

Linear Chain Structure of Four- α Clusters in ^{16}O

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We investigate the linear chain configurations of four- α clusters in ^{16}O using a Skyrme cranked Hartree-Fock method and discuss the relationship between the stability of such states and angular momentum. We show the existence of a region of angular momentum ($13\text{--}18\hbar$) where the linear chain configuration is stabilized. For the first time we demonstrate that stable exotic states with a large moment of inertia ($\hbar^2/2\Theta \sim 0.06\text{--}0.08$ MeV) can exist.

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Strong nuclear deformations provide an excellent framework in which to investigate the fundamental properties of quantum many-body systems. Strongly deformed nuclei have been identified by the observation of γ -ray cascades typical of rotational bands. Since the first observation of such bands [1], strongly deformed states with an aspect ratio 1:2 have been found in various nuclei. These bands are called superdeformed bands. Furthermore, the hyperdeformed bands, in which the deformation is around 1:3, have been reported in several experiments [2]. At first, those strongly deformed states were found in the heavy nuclei. Such new data have triggered interest in whether more exotic states exist in light nuclei where a strong deformation above 1:3 could be possible due to α -cluster structure.

Experimental candidates for strongly deformed states with an aspect ratio above 1:3 have been suggested in light $4N$ nuclei. One candidate is the four- α linear chain band starting around the four- α threshold energy region in ^{16}O suggested by Chevallier *et al.* [3] in the $^{12}\text{C}(\alpha, ^8\text{Be})^8\text{Be}$ reaction and was supported by Suzuki *et al.* [4]. Freer *et al.* [5] performed the $^{12}\text{C}(^{16}\text{O}, 4\alpha)$ reaction and obtained a smaller moment of inertia, about 2/3 of Ref. [3]. Recently it has been suggested that these have a loosely coupled four- α structure [6], in connection with the gaslike 0_2^+ (Hoyle) state in ^{12}C [7]. Another candidate is a six- α linear chain state, which has been extensively studied both theoretically and experimentally [8–11]. Wuosmaa *et al.* [10] and Rae *et al.* [11] suggested that the molecular resonance state observed in the inelastic reaction $^{12}\text{C}(^{12}\text{C}, ^{12}\text{C}(0_2^+))^{12}\text{C}(0_2^+)$ might be a candidate for the six- α linear chain state. Hirabayashi *et al.* [12] claimed that this has a loosely coupled $3\alpha + 3\alpha$ configuration rather than a linear chain. The seven- α linear chain state in ^{28}Si was not observed [13]. Despite many efforts no clear experimental evidence of a stable α linear chain structure has been confirmed and its existence remains an open problem.

The stability of such linear chain states has often been studied through the analysis of small vibrations around the

equilibrium configuration and with the axial symmetry [9,14]. However, it was shown that bending motion is an essential path for the transition to low-lying states [15]. Thus it is necessary to calculate the stability in a wide model space, which contains lower excited states. Two mechanisms are important for stabilizing the linear chain state. The first mechanism is the quantum-mechanical orthogonality condition to other low-lying states. The second is, as discussed by Wilkinson [16], the competition between the nuclear attractive and centrifugal forces due to rotation of the system: a large moment of inertia such as in the linear chain configuration is favored with a large angular momentum. On the other hand, high angular momenta would lead to fission of the linear chain due to the strong centrifugal force. Detailed investigations are necessary for the existence of a region of angular momentum where the linear chain configuration is stabilized.

In this Letter we show that a region of angular momentum ($13\text{--}18\hbar$) where the four- α linear chain configuration is stabilized exists in ^{16}O . The cranked Hartree-Fock (HF) method is used to investigate the stability of the configuration and the moment of inertia. The mechanism of the stabilization against the decay with respect to bending motion and fission and its angular-momentum dependence is clarified.

