{\alpha}_{\mu}$, and $\mathcal{L}_{\rm GR}$ is the linearized Einstein-Hilbert action for $\hat{h}_{\mu\nu}$. The terms containing the scalar field π are known as Galileon terms, which give rise to the second order differential equations, as explained in the Introduction. For the spherically symmetric case, the equation of motion for π simplifies to [12,17]

$$3\left(\frac{\pi'}{\rho}\right) + \frac{6}{\Lambda_3^3}\left(\frac{\pi'}{\rho}\right)^2 + \frac{2}{\Lambda_3^6}\left(\frac{\pi'}{\rho}\right)^3 = \frac{M}{4\pi M_{\rm Pl}\rho^3}, \quad (22)$$

where the integration constant is again chosen so that M is a mass of a particle at the origin. Using the relation between π and h, $h = -\pi'/(m^2 M_{\rm Pl}\rho)$, we can show that the solutions for $\tilde{f}$, n, and h given by Eq. (17) agree with the solutions Eqs. (20) and (22).

We have shown that the weak field solutions for the metric Eq. (11) with $D(r) = 0$ have three phases (see Fig. 1). On the largest scales, $m^{-1} \ll \rho$, beyond the Compton wavelength, the gravitational interactions decay exponentially due to the mass of graviton; see Eq. (16) and region 3 in Fig. 1. In the intermediate region $\rho_V < \rho < m^{-1}$, we obtain the $1/r$ gravitational potential, but Newton's constant is rescaled $G \to 4G/3$. Moreover, the post-Newtonian parameter γ is $\gamma = \tilde{f}/(2n) = (1/2)(1 + m\rho)$, which reduces to $\gamma = 1/2$ in the $m \to 0$ limit, instead of $\gamma = 1$ of GR, showing the vDVZ discontinuity (see region 2 in Fig. 1). Finally, below the Vainshtein radius $\rho < \rho_V$, the GR solution is recovered due to the strong coupling of the π mode [see Eq. (18) and region 1 in Fig. 1]. This background solution provides us with a testing ground for the BD ghost. Instead of expanding the action in $H_{\mu\nu}$ around the Minkowski spacetime perturbatively, one can study linear perturbations around this nonperturbative solution using the complete potential Eq. (7). In order to obtain the fully nonlinear solution, a numerical approach is necessary. In the next section, we consider the second branch of solutions for this theory, which can instead be obtained analytically.

Nondiagonal-metric branch: de Sitter–Schwarzschild solutions.—Next, we analyze the second branch of vacuum solutions that solve Eq. (13), where $B(r) = 4r^2/9$. Interestingly, this branch leads to asymptotically de Sitter configurations. There is another identity $C(r)T^{\mathcal{U}}_{rr} + A(r)T^{\mathcal{U}}_{tt} = 0$, which leads to the condition

$$\Delta(r) = A(r)C(r) + D^2(r) \equiv \Delta_0 = \text{const.} \quad (23)$$

The remaining Einstein equations provide the following unique solution (see Ref. [16] for detailed derivations):

$$A(r) = \frac{9\Delta_0}{4}[p(r) + \alpha + 1], \qquad B(r) = \frac{4}{9}r^2,$$
$$C(r) = \frac{9\Delta_0}{4}[1 - p(r)], \qquad D(r) = \frac{9}{4}\Delta_0\sqrt{p(r)[p(r) + \alpha]}, \quad (24)$$

where

$$p(r) = \frac{2\mu}{r} + \frac{m^2 r^2}{9}, \qquad \alpha = \frac{16}{81\Delta_0} - 1, \quad (25)$$

with arbitrary μ and Δ_0. This solution is similar to that in [18], up to numerical factors. Notice that this configuration depends on two integration constants. A sufficient condition to ensure that $D(r)$ is real is to choose $\mu \ge 0$ and $0 < \sqrt{\Delta_0} \le 4/9$. The form of metric coefficients as in Eq. (24) do not allow a manifest comparison with de Sitter spacetime, since we have already chosen the unitary gauge and cannot do a further coordinate transformation without exciting components of π^{μ}. However, if we allow for a vector π^{μ} of the form $\pi^{\mu} = (\pi_0(r), 0, 0, 0)$, the metric can be rewritten in a diagonal form as

