Robustness of Non-Gaussian Entanglement against Noisy Amplifier and Attenuator Environments

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The recently developed Kraus representation for bosonic Gaussian channels is employed to study analytically the robustness of non-Gaussian entanglement against evolution under noisy attenuator and amplifier environments, and compare it with the robustness of Gaussian entanglement. Our results show that some non-Gaussian states with one ebit of entanglement are more robust than all Gaussian states, even the ones with arbitrarily large entanglement, a conclusion of direct consequence to the recent conjecture by Allegra *et al.* [Phys. Rev. Lett. **105**, 100503 (2010)].

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Early developments in quantum information technology of continuous variable (CV) systems largely concentrated on Gaussian states and Gaussian operations [1]. The symplectic group of linear canonical transformations [2] is available as a handy and powerful tool in this Gaussian scenario, leading to an elegant classification of permissible Gaussian processes or channels [3]. The fact that states in the non-Gaussian sector could offer an advantage for several quantum information tasks has resulted more recently in considerable interest in non-Gaussian states, both experimental [4] and theoretical [5].

Since noise is unavoidable in any actual realization of these information processes [6], robustness of entanglement and other nonclassical effects against noise becomes an important consideration. Allegra *et al.* [7] have thus studied the evolution of what they call photon number entangled states (PNES) (i.e., two-mode states of the form $|\psi\rangle = \sum c_n|n, n\rangle$) in a noisy attenuator environment. They conjectured based on numerical evidence that, for a given energy, Gaussian entanglement is more robust than the non-Gaussian ones. Earlier Agarwal *et al.* [8] had shown that entanglement of the NOON state is more robust than Gaussian entanglement in the quantum-limited amplifier environment. More recently, Nha *et al.* [9] have shown that nonclassical features, including entanglement, of several non-Gaussian states survive a quantum-limited amplifier environment much longer than Gaussian entanglement. Since the conjecture of Ref. [7] refers to the noisy environment and the analysis in Refs. [8,9] to the noiseless or quantum-limited case, the conclusions of the latter do not necessarily amount to refutation of the conjecture of Ref. [7]. Indeed, Adesso has argued very recently [10] that the well-known extremality [11] of Gaussian states implies proof and rigorous validation of the conjecture of Ref. [7].

In the present work we employ the recently developed [12] Kraus representation of bosonic Gaussian channels to study analytically the behavior of non-Gaussian states in noisy attenuator and amplifier environments. Both NOON states and a simple form of PNES are considered. Our results show conclusively that the conjecture of Ref. [7] is too strong to be maintainable.

Noisy attenuator environment.—Under evolution through a noisy attenuator channel $\mathcal{C}_1(\kappa, a)$, $\kappa \leq 1$, an input state $\rho^{\rm in}$ with characteristic function (CF) $\chi_W^{\rm in}(\xi)$ goes to state $\rho^{\rm out}$ with CF

$$\chi_W^{\rm out}(\xi) = \chi_W^{\rm in}(\kappa\xi)e^{-(1/2)(1-\kappa^2+a)|\xi|^2}, \tag{1}$$

where κ is the attenuation parameter [3]. In this notation, quantum-limited channels [9] correspond to $a = 0$, and so the parameter a stands for the additional Gaussian noise. Thus, $\rho^{\rm in}$ is taken under the two-sided symmetric action of $\mathcal{C}_1(\kappa, a)$ to $\rho^{\rm out} = \mathcal{C}_1(\kappa, a) \otimes \mathcal{C}_1(\kappa, a)(\rho^{\rm in})$ with CF

$$\chi_W^{\rm out}(\xi_1, \xi_2) = \chi_W^{\rm in}(\kappa\xi_1, \kappa\xi_2)e^{-(1/2)(1-\kappa^2+a)(|\xi_1|^2+|\xi_2|^2)}. \tag{2}$$

To test for separability of $\rho^{\rm out}$ we may implement the partial transpose test on $\rho^{\rm out}$ in the Fock basis or on $\chi_W^{\rm out}(\xi_1, \xi_2)$. The choice could depend on the state.