Until now, most of the theoretical analyses of the linear chain structure have been performed using the conventional cluster model with effective interactions, whose parameters are determined to reproduce the binding energies and scattering phase shifts of the clusters. Thus it is highly desirable to study the presence of exotic cluster configurations based on different approaches, such as mean-field models. The effective interactions used in the mean-field models are determined in a completely different way; they are designed to reproduce various properties of nuclei in a wide mass range. The appearance of cluster structure as a result of calculations with such interactions and model spaces would give more confidence in their

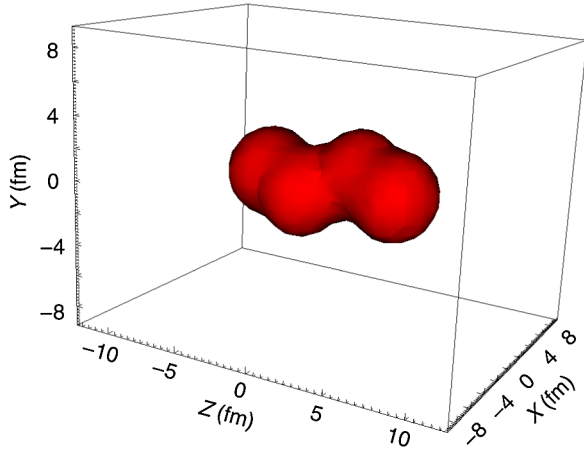


FIG. 1 (color online). Surface of the total nucleon density distribution for the initial twisted wave function. We chose the surface as half of the total density.

presence. Recent developments of three-dimensional calculations with Skyrme forces enable us to describe both shell-like and clusterlike configurations, and the interplay between these structures based on this approach has been successfully investigated [17].

To investigate the four- α linear chain state in the rotational frame, we perform cranked HF calculations. We self-consistently calculate the cranked HF equation, given by $\delta\langle H - \omega J \rangle = 0$, where H is the total Hamiltonian, ω is the rotational frequency, and J is the angular momentum around the y axis. We represent the single-particle wave functions on a Cartesian grid with a grid spacing of 0.8 fm. The grid size is typically 24^3 for ground states and 32×24^2 for superdeformed states. This accuracy was seen to be sufficient to provide converged configurations. The numerical procedure is the damped-gradient iteration method [18], and all derivatives are calculated using the Fourier transform method.

We take three different Skyrme forces which all perform very well concerning nuclear bulk properties but differ in details: SLy6 as a recent fit which includes information on

isotopic trends and neutron matter [19] and SkI3 as well as SkI4 as recent fits which map the relativistic isovector structure of the spin-orbit force [20]. The force SkI3 contains a fixed isovector part analogous to the relativistic mean-field model, whereas SkI4 is adjusted allowing free variation of the isovector spin-orbit term. Thus all forces differ somewhat in their actual shell structure. In addition to the effective mass, the bulk parameters (equilibrium energy and density, incompressibility, and symmetry energy) are comparable.

Here we discuss the stability of the four- α linear chain configuration in the rotating frame for ^{16}O . To this end, we perform the cranked HF calculations with various rotational frequencies ω . For the initial wave function, we choose the z axis as the principal axis and use the twisted four- α configuration, as shown in Fig. 1. We also show the corresponding two-dimensional plot in Fig. 2(a). Note that this initial configuration is a three-dimensional four- α one, which facilitates the transition of the initial state to low-lying states including the ground state during the convergence process. This was demonstrated for the carbon chain states in Refs. [21,22]. We calculate the rigid-body moment of inertia Θ using the total nucleon density at each iteration step. We only consider rotation around the y axis (perpendicular to the deformation axis z).

