## <span id="page-0-0"></span>Spin-Canting-Induced Improper Ferroelectricity and Spontaneous Magnetization Reversal in  $SmFeO<sub>3</sub>$

<span id="page-0-1"></span>Jung-Hoon Lee,<sup>1</sup> Young Kyu Jeong,<sup>1</sup> Jung Hwan Park,<sup>1</sup> Min-Ae Oak,<sup>1</sup> Hyun Myung Jang,<sup>1,2[,\\*](#page-3-0)</sup> Jong Yeog Son, $3$  and James F. Scott<sup>4[,†](#page-3-1)</sup>

 ${}^{1}$ Department of Materials Science and Engineering, and Division of Advanced Materials Science, Pohang University of Science and Technology, Pohang 790-784, Republic of Korea <sup>2</sup>

 $2$ Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Republic of Korea

 $3$ Department of Applied Physics, College of Applied Science, Kyung Hee University, Suwon 446-701, Republic of Korea

Department of Physics, Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom

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SmFeO<sub>3</sub>, a family of centrosymmetric rare-earth orthoferrites, is known to be nonferroelectric. However, we have found that  $SmFeO<sub>3</sub>$  is surprisingly ferroelectric at room temperature with a small polarization along the  $b$  axis of P $bnm$ . First-principles calculations indicate that the canted antiferromagnetic ordering with two nonequivalent spin pairs is responsible for this extraordinary polarization and that the reverse Dzyaloshinskii-Moriya interaction dominates over the exchange-striction mechanism in the manifestation of the improper ferroelectricity.  $SmFeO<sub>3</sub>$  further exhibits an interesting phenomenon of spontaneous magnetization reversal at cryogenic temperatures. This reversal is attributed to the activation of the Sm-spin moment which is antiparallel to the Fe-spin moment below  $\sim$  5 K.

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Multiferroics exhibit simultaneous ferroic properties with coupled electric, magnetic, and structural orders. Over the past decade, there has been a resurgence of interest in understanding and applications of multiferroic materials [\[1](#page-3-2)–[8\]](#page-3-3). A variety of theoretical models were proposed to account for the origin of magnetically induced ferroelectricity in antiferromagnetic oxides [[9](#page-3-4)[–16](#page-4-0)]. Among these, a spin-current-induction model [\[12](#page-4-1)] of the polarization of a  $S_i \times S_j$  type in noncollinear magnets is particularly appealing as this model suggests the possibility of the occurrence of ferroelectricity in  $ABO<sub>3</sub>$ -type orthoferrites which exhibit their canted antiferromagnetic (AFM) orderings well above room temperature [\[17](#page-4-2)[,18](#page-4-3)]. According to this vector-field model based on the spin-orbit-coupling-driven reverse Dzyaloshinskii-Moriya interaction, a small improperlike polarization can appear in conjunction with a para-to-AFM transition in a centrosymmetric crystal having a noncollinear spin structure. Conversely, a ferroelectric distortion can induce spin-canted weak ferromagnetism [\[10\]](#page-3-5) if the Dzyaloshinskii-type invariant  $[\sim \mathbf{P} \bullet (\mathbf{M} \times \mathbf{L})]$  is allowed in the free-energy expansion [\[15](#page-4-4),[19\]](#page-4-5).

Accordingly, we have been exploring a suitable AFM material which is characterized by a noncollinear spin structure with its Néel temperature  $(T_N)$  substantially higher than 300 K and insulating property up to  $T_N$ . We find that  $SmFeO<sub>3</sub>$  (SFO hereafter), a family of rare-earth orthoferrites, meets these requirements. It is known to be antiferromagnetic below  $\sim$  670 K [[17\]](#page-4-2) and is electrically resistive up to  $T_N$ . We will show that SFO is surprisingly ferroelectric with a substantial degree of the piezoelectricity at room temperature. On the basis of first-principles calculations, we conclude that the noncollinear AFM ordering with two nonequivalent spin pairs is primarily responsible for this extraordinary ferroelectricity. In addition to the magnetically induced ferroelectricity, SFO exhibits an interesting phenomenon of spontaneous magnetization reversal at cryogenic temperatures.