Consider first the Gaussian case, and, in particular, the two-mode squeezed state $|\psi(\mu)\rangle = {\rm sech}\mu \sum_{n=0}^{\infty} \times \tanh^n\mu|n, n\rangle$ with variance matrix $V_{\rm sq}(\mu)$. Under the two-sided action of noisy attenuator channels $\mathcal{C}_1(\kappa, a)$, the output two-mode Gaussian state $\rho^{\rm out}(\mu) = \mathcal{C}_1(\kappa, a) \otimes \mathcal{C}_1(\kappa, a)(|\psi(\mu)\rangle\langle\psi(\mu)|)$ has variance matrix

$$V^{\rm out}(\mu) = \kappa^2 V_{\rm sq}(\mu) + (1 - \kappa^2 + a)\mathbb{1}_4,$$
$$V_{\rm sq}(\mu) = \begin{pmatrix} c_{2\mu}\mathbb{1}_2 & s_{2\mu}\sigma_3 \\ s_{2\mu}\sigma_3 & c_{2\mu}\mathbb{1}_2 \end{pmatrix}, \tag{3}$$

where $c_{2\mu} = \cosh 2\mu$, $s_{2\mu} = \sinh 2\mu$. Note that our variance matrix differs from that of some authors by a factor 2; in particular, the variance matrix of vacuum is the unit matrix in our notation. Partial transpose test [13] shows that $\rho^{\rm out}(\mu)$ is separable iff $a \geq \kappa^2(1-e^{-2\mu})$. The "additional noise" a required to render $\rho^{\rm out}(\mu)$ separable is an increasing function of the squeeze (entanglement) parameter μ and saturates at κ^2. In particular, $|\psi(\mu_1)\rangle$, $\mu_1 \approx 0.5185$ corresponding to one ebit of entanglement is rendered separable when $a \geq \kappa^2(1-e^{-2\mu_1})$. For $a \geq \kappa^2$, $\rho^{\rm out}(\mu)$ is separable, independent of the initial squeeze parameter μ. Thus $a = \kappa^2$ is the additional noise that renders separable all Gaussian states.

Behavior of non-Gaussian entanglement may be handled directly in the Fock basis using the recently developed Kraus representation of Gaussian channels [12]. In this basis quantum-limited attenuator $\mathcal{C}_1(\kappa;0)$, $\kappa \leq 1$ and quantum-limited amplifier $\mathcal{C}_2(\kappa;0)$, $\kappa \geq 1$ are described, respectively, by Kraus operators [12]

$$B_\ell(\kappa) = \sum_{m=0}^{\infty} \sqrt{{}^{m+\ell}C_\ell}\left(\sqrt{1-\kappa^2}\right)^\ell \kappa^m |m\rangle\langle m+\ell|,$$
$$A_\ell(\kappa) = \frac{1}{\kappa}\sum_{m=0}^{\infty} \sqrt{{}^{m+\ell}C_\ell}\left(\sqrt{1-\kappa^{-2}}\right)^\ell \frac{1}{\kappa^m}|m+\ell\rangle\langle m|,$$