We first investigate the convergence behavior of the HF iterations. To check this, we calculate the coefficient of the rotational energy, given by $\hbar^2/2\Theta$, at each iteration step. Figure 3 shows the calculated results with various rotational frequencies versus the iterations in the case of the SkI3 interaction. The initial state with the twisted linear chain configuration is not the true ground state of the HF model space and the solution changes into the true ground state after some large number of iterations; however, the situation depends on the value of the rotational frequency ω . In Fig. 3, we see that the rotational frequencies $\omega = 0.5, 1.0,$ and $1.5 \text{ MeV}/\hbar$ (the dashed, dotted, and dot-dashed lines, respectively) lead to the true ground state. Note that the rigid-body moment of inertia Θ is a classical value, which does not become zero even with spherical

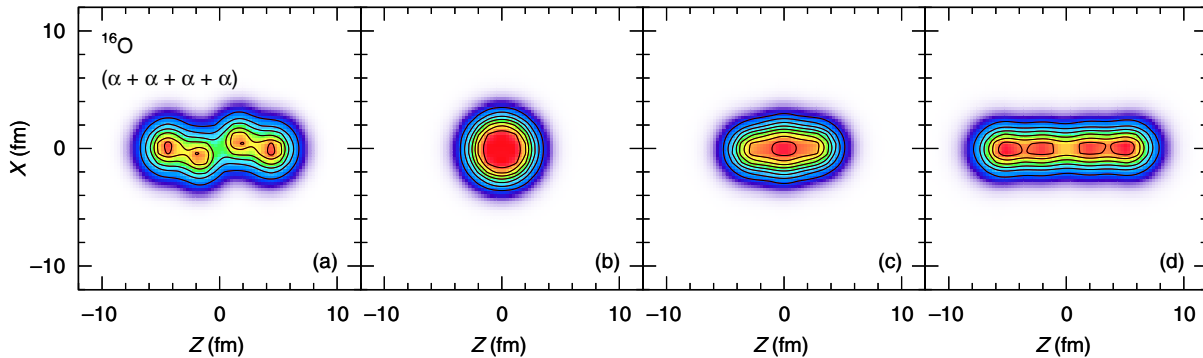


FIG. 2 (color online). Total nucleon density distribution calculated using the cranking method for (a) the initial wave function, (b) the ground state, (c) the quasistable state, and (d) the four- α linear chain state. The isolines correspond to multiples of 0.02 fm^{-3} . We normalize the color to the density distribution at the maximum of each plot.

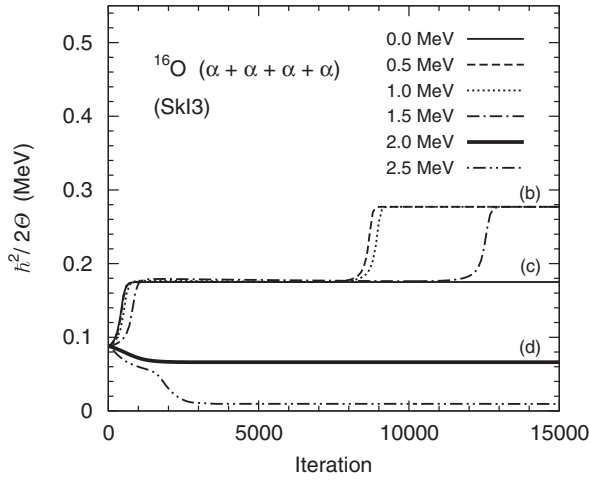


FIG. 3. Coefficient of the rotational energy $\hbar^2/2\Theta$ calculated using the cranking method versus the HF iterations with various rotational frequencies ω . The symbols (b), (c), and (d) correspond to the density distributions given in Fig. 2.

shapes. The corresponding density distribution at the 15000th iteration is plotted in Fig. 2(b). The frequency $\omega = 0.0 \text{ MeV}/\hbar$ (the solid line in Fig. 3) leads to the quasistable state [see Fig. 2(c)]. At around $\omega = 2.0 \text{ MeV}/\hbar$, we obtain the state (the thick solid line in Fig. 3) with the four- α linear chain configuration, as shown in Fig. 2(d), whereas fission occurs above those rotational frequencies (the dot-dot-dashed line in Fig. 3).