SFO is characterized by orthorhombic *Pbnm* (or *Pnma*) structure [Fig. [1\(a\)\]](#page-1-0). To understand the ground-state spin configuration of the Fe-spin subsystem, we have carried out density-functional theory (DFT) calculations on the basis of the generalized gradient approximation [[20](#page-4-6)] implemented with the projector augmented wave pseudopotential [\[21\]](#page-4-7) using the Vienna ab initio simulation package. According to the DFT calculations, the orthorhombic SFO is characterized by two nonequivalent canted AFM Fe-spin pairs,  $(S_2, S_3) = \alpha$  and  $(\hat{S}_1, S_4) = \beta$ , as<br>schematically shown in Fig. 1(a). The ground-state AFM schematically shown in Fig. [1\(a\).](#page-1-0) The ground-state AFM ordering direction (Néel vector) of each Fe-spin subsystem is predicted to be parallel to [001] of the Pbnm setting but with a small value of the net canted moment (M) along [100]. More specifically, our DFT calculations predict the following relation for M (net magnetization of the Fe spin) in the  $\Gamma_2$ -spin structure [[22](#page-4-8)] which is known to be stable for a wide temperature range including 300 K:  $M \equiv$  $\mathbf{M}_{\alpha} + \mathbf{M}_{\beta} = (\mathbf{S}_2 + \mathbf{S}_3) + (\mathbf{S}_1 + \mathbf{S}_4) = (0.076\hat{\mathbf{x}} - 0.054\hat{\mathbf{y}}) +$ <br>  $(0.076\hat{\mathbf{x}} + 0.054\hat{\mathbf{y}}) = 0.152\hat{\mathbf{x}}$  with  $\mathbf{J} = (\mathbf{S}_1 - \mathbf{S}_2) =$  $(0.076\hat{x} + 0.054\hat{y}) = 0.152\hat{x}$ , with  $\mathbf{L}_{\alpha} \equiv (\mathbf{S}_{2} - \mathbf{S}_{3}) =$ <br>  $-7.10\hat{z}$  and  $\mathbf{L}_{\alpha} \equiv (\mathbf{S}_{1} - \mathbf{S}_{2}) = +7.10\hat{z}$  where  $\hat{x}$  for example, designates a unit vector along  $[100]$  (i.e., along the 7.10 $\hat{z}$  and  $\mathbf{L}_{\beta} = (\mathbf{S}_1 - \mathbf{S}_4) = +7.10\hat{z}$ , where  $\hat{x}$ , for exa axis), and  $L_{\alpha}$  and  $L_{\beta}$  denote the Néel vector for  $\alpha$  and  $\beta$ subsystems, respectively [[23](#page-4-9)]. Here numerical values are given in the unit of Bohr magneton  $(\mu_B)$ .

Fe  $L_{2,3}$ -edge x-ray magnetic circular dichroism (XMCD) spectra  $(\rho^+, \rho^-)$  of the flux-grown SFO single



<span id="page-1-0"></span>FIG. 1 (color online). Crystal structure and magnetic spectra of orthorhombic SmFeO3. (a) A unit-cell crystal structure of SmFeO<sub>3</sub> in *Pbnm* setting (left) and a schematic  $\Gamma_2$ -spin structure showing two nonequivalent spin pairs (right). (b) Fe  $L<sub>2,3</sub>$ -edge x-ray magnetic circular dichroism (XMCD) spectra obtained at 360 K. Bottom panel: MCD signals,  $\triangle \rho$  and  $\Sigma(\triangle \rho)$ , over the entire  $I_{\odot \delta}$  region entire  $L_{2,3}$  region.

crystal [Fig. [1\(b\)\]](#page-1-0) also demonstrate that the magnetic easy axis is parallel to [100] below  $T_{SR}$ . The four major dichroism  $(\Delta \rho = \rho^+ - \rho^-)$  peaks that appeared at the  $L_3$ <br>region with two positive  $(\Delta \rho > 0)$  and two pegative region with two positive ( $\Delta \rho > 0$ ) and two negative  $(\triangle \rho < 0)$  peaks indicate that two of the four nonequivalent spin moments  $(S_2, S_3)$  form one set of the spin subsystem  $(\mathbf{M}_{\alpha})$  while the other two spin moments  $(\mathbf{S}_1, \mathbf{S}_4)$ form the other set of the spin subsystem  $(M_\beta)$ . Thus, the XMCD result supports the prediction of first-principles calculations. The integration of the dichroism over the entire  $L_{2,3}$  absorption region [bottom panel of Fig. [1\(b\)\]](#page-1-0) indicates that, unlike  $GaFeO<sub>3</sub>$  [[24](#page-4-10)], the orbital magnetic moment of Fe ions is negligibly small, as compared with the spin magnetic moment.

