Entanglement Distillation by Dissipation and Continuous Quantum Repeaters

Karl Gerd H. Vollbrecht, Christine A. Muschik, and J. Ignacio Cirac
Max-Planck–Institut für Quantenoptik, Hans-Kopfermann-Strasse, D-85748 Garching, Germany
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Even though entanglement is very vulnerable to interactions with the environment, it can be created by purely dissipative processes. Yet, the attainable degree of entanglement is profoundly limited in the presence of noise sources. We show that distillation can also be realized dissipatively, such that a highly entangled steady state is obtained. The schemes put forward here display counterintuitive phenomena, such as improved performance if noise is added to the system. We also show how dissipative distillation can be employed in a continuous quantum repeater architecture, in which the resources scale polynomially with the distance.

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Entanglement plays a central role in applications of quantum information science such as quantum computation, simulation, metrology, and communication. However, any quantum technology is challenged by dissipation. The interaction of the system with its environment is regarded a major obstacle, and the degradation of entangled states due to dissipation is typically considered to be a key problem. Contrary to this belief, new approaches aim at utilizing dissipation for quantum information processes [1] including quantum state engineering [2–4], quantum computing [4], quantum memories [5], the creation of entangled states [6], and error correction [7].

Entanglement generated by dissipation has been demonstrated [8] following a recent theoretical proposal [6]. The main advantage of this scheme lies in the fact that entangled states are generated in a steady state. Furthermore, as opposed to standard methods, the desired state is reached independently of the initial one. By coupling two quantum systems to a common environment (e.g., the electromagnetic field [8]) a robust entangled steady state can be quickly generated and maintained for an arbitrary long time without the need for error correction such that entanglement is available any time.

As other schemes, dissipative protocols are exposed to noise sources, which degrade the quality of the produced state and render it inapplicable for many important applications in quantum information, like quantum communication where noise effects increase dramatically with the distance. By means of distillation [9], entanglement can be improved at the expense of using several copies. In combination with teleportation, this allows for the construction of quantum repeaters [10], which enable the distribution of high-quality entanglement for long distance quantum communication with a favorable scaling of resources. Unfortunately, existing schemes for distillation and teleportation are incompatible with protocols generating steady state entanglement, as they require the decoupling of the system from the environment, such that the above-mentioned advantages of the dissipative approach (e.g., the inherent robustness) are lost. Hence, new procedures which are suitable to accommodate dissipative methods such that all advantages can be retained and used for quantum repeaters are highly desirable.

We introduce and analyze different dissipatively driven distillation protocols, which allow for the production of highly entangled steady states independent of the initial one and present a novel quantum repeater scheme featuring the same properties. This protocol continuously produces high-quality long-range entanglement and the required resources scale only polynomially in the distance. Once the system is operating in steady state, the resulting entangled link can be used for applications. Remarkably, the time required to drive a new pair into a highly entangled steady state is independent of the length of the link such that this setup provides a continuous supply of long-range entanglement [10]. Moreover, the proposed distillation protocols exhibit intriguing features. We present a method which allows for distillation in steady state where none of the individual source pairs is entangled and describe another one whose performance can be improved by adding noise to the system.

In the following, we introduce two types of dissipative distillation protocols suitable for different situations. We first explain scheme I, which is physically motivated, and consider the situation shown in Fig. 1. Two parties, Alice and Bob, share two source qubit pairs s_1 and s_2, which are each dissipatively driven into an entangled steady state and

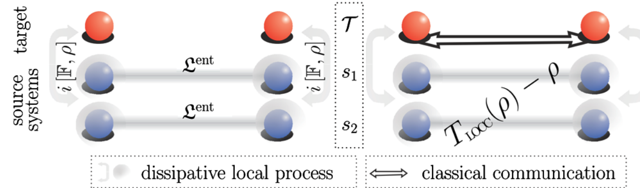

FIG. 1 (color online). Entanglement distillation by dissipation (a) Distillation setup without communication. (b) Distillation setup including classical communication.