We next estimate the range of the rotational frequencies where the four- α linear chain configuration can be stabilized. Figure 4 shows the coefficient of the rotational energy $\hbar^2/2\Theta$ versus the rotational frequency ω with various Skyrme interactions. We find stable states for the four- α linear chain configuration for all of the interactions. For the SkI3 interaction, we obtain the lower and upper bounds of the rotational frequencies as 1.8 and 2.2 MeV/\hbar . Between these the four- α linear chain configuration is stabilized. The values are 1.9 and 2.2 MeV/\hbar for the SkI4 interaction and 2.0 and 2.1 MeV/\hbar for the SLy6 interaction, respectively. In these frequency regions where the linear chain configuration is stabilized, we can define the rigid-body moments of inertia, which are calculated as 0.065 MeV for the SkI3 and SkI4 interactions and 0.06 MeV for the SLy6 interaction. These values are consistent with 0.063 in a naive picture of rigid-body four- α 's laid in linear chain with 12 fm as in Fig. 2(d).

We also estimate the corresponding angular momentum where the four- α linear chain configuration is stabilized. We calculate the angular momentum using the rigid-body moment of inertia obtained and compare it with the value calculated by the cranking method. The angular momentum with the rigid-body moment of inertia J_{rid} is calculated as $J_{\text{rid}} = \Theta\omega$. The angular momentum calculated using the cranking method J_{cra} is given by $J_{\text{cra}} = \langle J \rangle$, where $\langle J \rangle$ is the expectation value of the angular momentum in the

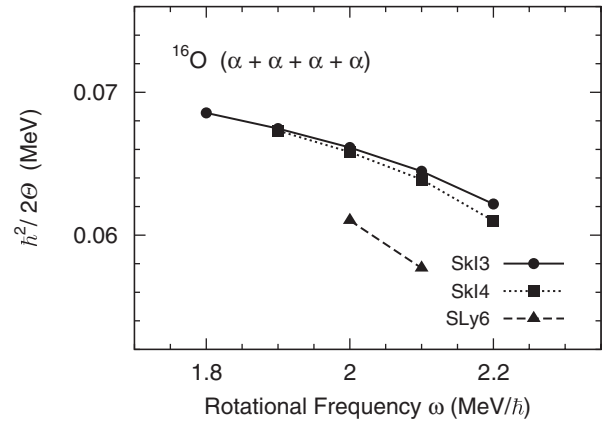


FIG. 4. Coefficient of the rotational energy $\hbar^2/2\Theta$ as a function of rotational frequency ω . The lines correspond to the different Skyrme forces as indicated.

cranking equation. Figure 5 shows the angular momentum obtained versus the rotational frequency. We see that the calculated angular momentum using the rigid-body moment of inertia agrees well with that of the cranking method, indicating that the rigid-body approximation is reasonable for the four- α linear chain states. We find that the lower and upper bounds of the angular momentum where the four- α linear chain configuration is stabilized are about 13 and 18 \hbar for the SkI3 interaction, 14 and 18 \hbar for the SkI4 interaction, and 16 and 18 \hbar for the SLy6 interaction, respectively. With such a high angular momentum, a very exotic configuration of the four- α linear chain can be stabilized. Fission occurs beyond this angular-momentum region. Furthermore, it is possible that states with even lower angular momenta are stabilized when the coupling effect with low-lying states is taken into account.

Finally, in Fig. 6 we show the excitation energies of calculated four- α linear chain states with the SkI3 force