The two prominent features of the magnetization-field  $(M-H)$  curve [Fig. [2\(a\)\]](#page-1-1) are (i) a pronounced magnetic anisotropy and (ii) a small value of the coercive field  $(\sim 30 \text{ Oe})$ . These features can be practically exploited in the development of an ultralow-field magnetic switching device that requires an extremely small coercive field with a prominent anisotropy. The M-H curve also indicates that the magnetic easy axis of SFO is nearly parallel to the a axis of Pbnm (equivalently, parallel to the c axis of Pnma) [\[22\]](#page-4-8). This observation is consistent with our theoretical



<span id="page-1-1"></span>FIG. 2 (color online). Magnetic properties of the  $SmFeO<sub>3</sub>$  single crystal. (a) Room-temperature magnetization-field (M-H) hysteresis curves, showing the effect of the applied magnetic-field direction on the magnetization response. (b) Temperature-dependent magnetization,  $M(T)$ , curves showing a [001]-to-[100] spin-reorientation transition at  $\sim$  480 K. On the other hand, we did not find any meaningful [010] magnetization. (c)  $M(T)$  curves for temperatures below 300 K, showing a thermally induced spontaneous magnetization reversal at 5 K. We measured  $M(T)$  curves for both (+) and  $(-)$  cooling fields. The inset presents a magnified view of  $M(T)$  at cryogenic temperatures cryogenic temperatures.

prediction that  $M = 0.152\hat{x}$  for the  $\Gamma_2$ -spin structure, as discussed previously.

The temperature-dependent magnetization curve demonstrates an AFM ordering at 670 K  $(T_N)$ . As shown in Fig. [2\(b\)](#page-1-1), there rapidly occurs a spin-reorientation (SR) transition at  $\sim$ 480 K, switching the magnetic easy axis of [100] to [001] with increasing temperature. This corresponds to a  $\Gamma_2$ -to- $\Gamma_4$  transition in the AFM spin configuration. According to our DFT calculations, M (the net magnetization of the Fe spin) in the  $\Gamma_4$ -spin structure, which is stable for the temperature range between  $T_N$ (670 K) and  $T_{SR}$  (480 K), is described by the following relation [[23](#page-4-9)]:  $\mathbf{M} \equiv \mathbf{M}_{\alpha} + \mathbf{M}_{\beta} \equiv (\mathbf{S}_2 + \mathbf{S}_3) + (\mathbf{S}_1 + \mathbf{S}_4) =$ <br>  $(-0.045\hat{s} - 0.059\hat{s}) + (+0.045\hat{s} - 0.059\hat{s}) = -0.118\hat{s}$ This prediction is consistent with the observation of the  $0.045\hat{y} - 0.059\hat{z}$  +  $(+0.045\hat{y} - 0.059\hat{z}) = -0.118\hat{z}$ .<br>is prediction is consistent with the observation of the [100]-to-[001] spin reorientation at  $T_{SR}$  [480 K; Fig. [2\(b\)\]](#page-1-1).

Upon cooling to cryogenic temperatures, SFO further shows an interesting phenomenon of spontaneous magnetization reversal [Fig. [2\(c\)\]](#page-1-1). This suggests a long-range ordering of the rare-earth  $Sm^{3+}$ -spin moment. In this case, the direction of the net  $Sm^{3+}$  moment should be opposite to that of the canted  $Fe^{3+}$ -spin moment (+a) in the  $\Gamma_2$ -spin structure. According to our DFT calculations, the net magnetization is antiparallel to the  $a$  axis of  $Pbnm$ with its value of  $-0.061\mu_B$  per f.u. We are now<br>able to delineate the evolution of the spin structure with able to delineate the evolution of the spin structure with decreasing temperature: (i) para-to-AFM ordering at 670 K ( $T_N$ ), (ii)  $\Gamma_4$ -to- $\Gamma_2$  spin reorientation at 480 K, and (iii) spontaneous magnetization reversal caused by a longrange ordering of the rare-earth  $Sm^{3+}$ -spin moment along -figuration with the variation of temperature are pictorially  $-a$  of *Pbnm*. These sequential changes in the spin conpresented in Fig. [3.](#page-2-0)

Let us now examine the possibility of the occurrence of spin-canting-driven ferroelectricity. Since SFO is