used as resource for creating a single highly entangled pair in target system $\mathcal{T}$. Assuming Markov dynamics, the time evolution of the density matrix ρ can be described by a master equation of Lindblad form $\dot{\rho} = \gamma[O\rho O^\dagger - \frac{1}{2}\times(\rho O^\dagger O + O^\dagger O\rho)]$ with rate γ and will be abbreviated by the short hand notation $\dot{\rho} = \gamma\mathfrak{L}^O(\rho)$. The entangling dissipative process acting on the source qubits [11] corresponds to the master equation $\dot{\rho} = \mathfrak{L}^{\mathrm{ent}}(\rho) = \gamma[\mathfrak{L}^A(\rho) + \mathfrak{L}^B(\rho)]$ with $A = \cosh(r)\sigma^-_{\mathrm{Alice}} + \sinh(r)\sigma^+_{\mathrm{Bob}}$ and $B = \cosh(r)\sigma^-_{\mathrm{Bob}} + \sinh(r)\sigma^+_{\mathrm{Alice}}$, where $\sigma^- = |0\rangle\langle 1|$ and $\sigma^+ = |1\rangle\langle 0|$. The unique steady state of this evolution is the pure entangled state $|\psi\rangle = (|00\rangle - \lambda|11\rangle)/\sqrt{1+\lambda^2}$, where $\lambda = \tanh(r)$. The system is subject to local cooling, heating and dephasing noise described by $\mathfrak{L}^{\mathrm{noise}}(\rho) = \varepsilon_c[\mathfrak{L}^{\sigma^-_{\mathrm{Alice}}}(\rho) + \mathfrak{L}^{\sigma^-_{\mathrm{Bob}}}(\rho)] + \varepsilon_h[\mathfrak{L}^{\sigma^+_{\mathrm{Alice}}}(\rho) + \mathfrak{L}^{\sigma^+_{\mathrm{Bob}}}(\rho)] + \varepsilon_d[\mathfrak{L}^{|1\rangle\langle 1|_{\mathrm{Alice}}}(\rho) + \mathfrak{L}^{|1\rangle\langle 1|_{\mathrm{Bob}}}(\rho)]$.

We assume that the entangling dynamics acting on s_1 and s_2 is noisy, while the target system is protected (this assumption will be lifted below). The source qubits are locally coupled to $\mathcal{T}$ such that

$$\dot{\rho} = \mathfrak{L}^{\mathrm{ent}}_{s_1}(\rho) + \mathfrak{L}^{\mathrm{ent}}_{s_2}(\rho) + \mathfrak{L}^{\mathrm{noise}}_{s_1}(\rho) + \mathfrak{L}^{\mathrm{noise}}_{s_2}(\rho) + \mathfrak{L}_{\mathrm{Alice}}(\rho) + \mathfrak{L}_{\mathrm{Bob}}(\rho),$$

where $\mathfrak{L}_{\mathrm{Alice}}(\mathfrak{L}_{\mathrm{Bob}})$ acts only on Alice's (Bob's) side. We choose $\mathfrak{L}_{\mathrm{Alice}}(\rho) = -\mathfrak{L}_{\mathrm{Bob}}(\rho) = i\delta_{\mathbb{F}}[\mathbb{F},\rho]$, corresponding to the unitary evolution with respect to the Hamiltonian $\mathbb{F} = \sum_{i,j}|j_t\hat{i}_s\rangle\langle i_t\hat{j}_s|$, where $|\hat{0}_s\rangle = |0_{s_1}1_{s_2}\rangle$ and $|\hat{1}_s\rangle = |1_{s_1}0_{s_2}\rangle$. Note that this protocol does not require any classical communication or predefined correlations. The efficiency is mainly determined by the mixedness of the source states [see Fig. 2(a)] rather than their entanglement. In the absence of errors, the target system reaches a maximally entangled state.