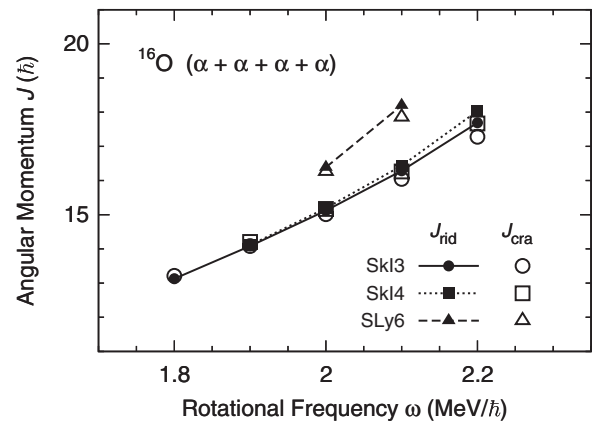


FIG. 5. Angular momentum as a function of rotational frequency ω for the Skyrme forces. The lines with solid symbols denote the calculated results for the rigid-body moment of inertia, while the open symbols denote the results for the cranking method.

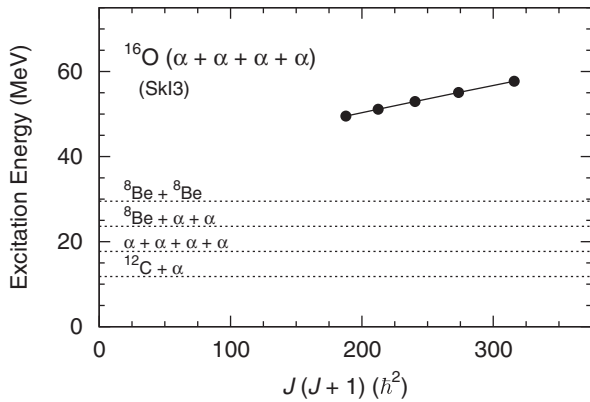


FIG. 6. Calculated excitation energies of the four- α linear chain states with the SkI3 force versus the angular momentum. The dotted lines denote the corresponding cluster-decomposition threshold energies.

versus the angular momentum J measured from the ground state of ^{16}O . We also calculate the cluster-decomposition thresholds using the ground state energies taken from Ref. [17]. Note that the HF calculations with the Skyrme force often underestimate the binding energy of ^8Be . To improve this, the angular-momentum projection would be necessary. The calculated cluster-decomposition threshold energies for $^{12}\text{C} + \alpha$, $\alpha + \alpha + \alpha + \alpha$, $^8\text{Be} + \alpha + \alpha$, and $^8\text{Be} + ^8\text{Be}$ are 11.8, 17.7, 23.6, and 29.5 MeV, respectively (the dotted lines in Fig. 6). The excitation energy is very high, at around 50 MeV with $J \sim 14$. If we extrapolate the energy to lower angular momentum, the band head energy at $J = 0$ is estimated to be around 38 MeV, which is much higher than the value suggested in Ref. [3]. This rather high excitation energy in the Skyrme forces is also reported in Ref. [9].

In summary, we have investigated the stability of the four- α linear chain configuration in ^{16}O using the Skyrme mean-field method with cranking. Even if the bending path is opened in the three-dimensional space, we obtained regions of rotational frequency where the linear chain configuration is stabilized. Below this region, the state converges to low-lying configurations, with fission occurring beyond this region. The frequency range corresponds to angular momenta of 13–18 \hbar . Furthermore, when the coupling effect with low-lying states is taken into account there is a possibility that states with even lower angular momentum are stabilized. The coefficient of the rotation ($\hbar^2/2\Theta$) of the four- α linear chain configuration obtained is around 0.06–0.08 MeV. We have, for first time, shown that states with such large moments of inertia are possible in light nuclei under conditions of large angular momenta.

As shown in this Letter, the exotic four- α linear chain state can indeed exist in ^{16}O . We also investigated whether such a state can be accessible in the $^8\text{Be} + ^8\text{Be}$ fusion reaction using the time-dependent HF method with the same Skyrme force as the present study [23]. We obtained a quasistable state with a similar moment of inertia, as shown here. The HF method is a powerful tool for

investigating both the static and dynamical properties of nuclei in the consistent framework. This method is a promising tool to reveal the existence of states with more exotic geometric configurations, such as longer linear chain [24] and polygons [16], which is still an open question for their existence.

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