$\ell = 0, 1, 2, \cdots$. In either case, the noisy channel $\mathcal{C}_j(\kappa;a)$, $j = 1, 2$ can be realized in the form $\mathcal{C}_2(\kappa_2;0)\circ\mathcal{C}_1(\kappa_1;0)$, so that the Kraus operators for the noisy case is simply the product set $\{A_{\ell'}(\kappa_2)B_\ell(\kappa_1)\}$. Indeed, the composition rule $\mathcal{C}_2(\kappa_2;0)\circ\mathcal{C}_1(\kappa_1;0) = \mathcal{C}_1(\kappa_2\kappa_1;2(\kappa_2^2-1))$ or $\mathcal{C}_2(\kappa_2\kappa_1;2\kappa_2^2(1-\kappa_1^2))$ according as $\kappa_2\kappa_1 \leq 1$ or $\kappa_2\kappa_1 \geq 1$ implies that the noisy attenuator $\mathcal{C}_1(\kappa;a)$, $\kappa \leq 1$ is realized by the choice $\kappa_2 = \sqrt{1+a/2} \geq 1$, $\kappa_1 = \kappa/\kappa_2 \leq \kappa \leq 1$, and the noisy amplifier $\mathcal{C}_2(\kappa;a)$, $\kappa \geq 1$ by $\kappa_2 = \sqrt{\kappa^2+a/2} \geq \kappa \geq 1$, $\kappa_1 = \kappa/\kappa_2 \leq 1$ [12]. Note that one goes from realization of $\mathcal{C}_1(\kappa;a)$, $\kappa \leq 1$ to that of $\mathcal{C}_2(\kappa;a)$, $\kappa \geq 1$ simply by replacing $(1+a/2)$ by $(\kappa^2+a/2)$; this fact will be exploited later.

Under the action of $\mathcal{C}_j(\kappa;a) = \mathcal{C}_2(\kappa_2;0)\circ\mathcal{C}_1(\kappa_1;0)$, $j = 1, 2$ the elementary operators $|m\rangle\langle n|$ go to

$$\begin{aligned}\mathcal{C}_2(\kappa_2;0)&\circ\mathcal{C}_1(\kappa_1;0)(|m\rangle\langle n|)\\ &= \kappa_2^{-2}\sum_{j=0}^{\infty}\sum_{\ell=0}^{\min(m,n)}[{}^{m-\ell+j}C_j\,{}^{n-\ell+j}C_j\,{}^mC_\ell\,{}^nC_\ell]^{1/2}\\ &\quad\times(\kappa_2^{-1}\kappa_1)^{(m+n-2\ell)}(1-\kappa_2^{-2})^j(1-\kappa_1^2)^\ell|m-\ell+j\rangle\\ &\quad\times\langle n-\ell+j|. \qquad (4)\end{aligned}$$

Substitution of $\kappa_2 = \sqrt{1+a/2}$, $\kappa_1 = \kappa/\kappa_2$ gives realization of $\mathcal{C}_1(\kappa;a)$, $\kappa \leq 1$ while $\kappa_2 = \sqrt{\kappa^2+a/2}$, $\kappa_1 = \kappa/\kappa_2$ gives that of $\mathcal{C}_2(\kappa;a)$, $\kappa \geq 1$.

As our first non-Gaussian example we study the NOON state $|\psi\rangle = (|n0\rangle + |0n\rangle)/\sqrt{2}$ with density matrix

$$\begin{aligned}\rho = \tfrac{1}{2}(&|n\rangle\langle n|\otimes|0\rangle\langle 0| + |n\rangle\langle 0|\otimes|0\rangle\langle n|\\ &+ |0\rangle\langle n|\otimes|n\rangle\langle 0| + |0\rangle\langle 0|\otimes|n\rangle\langle n|). \qquad (5)\end{aligned}$$

The output state $\rho^{\rm out} = \mathcal{C}_1(\kappa;a)\otimes\mathcal{C}_1(\kappa;a)(\rho)$ can be detailed in the Fock basis through use of Eq. (4).

To test for inseparability, we project $\rho^{\rm out}$ onto the 2×2 subspace spanned by the four bipartite vectors $\{|00\rangle, |0n\rangle, |n0\rangle, |nn\rangle\}$, and test for entanglement in this subspace; this simple test proves sufficient for our purpose. The matrix elements of interest are $\rho^{\rm out}_{00,00}$, $\rho^{\rm out}_{nn,nn}$, and $\rho^{\rm out}_{0n,n0} = \rho^{\rm out*}_{n0,0n}$. Negativity of $\delta_1(\kappa,a) \equiv \rho^{\rm out}_{00,00}\rho^{\rm out}_{nn,nn} - |\rho^{\rm out}_{0n,n0}|^2$ will prove for $\rho^{\rm out}$ not only NPT (nonpositive under partial transpose) entanglement, but also one-copy distillability [14].