In order to allow also for noise acting on $\mathcal{T}$, we include now classical communication. As shown in the appendix, any Lindblad operator of the form $\mathfrak{L}^{T_{\mathrm{LOCC}}}(\rho) = [T_{\mathrm{LOCC}}(\rho) - \rho]$, where T_{LOCC} is an arbitrary LOCC channel [12], can be realized using local dissipative processes and classical communication [13]. In particular, this allows for the stabilization of the distillation schemes discussed below against errors acting on the target system by running them using m blocks of source pairs, which are all coupled to the same target state (see Sec. 3 in [14]) as shown in Fig. 3(a). If sufficiently many source-blocks, m, are used, the dynamics is dominated by the desired processes. For clarity, we discuss the following schemes in the absence of target errors, which corresponds exactly to the limit $m \to \infty$ and consider the master equation

$$\dot{\rho} = \mathfrak{L}^{\mathrm{ent}}_{s_1}(\rho) + \mathfrak{L}^{\mathrm{ent}}_{s_2}(\rho) + \mathfrak{L}^{\mathrm{noise}}_{s_1}(\rho) + \mathfrak{L}^{\mathrm{noise}}_{s_2}(\rho) + \delta_{\mathbb{F}}[T_{\mathbb{F}}(\rho) - \rho].$$

The LOOC map $T_{\mathbb{F}}(\rho)$ is defined by the four Kraus operators $\mathbb{F}_A \otimes \mathbb{F}_B$, $P^\perp_A \otimes P_B$, $P_A \otimes P^\perp_B$, $P^\perp_A \otimes P^\perp_B$, where P, $P^\perp$ are the projection onto the one excitation subspace and its orthogonal complement. Alice and Bob measure the number of excitations on their side. After successful projection onto the subspace with one excitation $P_A \otimes P_B$, a flip operation $\mathbb{F}$ is performed. In the unsuccessful case no operation is carried out. We will repeatedly discuss Lindblad terms in the language of quantum jumps, i.e., as a measurement followed by an action, e.g., a unitary. However, note that the actual (dissipative) implementation is a continuous probabilistic process due to a designed

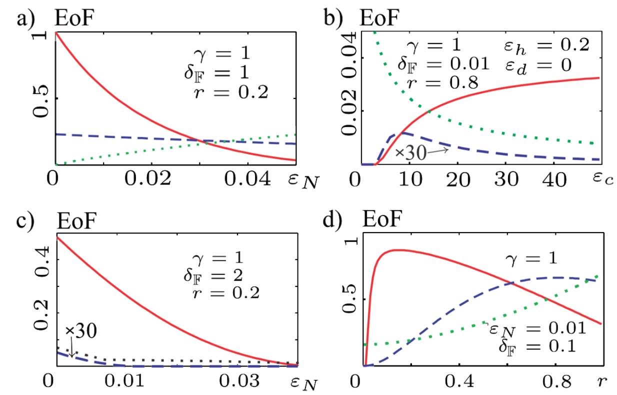

FIG. 2 (color online). Dissipative distillation according to scheme I without communication [panel (a)] and including classical communication [panels (b)–(d)]. The full red lines show the steady state entanglement of formation (Eof) of system $\mathcal{T}$. The dashed blue lines depict the steady state Eof of the source state s_1 if no distillation is performed (a),(c),(d) and during the protocol (b). For better visibility the blue dashed curve is multiplied by a factor 30 in panels b and c. The dotted green lines show the entropy of s_1 which is a measure of its mixedness. (a) EoF attainable without communication versus error rate $\varepsilon_{\mathrm{N}} \equiv \varepsilon_h = \varepsilon_c = \varepsilon_d$. (b) EoF versus the noise parameter ε_c. (c) EoF versus error rate ε_{N}. The black dotted curve represents the entanglement of the total source system measured in log negativity. (d) EoF versus the parameter r.