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FIG. 3 (color online). Sequential changes in the spin configuration with the variation of temperature. The spontaneous magnetization reversal observed at 5 K ( $T_{MR}$ ) is attributed to a long-range ordering of the  $Sm^{3+}$ -spin moment which is opposite to the canted Fe<sup>3+</sup>-spin moment ( + a) in the  $\Gamma_2$ -spin structure of orthorhombic SmFeO<sub>3</sub>.

characterized by the two nonequivalent canted AFM subsystems,  $\alpha = (\mathbf{S}_2, \mathbf{S}_3)$  and  $\beta = (\mathbf{S}_1, \mathbf{S}_4)$ , the noncollinear<br>spin-canting-induced polarization has two distinct compospin-canting-induced polarization has two distinct components,  $P_{\alpha}$  and  $P_{\beta}$ , if they are nonzero. According to the spin-current-induction model [\[12\]](#page-4-1),  $P_\alpha$  can be formally written as  $P_{\alpha} = e_{23} \times (S_2 \times S_3)$ . Our DFT computations using the Berry-phase method [[25](#page-4-11)] now read:  $P_\alpha = e_{23} \times$  $(S_2 \times S_3) = +P_x \hat{x} - P_y \hat{y}$ , where  $P_x/P_y = 1.40$  and  $P_y = +50.0 \mu C/m^2$  for the E-spin structure Similarly DET +50.0  $\mu$ C/m<sup>2</sup> for the  $\Gamma_2$ -spin structure. Similarly, DFT calculations predict:  $P_{\beta} = e_{14} \times (S_1 \times S_4) = -P_x \hat{x} - P_y \hat{y}$ .<br>Thus the net spin-canting-induced polarization (**P** Thus, the net spin-canting-induced polarization ( $P_{ind} \equiv$  $\mathbf{P}_{\alpha} + \mathbf{P}_{\beta}$ , which is parallel to the *b* axis, is predicted<br>to be  $\vert -2P_0 \rangle = +100.0 \mu C/m^2$  for the *C*-spin structo be  $|-2P_y\hat{y}| = +100.0 \mu C/m^2$  for the  $\Gamma_2$ -spin structure. Essentially the same prediction can be made for the  $\Gamma_4$ -spin structure which is known to be stable for the temperature range between  $T_{SR}$  and  $T_N$ :  $P_{ind}$  =  $\mathbf{P}_{\alpha} + \mathbf{P}_{\beta} = \{-P_{y}^{T}\hat{y} - P_{z}^{T}\hat{z}\} + \{-P_{y}^{T}\hat{y} + P_{z}^{T}\hat{z}\} = -2P_{y}^{T}\hat{y}$ <br>parallel to [010] with  $P_{z}^{T}/P_{z} = 1.33$  [23]. Accordingly parallel to [010] with  $P_z^{\prime}/P_y^{\prime} = 1.33$  [\[23](#page-4-9)]. Accordingly, we now deduce one important conclusion on the direction of  $P_{ind}$  that the net noncollinear spin-canting-induced polarization ( $P_{ind}$ ) remains to be parallel to [010], irrespective of the spin reorientation at  $T_{SR}$ .

Though the noncollinear spin-canting-induced polarization  $(P_{ind})$  corresponds to the reverse Dzyaloshinskii-Moriya interaction mechanism, it can be shown that the two Dzyaloshinskii-type invariants, i.e.,  $\{P_\alpha \bullet (M_\alpha \times L_\alpha)\}$  $\mathbf{P}_{\beta} \bullet (\mathbf{M}_{\beta} \times \mathbf{L}_{\beta})$ , cancel each other out [\[23\]](#page-4-9) and, thus, do<br>not contribute to the free energy density which is an not contribute to the free-energy density, which is an interesting exception to Fennie's rule [[15](#page-4-4)]. This indicates that a  $P \bullet (M \times L)$ -type trilinear coupling [[19](#page-4-5)] is not effective in the orthorhombic SFO because of the presence of the two nonequivalent AFM subsystems.