To evaluate $\rho^{\rm out}_{00,00}$, $\rho^{\rm out}_{0n,n0}$, and $\rho^{\rm out}_{nn,nn}$, it suffices to evolve the four single-mode operators $|0\rangle\langle 0|$, $|0\rangle\langle n|$, $|n\rangle\langle 0|$, and $|n\rangle\langle n|$ through the noisy attenuator $\mathcal{C}_1(\kappa;a)$ using Eq. (4), and then project the output onto one of these operators. For our purpose we need only the following single-mode matrix elements:

$$\begin{aligned}x_1 &\equiv \langle n|\mathcal{C}_1(\kappa;a)(|n\rangle\langle n|)|n\rangle\\ &= (1+a/2)^{-1}\sum_{\ell=0}^{n}[{}^nC_\ell]^2[\kappa^2(1+a/2)^{-2}]^\ell\\ &\qquad\times[(1-\kappa^2(1+a/2)^{-1})(1-(1+a/2)^{-1})]^{n-\ell},\\ x_2 &\equiv \langle 0|\mathcal{C}_1(\kappa;a)(|n\rangle\langle n|)|0\rangle\\ &= (1+a/2)^{-1}[1-\kappa^2(1+a/2)^{-1}]^n,\\ x_3 &\equiv \langle 0|\mathcal{C}_1(\kappa;a)(|0\rangle\langle 0|)|0\rangle = (1+a/2)^{-1},\\ x_4 &\equiv \langle n|\mathcal{C}_1(\kappa;a)(|0\rangle\langle 0|)|n\rangle\\ &= (1+a/2)^{-1}[1-(1+a/2)^{-1}]^n,\\ x_5 &\equiv \langle n|\mathcal{C}_1(\kappa;a)(|n\rangle\langle 0|)|0\rangle = \kappa^n(1+a/2)^{-(n+1)},\\ &\equiv \langle 0|\mathcal{C}_1(\kappa;a)(|0\rangle\langle n|)|n\rangle^*. \qquad (6)\end{aligned}$$

One finds $\rho^{\rm out}_{00,00} = x_2x_3$, $\rho^{\rm out}_{nn,nn} = x_1x_4$, and $\rho^{\rm out}_{0n,n0} = x_5^2/2$, and therefore

$$\delta_1(\kappa,a) = x_1x_2x_3x_4 - (|x_5|^2/2)^2. \qquad (7)$$

Let $a_1(\kappa)$ be the solution to $\delta_1(\kappa,a) = 0$. This means that entanglement of our NOON state survives all values of noise $a < a_1(\kappa)$. The curve labeled N_5 in Fig. 1 shows, in the (a, κ) space, $a_1(\kappa)$ for the NOON state with $n = 5$: entanglement of $(|50\rangle + |05\rangle)/\sqrt{2}$ survives all noisy attenuators below N_5. The straight line denoted g_∞ corresponds to $a = \kappa^2$: channels above this line break entanglement of all Gaussian states, even the ones with arbitrarily large entanglement. The line g_1 denotes $a = \kappa^2(1-e^{-2\mu_1})$, where $\mu_1 = 0.5185$ corresponds to 1 ebit of Gaussian entanglement: Gaussian entanglement ≤ 1 ebit does not survive any of the channels above this line. The region R of channels above g_∞ but below N_5 are distinguished in this sense: no Gaussian entanglement survives the channels in this region, but the entanglement of the NOON state $(|50\rangle + |05\rangle)/\sqrt{2}$ does.