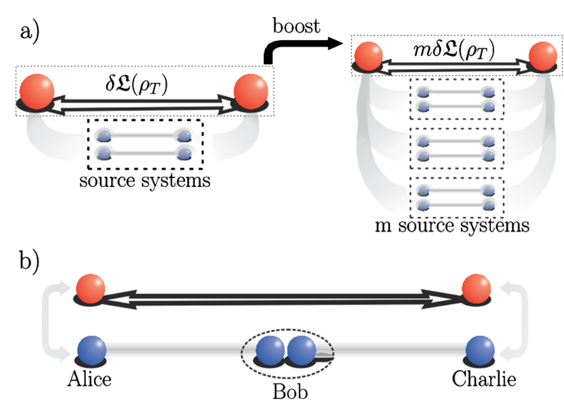

FIG. 3 (color online). Building blocks of a dissipative quantum repeater. (a) Noise resistant distillation setup. The process acting on the target system is boosted using several copies of the source system. (b) Continuous entanglement swapping procedure.

coupling to an environment described by a master equation. As shown in Fig. 2(b), the scheme is robust against local noise of cooling-type $[\mathfrak{L}^{\sigma^-}(\rho)]$. This kind of noise can even be used to enhance the performance of the distillation protocol in the steady state at the cost of a lower convergence rate. Thus, counterintuitively, it can be beneficial to add noise to the system in order to increase the distilled entanglement. Moreover, the steady state entanglement of the source pairs is zero in the absence of cooling noise for the parameters considered in Fig. 2(b), if no distillation scheme is performed. For increasing ϵ_c, the entanglement in s_1 and s_2 increases, reaches an optimal point an decreases again. Yet, the entanglement that can be distilled from these pairs is monotonically increasing and displays a boost effect. Because of the noise, the system is driven into a less entangled but more pure steady state for wich the scheme is more efficient. Panel c also hints at another counterintuitive effect, namely, that entanglement can be distilled even though none of the source pairs is (individually) entangled in the steady state. This can be explained by noticing that the two-copy entanglement can be maintained for high noise rates when the single-copy entanglement is already vanishing. Figure 2(d) shows that the distilled entanglement increases considerably for small values of r despite the decrease in the entanglement of the source pairs. This is due to the fact that the protocol is most efficient for source states close to pure states.

If the source states can be highly mixed, another distillation scheme (scheme II hereafter) is the method of choice and will be explained in the following. We analyze a generic model, which can be solved exactly and allows one to reduce the discussion to the essential features of dissipative entanglement distillation. As in standard distillation schemes, we study the general problem in terms of Werner states [15], since many situations can be described this way and a wide range of processes can be cast in this form by twirling [15]. Werner states are of the simple form $\rho_W(f) = f\Omega + (1-f)(\mathbb{1} - \Omega)/3$, where Ω is a projector onto the maximally entangled state $|00\rangle + |11\rangle$, and $\mathbb{1}$ the identity operator. We assume a process driving each source pair into the state Ω, $\dot{\rho} = \gamma[\mathrm{tr}(\rho)\Omega - \rho] \equiv \gamma E(\rho)$. Local depolarizing noise is added in the form of the term $N(\rho) \equiv (\rho_{\mathrm{Alice}} \otimes \mathbb{1} - \rho) + (\mathbb{1} \otimes \rho_{\mathrm{Bob}} - \rho)$, where $\rho_{\mathrm{Alice(Bob)}}$ denotes Alice's (Bob's) reduced density matrix and $\mathbb{1}$ the normalized identity. This term describes the continuous replacement of the initial state by the completely mixed one. The source system reaches the steady state $\rho_s \propto \gamma\Omega + \varepsilon\mathbb{1}$ of the total master equation $\dot{\rho} = \gamma E(\rho) + \frac{\varepsilon}{2}N(\rho)$ at least exponentially fast in γ (see Sec. 4 in [14]). A continuous distillation process based on a standard protocol [16] can be constructed using n source pairs which are independently driven into the steady state ρ_s and a target system $\mathcal{T}$. $\mathcal{T}$ is coupled to the source pairs by a dissipative dynamics of the form $\dot{\rho} = \delta_D[\mathrm{tr}(\rho)T_D(\rho) - \rho]$, where the completely positive map $T_D(\rho)$ corresponds to a process which acts on the n source pairs and distills a single higher entangled copy. The output state is written on $\mathcal{T}$ and the n source pairs are reinitialized in the state $\mathbb{1}$. The total master equation is given by $\dot{\rho} = \sum_{i=1}^{n}[\gamma E_i(\rho) + \frac{\varepsilon}{2}N_i(\rho)] + \delta_D[T_D(\rho) - \rho]$ where E_i, N_i denote entangling and noise processes on the ith source qubit pair. The steady state has a fidelity of $f = \int_0^1 dx f_D[f_s - (f_s - 0.25)x^{(\gamma+\varepsilon)/(\delta_D)}]$, where f_s and $f_D(f) = \mathrm{tr}[\Omega T_D(\rho_W(f)^{\otimes n})]$ are the fidelity of ρ_s and the output of the distillation protocol with n input states of fidelity f. High fidelities require low values of δ_D. However, the solution $\rho(t)$ (see [14], Sec. 4) shows that fast convergence requires high values of this parameter. A low convergence speed on the target system is extremely disadvantageous if noise is acting on $\mathcal{T}$. Therefore, a boost of the process as illustrated in Fig. 3(a) is required, which can be achieved by coupling m source systems individually to the same target. This way, the new convergence rate is given by $m\delta_D$ while the backaction on each source system remains unchanged (see [14], Sec. 3).