$$ds^2 = -C(r)dt^2 + \tilde{A}(r)dr^2 + B(r)d\Omega^2. \quad (26)$$

Then we can write down the action in terms of C, $\tilde{A}$, B, and π_0, considering them as fields. It is possible to show that the following configuration solves the corresponding equations of motion,

$$\tilde{A}(r) = \frac{4}{9}\frac{1}{1 - p(r)}, \quad \pi_0'(r) = -\frac{\sqrt{p(r)[p(r) + \alpha]}}{1 - p(r)}, \quad (27)$$

while $C(r)$ and $B(r)$ are the same as in Eq. (24). The resulting metric has then a manifestly de Sitter–Schwarzschild form by making a time rescaling $t \to (4/9\Delta_0^{1/2})t$. However, we should note that this time rescaling cannot be done without introducing an additional time dependent contribution to π_0. As expected, the metric in Eq. (26) can be obtained by making the following transformation of the time coordinate $d\tilde{t} \equiv dt + \pi_0' dr$ to the metric (11); this produces a nonzero time component of π^{μ} that does not vanish even in the $m \to 0$ limit for any allowed value of Δ_0. There are two integration constants, μ and Δ_0, in this solution. In GR, μ corresponds to the mass of a source at the origin, but a careful analysis including a matter source is necessary to fully understand the role of these integration constants. Note that there is an apparent singularity at the horizon $p(r) = 1$, both for the metric and for π_0.

We can further make a coordinate transformation at the expense of exciting further components of π^{μ}. For example, by setting $\mu = 0$ and making the following coordinate transformations $t = F_t(\tau, \rho)$, $r = F_r(\tau, \rho)$ with

$$F_t(\tau,\rho) = \frac{4}{3\Delta_0^{1/2} m} \operatorname{arctanh}\left(\frac{\sinh(m\tau/2) + (m^2\rho^2/8)e^{m\tau/2}}{\cosh(m\tau/2) - (m^2\rho^2/8)e^{m\tau/2}}\right),$$
$$F_r(\tau,\rho) = \frac{3}{2}\rho e^{m\tau/2}, \tag{28}$$

the metric becomes that of flat slicing of de Sitter,

$$ds^2 = -d\tau^2 + e^{m\tau}(d\rho^2 + \rho^2 d\Omega^2), \tag{29}$$

where the Hubble parameter is given by $m/2$. The Stückelberg fields π^μ are now given by $\pi^\mu = (\pi^\tau(\tau,\rho), \pi^\rho(\tau,\rho), 0, 0)$, $\pi^\tau = \pi_0 + \tau - F_t(\tau,\rho)$, $\pi^\rho = \rho - F_r(\tau,\rho)$ This is an interesting solution in which the acceleration of the Universe is determined by the graviton mass and the Hubble parameter is given by $m/2$. For $\Delta_0 = 16/81$, this solution reduces to the "self-accelerating" solution obtained in the decoupling limit in Ref. [19].

Conclusions.—The solutions obtained in the nonlinear covariant massive gravity are remarkably similar to those in the DGP braneworld model including the existence of the self-accelerating de Sitter solution without cosmological constant [20] although there are differences in detail. There are a number of important issues. Firstly, we should confirm that there is no BD ghost in this theory by studying perturbations around the nonperturbative solution obtained in this Letter. In the DGP model, the self-accelerating solution suffers from a ghost instability [10,11,21], which is related to the ghost in the FP theory on a de Sitter background. Secondly, it is crucial to study the stability of the de Sitter solution in this model. In fact Ref. [19] showed that there exists a ghost in this self-accelerating background in the decoupling limit for a particular value of the second integration constant Δ_0. They argue that this ghost can be cured by adding higher order corrections in $\mathcal{K}$ to the potential. Our formalism is ready to be applied to this extended model. However, we believe a more complete analysis of perturbations about our exact solution is needed. Once these issues are clarified, the massive gravity model presented here provides an interesting playground to study large distance modifications of general relativity.

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