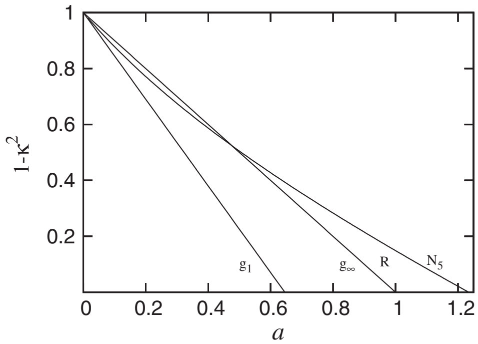

FIG. 1. Comparison of the robustness of the entanglement of a NOON state with that of two-mode Gaussian states under the two-sided action of symmetric noisy attenuator.

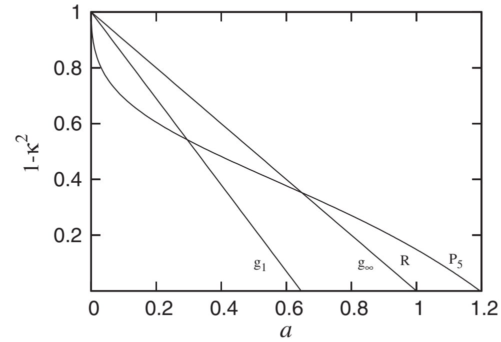

FIG. 2. Comparison of the robustness of the entanglement of a PNES state with that of two-mode Gaussian states under the action of two-sided symmetric noisy attenuator.

As a second non-Gaussian example we study the PNES $|\psi\rangle = (|00\rangle + |nn\rangle)/\sqrt{2}$ with density matrix

$$\rho = \frac{1}{2}(|0\rangle\langle 0| \otimes |0\rangle\langle 0| + |0\rangle\langle n| \otimes |0\rangle\langle n| + |n\rangle\langle 0| \otimes |n\rangle\langle 0| + |n\rangle\langle n| \otimes |n\rangle\langle n|). \tag{8}$$

The output state $\rho^{\text{out}} = \mathcal{C}_1(\kappa; a) \otimes \mathcal{C}_1(\kappa; a)(\rho)$ can be detailed in the Fock basis through use of Eq. (4).

Now to test for entanglement of ρ^{out}, we again project ρ^{out} onto the 2×2 subspace spanned by the vectors $\{|00\rangle, |0n\rangle, |n0\rangle, |nn\rangle\}$, and see if it is (NPT) entangled in this subspace. Clearly, it suffices to evaluate the matrix elements $\rho^{\text{out}}_{0n,0n}$, $\rho^{\text{out}}_{n0,n0}$, and $\rho^{\text{out}}_{00,nn}$, for if $\delta_2(\kappa, a) \equiv \rho^{\text{out}}_{0n,0n}\rho^{\text{out}}_{n0,n0} - |\rho^{\text{out}}_{00,nn}|^2$ is negative then ρ^{out} is NPT entangled, and one-copy distillable.

Once again, the matrix elements listed in (6) prove sufficient to determine $\delta_2(\kappa, a)$: $\rho^{\text{out}}_{0n,n0} = \rho^{\text{out}}_{n0,n0} = (x_1x_2 + x_3x_4)/2$, and $\rho^{\text{out}}_{00,nn} = |x_5|^2/2$, and so

$$\delta_2(\kappa, a) = ((x_1x_2 + x_3x_4)/2)^2 - (|x_5|^2/2)^2. \tag{9}$$

Let $a_2(\kappa)$ denote the solution to $\delta_2(\kappa, a) = 0$. That is, entanglement of our PNES survives all $a \leq a_2(\kappa)$. This $a_2(\kappa)$ is shown as the curve labeled P_5 in Fig. 2 for the PNES $(|00\rangle + |55\rangle)/\sqrt{2}$. The lines g_1 and g_∞ have the same meaning as in Fig. 1. The region R above g_∞ but below P_5 corresponds to channels (κ, a) under whose action all two-mode Gaussian states are rendered separable, while entanglement of the non-Gaussian PNES $(|00\rangle + |55\rangle)/\sqrt{2}$ definitely survives.