The distribution of long distance entanglement is a big challenge in quantum information. In repeater schemes, entanglement is generated over short distances with high accuracy and neighboring links are connected by entanglement swapping. This allows one to double the length of the links, but comes at the cost of a decrease in entanglement for nonmaximally entangled states or noisy operations. Therefore, a distillation scheme has to be applied before proceeding to the next stage, which consists again of entanglement swapping and subsequent distillation. The setup for a continuous entanglement swapping procedure is sketched in Fig. 3(b). It consists of three nodes operated by Alice, Bob and Charlie, where Alice and Bob as well as Bob and Charlie share an entangled steady state. By performing a teleportation, an entangled link is established between Alice and Charlie and mapped to the target system, while the source systems are reinitialized in the state $\mathbb{1}$. This corresponds to a LOCC operation $T_{sw}(\rho)$. The dynamics is described by $\dot{\rho} = \sum_{i=1}^{2}[\gamma E_i(\rho) + \frac{\varepsilon}{2}N_i(\rho)] + \delta_{\mathrm{sw}}[T_{\mathrm{sw}}(\rho) - \rho]$. The steady state has a target fidelity of $f = \frac{2\gamma^2}{(2\gamma+\delta_{sw})(\gamma+\delta_{sw})}[f_{sw}(f_s) - \frac{1}{4}] + \frac{1}{4}$, where $f_{sw}(f_s)$ is the output fidelity of the entanglement swapping protocol for two input states with fidelity f_s (see [14], Sec. 5).

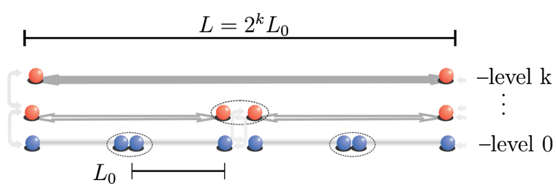

FIG. 4 (color online). Nested steady state quantum repeater scheme. In contrast to common repeater schemes, the levels of the nested protocol are physically present all the time and connected via dissipative processes.