Let us now consider experimental aspects of the canted AFM-ordering-induced ferroelectricity. The capacitancevoltage  $(C-V)$  hysteresis curve presented in Fig.  $4(a)$ clearly demonstrates a nonzero remanent polarization  $(P<sub>r</sub>)$  along the b axis of *Pbnm* at 300 K. The magnitude of the induced polarization was evaluated by measuring and integrating temperature-dependent pyroelectric current [\[23\]](#page-4-9). As shown in Fig. [4\(b\)](#page-3-6), the polarization developed along the b axis of Pbnm is  $\sim 93 \mu C/m^2$ . Figure [4\(b\)](#page-3-6) also shows that the onset of the para-to-ferroelectric transition coincides well with the AFM ordering temperature  $[670 \text{ K}, \text{Fig. 2(b)}]$ . This strongly supports the AFM-ordering-induced ferroelectricity in a centrosymmetric crystal. A nonzero value of the spontaneous polarization  $(P_s)$  at 300 K was further confirmed by carrying out positive-up and negative-down (PUND) pulse measurement [\[23\]](#page-4-9). The saturation  $P_r$  value obtained from the PUND measurement at 300 K is ~100  $\mu$ C/m<sup>2</sup>, which coincides well with the DFT<br>prediction of 100  $\mu$ C/m<sup>2</sup> for the  $\Gamma_2$ -spin structure. The consistency between the DFT prediction (with the spin-orbit coupling option) and the experimental observation supports the prevalence of the noncollinear spin-orbitcoupling-driven polarization of a  $S_i \times S_j$  type in SFO.



<span id="page-3-6"></span>FIG. 4 (color online). Ferroelectric and piezoelectric responses of the  $SmFeO<sub>3</sub>$  single crystal. (a) A room-temperature  $C-V$  curve of the flux-grown orthorhombic  $SmFeO<sub>3</sub>$  single crystal obtained at a 100 kHz measuring frequency with the probing electric field along [010]. (b) Temperature-dependent spontaneous polarization along [010]. No pyroelectric response was detected for an electrically poled  $SmFeO<sub>3</sub>$  crystal along two other principal directions,  $[100]$  and  $[001]$ . (c) Variations of  $b$ - and  $c$ -axis parameters under the electric field applied to [010] and [001] directions, respectively.

On the other hand, the computed polarization is strictly zero if we do not adopt the spin-orbit coupling in the Berry-phase calculations. This clearly indicates that the exchange-striction mechanism of a  $S \cdot S$  type [[8](#page-3-3)] is not relevant to the manifestation of the improper ferroelectricity in SFO.

If SFO is truly ferroelectric, it should show a certain degree of the piezoelectricity along the  $P_s$  (spontaneous polarization) direction. To examine this proposition, we have carried out *in situ* synchrotron x-ray microdiffraction (XRMD) experiments [[26](#page-4-12)]. According to the phenomenological thermodynamic prediction, the piezoelectric coefficient  $d_{22}$  (for both the elastic strain and the measuring field along the polarization direction,  $b$  axis of  $Pbnm$ ) is nonzero while the other two diagonal components,  $d_{11}$  and  $d_{33}$ , should be nearly zero [[23](#page-4-9)]. As shown in Fig. [4\(c\),](#page-3-6) the c-axis parameter evaluated using the branch of decreasing  $E$  field [\[26\]](#page-4-12) is nearly independent of the applied  $E$  field, indicating that  $d_{33} \approx 0$ . We also found that  $d_{11} \approx 0$ . On the contrary, the slope is nonzero for  $d_{22}$ , and we have evaluated  $d_{22}$  using the XRMD data (for the linear low E-field region) and the relation  $d_{22} = (\partial l_{y}/\partial E_{y})_{T,\sigma=0}$ , where  $l_{y}$ denotes the spontaneous elastic strain along the b axis. The estimated  $d_{22}$  (  $\sim$  22  $\pm$  4 pm/V) is slightly higher than  $d_{33}$ of the epitaxial BiFeO<sub>3</sub> thin film ( $\sim 15$  pm/V) having 4 mm symmetry [[26](#page-4-12)]. Thus, the XRMD study clearly supports that SFO is ferroelectric with its  $P_s$  along the b axis of Pbnm.

First-principles calculations indicate that the canted AFM ordering of a  $S_i \times S_j$  type is primarily responsible for the observed extraordinary ferroelectricity in SFO. Thus, a high AFM ordering temperature with a noncollinear canted spin structure is the main reason why SFO is an ambient multiferroic while most other oxides are not.

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<span id="page-3-1"></span><span id="page-3-0"></span>[\\*h](#page-0-0)mjang@postech.ac.kr [†](#page-0-1) jfs32@cam.ac.kr

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