Noisy amplifier environment.—We turn our attention now to the amplifier environment. Under the symmetric two-sided action of a noisy amplifier channel $\mathcal{C}_2(\kappa; a)$, $\kappa \geq 1$, the two-mode CF $\chi^{\text{in}}_W(\xi_1, \xi_2)$ is taken to

$$\chi^{\text{out}}_W(\xi_1, \xi_2) = \chi^{\text{in}}_W(\kappa\xi_1, \kappa\xi_2)e^{-(1/2)(\kappa^2-1+a)(|\xi_1|^2+|\xi_2|^2)}.$$

In particular, the two-mode squeezed vacuum state $|\psi(\mu)\rangle$ with variance matrix $V_{\text{sq}}(\mu)$ is taken to a Gaussian state with variance matrix

$$V^{\text{out}}(\mu) = \kappa^2 V_{\text{sq}}(\mu) + (\kappa^2 - 1 + a)\mathbb{1}_4. \tag{10}$$

The partial transpose test [13] readily shows that the output state is separable when $a \geq 2 - \kappa^2(1 + e^{-2\mu})$: the additional noise a required to render the output Gaussian state separable increases with the squeeze or entanglement parameter μ and saturates at $a = 2 - \kappa^2$; for $a \geq 2 - \kappa^2$ the output state is separable for every Gaussian input. The noise required to render the two-mode squeezed state $|\psi(\mu_1)\rangle$ with 1 ebit of entanglement ($\mu_1 \approx 0.5185$) separable is $a = 2 - \kappa^2(1 + e^{-2\mu_1})$.

Now we examine the behavior of the NOON state $(|n0\rangle + |0n\rangle)/\sqrt{2}$ under the symmetric action of noisy amplifiers $\mathcal{C}_2(\kappa; a)$, $\kappa \geq 1$. Proceeding exactly as in the attenuator case, we know that ρ^{out} is definitely entangled if $\delta_3(\kappa, a) \equiv \rho^{\text{out}}_{00,00}\rho^{\text{out}}_{nn,nn} - |\rho^{\text{out}}_{0n,n0}|^2$ is negative. As remarked earlier the expressions for $\mathcal{C}_1(\kappa; a)$, $\kappa \leq 1$ in Eq. (6) are valid for $\mathcal{C}_2(\kappa; a)$, $\kappa \geq 1$ provided $1 + a/2$ is replaced by $\kappa^2 + a/2$. For clarity we denote by x'_j the expressions resulting from x_j when $\mathcal{C}_1(\kappa; a)$, $\kappa \leq 1$ is replaced by $\mathcal{C}_2(\kappa; a)$, $\kappa \geq 1$ and $1 + a/2$ by $\kappa^2 + a/2$. For instance, $x'_5 \equiv \langle n|\mathcal{C}_2(\kappa; a)(|n\rangle\langle 0|)|0\rangle = \kappa^n(\kappa^2 + a/2)^{-(n+1)}$ and $\delta_3(\kappa; a) = x'_1x'_2x'_3x'_4 - (|x'_5|^2/2)^2$.

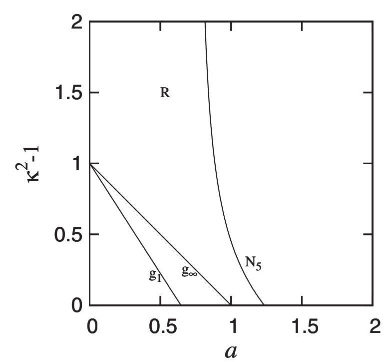

FIG. 3. Comparison of the robustness of the entanglement of a NOON state with that of all two-mode Gaussian states under the action of two-sided symmetric noisy amplifier.