The basic idea of a steady state quantum repeater is shown in Fig. 4. At the lowest level, entangled steady states are generated over a distance L_0. At each new level, two neighboring states are connected via continuous entanglement swapping and then written onto a target pair separated by twice the distance. The distillation and boost processes required at each level to keep the fidelity constant are not shown in this picture. The resources required for this repeater scheme can be estimated as follows. Entanglement swapping processes acting on source pairs of length l with fidelity f_l result in entangled target pairs of length $2l$, with degraded fidelity $f_{2l} < f_l$. This reduction is due to the swapping procedure, noise acting on the target system and the backaction from entanglement distillation. Stabilization against noise acting on the target systems is achieved by coupling each of them to m copies of the source system and requires therefore $2m$ source pairs of length l. In order to obtain a fidelity $f_{2l} \geq f_l$, n copies of these error stabilized links are used as input for a n to 1 distillation process. The distilled state is mapped to another target pair of length $2l$, which also needs to be stabilized using m copies of the blocks described. Hence, in total $2m^2n$ pairs of length l are required for a repeater stage which doubles the distance. For creating a link of length $L = L_0 2^k$, $(2m^2n)^k$ source pairs are needed, where k is the number of required iterations of the repeater protocol. Therefore, the resources scale polynomial with $(L/L_0)^{\mathrm{Log}_2(2m^2n)}$. In Sec. 5 in [14], we discuss a specific example scaling with $(L/L_0)^{16.4}$. The convergence time of the total system scales only logarithmically with the distance. Once the steady state is reached, the entanglement of the last target system can be used for quantum communication or cryptography. The underlying source systems are not effected by this process and remain in the steady state. Therefore, the target state is restored in constant time. Note that the error rate affecting the source systems is highly dependent on the length of the basic segments L_0. Because of the resulting trade-off in the use of resources, there exists an optimum length which depends on the implementation. In conclusion, we have shown how entanglement can be distilled in a steady state and distributed over long distances by means of a dissipative quantum repeater scheme serving as stepping stone for future work aiming at the optimization in view of efficiency and experimental implementations. The basic repetition rate is limited by the time required for classical communication, as is typically the case for quantum repeater schemes [17]. The development of concrete schemes for realistic physical realizations will be an important topic for future work. Possible implementations could, for example, be based on atoms in cavities by extending ideas put forward in [18].

Appendix.— The continuous exchange of classical communication is added in the framework of dissipative quantum information processing, by assuming that Alice and Bob have access to a system, which is used for communication only and considering the master equation $\dot{\rho} = \Gamma(\sum_i \langle i_{c_A}|\rho_{\mathrm{Alice}}|i_{c_A}\rangle\,|0_{c_A} i_{c_B}\rangle\langle 0_{c_A} i_{c_B}| - \rho) \equiv \Gamma\mathfrak{C}_{A\to B}(\rho)$. States referring to the communication system at Alice's and Bob's side are labeled by subscripts c_A and c_B. Alice's communication system is continuously measured in the computational basis yielding the quantum state $|i_{c_A}\rangle$ with probability $\langle i_{c_A}|\rho_{\mathrm{Alice}}|i_{c_A}\rangle$ and reset to the state $|0_{c_A}\rangle$, while the communication system on Bob's side is set to the measurement outcome. This way, classical information can be sent at a rate Γ, but no entanglement can be created [14]. As proven in Sec. 2 in [14], any operation that can be realized by means of local operations and classical communication (LOCC) can be constructed in a continuous fashion using communication processes $\mathfrak{C}_{A\to B}$ and $\mathfrak{C}_{B\to A}$, if the rate Γ is fast compared to all other relevant processes including the retardation due to back and forth communication.

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[11] The entangling dissipative process considered here is a discrete single particle version of the collective dynamics realized in [8] for continuous variables.

[12] LOCC channels are completely positive trace preserving maps that can be realized by means of Local Operations and Classical Communication.

[13] We assume, that the time scales for classical communication Γ^{-1} (see appendix) are sufficiently long such that retardation effects can be ignored.

[14] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.107.120502 for details of the proposed distillation schemes and the repeater architecture.
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