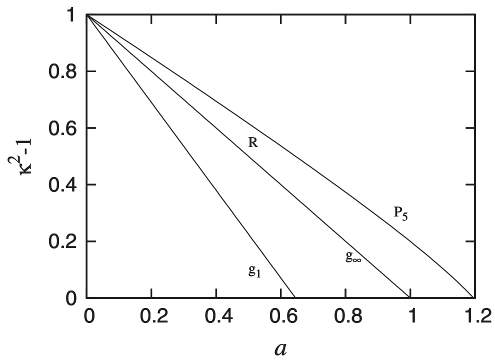

FIG. 4. Comparison of the robustness of the entanglement of a PNES state with that of all two-mode Gaussian states under the action of two-sided symmetric noisy amplifier.

Let $a_3(\kappa)$ be the solution to $\delta_3(\kappa, a) = 0$. This is represented in Fig. 3 by the curve marked N_5, for the case of the NOON state $(|05\rangle + |50\rangle)/\sqrt{2}$. This curve is to be compared with the line $a = 2 - \kappa^2$, denoted g_∞, above which no Gaussian entanglement survives, and with the line $a = 2 - \kappa^2(1 + e^{-2\mu_1})$, $\mu_1 = 0.5185$, denoted g_1, above which no Gaussian entanglement ≤ 1 ebit survives. In particular, the region R between g_∞ and N_5 corresponds to noisy amplifier channels against which entanglement of the NOON state $(|05\rangle + |50\rangle)/\sqrt{2}$ is robust, whereas no Gaussian entanglement survives.

Finally, we consider the behavior of the PNES $(|00\rangle + |nn\rangle)/\sqrt{2}$ in this noisy amplifier environment. The output, denoted ρ^{out}, is certainly entangled if $\delta_4(\kappa, a) \equiv \rho^{\text{out}}_{0n,0n}\rho^{\text{out}}_{n0,n0} - |\rho^{\text{out}}_{00,nn}|^2$ is negative. Proceeding as in the case of the attenuator, and remembering the connection between x_j's and the corresponding x'_j's, we have $\delta_4(\kappa, a) = ((x'_1x'_2 + x'_3x'_4)/2)^2 - (|x'_5|^2/2)^2$. The curve denoted P_5 in Fig. 4 represents $a_4(\kappa)$ forming solution to $\delta_4(\kappa, a) = 0$, for the case of the PNES $(|00\rangle + |55\rangle)/\sqrt{2}$. The lines g_∞ and g_1 have the same meaning as in Fig. 3. The region R between g_∞ and P_5 signifies the robustness of our PNES: for every $\kappa \geq 1$, the PNES is seen to endure more noise than Gaussian states with arbitrarily large entanglement.

We conclude with a pair of remarks. First, our conclusion following Eqs. (3) and (10) that entanglement of two-mode squeezed (pure) state $|\psi(\mu)\rangle$ does not survive, for any value of μ, channels (κ, a) which satisfy the inequality $|1 - \kappa^2| + a \geq 1$ applies to all Gaussian states. Indeed, for an arbitrary (pure or mixed) two-mode Gaussian state with variance matrix V_G it is clear from Eqs. (3) and (10) that the output Gaussian state has variance matrix $V^{\text{out}} = \kappa^2 V_G + (|1 - \kappa^2| + a)\mathbb{1}_4$. Thus $|1 - \kappa^2| + a \geq 1$ immediately implies, in view of nonnegativity of V_G, that $V^{\text{out}} \geq \mathbb{1}_4$, demonstrating separability of the output state for arbitrary Gaussian input [13].

Second, Gaussian entanglement resides entirely "in" the variance matrix, and hence disappears when environmental noise raises the variance matrix above the vacuum or quantum noise limit. That our chosen states survive these environments shows that their entanglement resides in the higher moments, in turn demonstrating that their entanglement is genuine non-Gaussian. Indeed, the variance matrix of our PNES and NOON states for $N = 5$ is 6 times that of the vacuum state.

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