# Visualization of Dimensional Effects in Collective Excitations of Optically Trapped Quasi-Two-Dimensional Bose Gases

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In quasi-two dimensions (quasi-2D), where excitations are frozen in one direction, the scattering amplitudes exhibit 2D features of the particle motion and a 3D to 2D dimensional crossover emerges in the behavior of scattering. We explore its physical consequences, capitalizing on a hidden connection between the Pitaevskii-Rosch dynamical symmetry and breathing modes. We find broken Pitaevskii-Rosch symmetry by arbitrarily small 2D effects, inducing a frequency shift in breathing modes. The predicted shift rises significantly from the order of 0.5% to more than 5% in transiting from the 3D-scattering to the 2D-scattering regime. Comparisons with other relevant effects suggest our results are observable within current experimental capabilities.

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One of the most fascinating aspects of many-body systems is the role of dimensionality [1,2]. In two dimensions (2D), remarkable phenomena arise including high-$T_c$ superconductivity [3] and Berezinskii-Kosterlitz-Thouless transition [4]. With the advent of ultracold gases in tight confinement [5], one has unprecedentedly new opportunities to explore low-D behaviors, as well as dimensional crossovers, in a highly controllable way [1].

Dimensional crossovers are characterized by hierarchical access to new energy and length scales. Bose gases become quasi-2D when energetic restriction to freeze axial excitations is reached [6,7]. Next, into the quasi-2D regime, a regularization of the coupling constant due to restricted kinematics leads to an appearance of a new length scale $a_{2D}$ competing with the 3D scattering length $a_{3D}$ [6]. Consequently, from accessing one extreme of $a_{3D}/a_{2D}$ to the other, two distinctive scattering regimes are further identified and another dimensional crossover emerges. One regime corresponds to $a_{3D}/a_{2D} \ll 1$ and features a 3D character in scattering. Whereas, the opposite $a_{3D}/a_{2D} \geq 1$ defines the 2D-scattering regime that shows strong density dependence of a coupling constant [6]. In passing, the kinematic reduction from 3D to quasi-2D in crossing the energy hierarchy has been the focus of most theoretical [7,8] and experimental efforts [5,9]. This work, on the other hand, concerns the dimensional crossover in the behavior of scattering induced by length-scale competitions. Considering that tight confinement may fundamentally alter binary atomic collisions; detailed analysis of such crossover would open a new perspective toward studying 2D many-body systems.

In this Letter, we are motivated to discuss visualizing dimensional effects in the scattering process on collective excitations of a quasi-2D Bose gas, all the way from the 3D-scattering to 2D-scattering regime with an increasing ratio of $a_{3D}/a_{2D}$. With this goal in mind, we invoke a hidden connection that exists in 2D among the scaling transformation, the Pitaevskii-Rosch (PR) dynamical symmetry and breathing modes [10,11].

*PR symmetry and breathing mode in (quasi-)2D.*—As pointed out in Ref. [10], a 2D Bose gas interacting via $g\delta^2(\mathbf{r})$ potential, with $g$ being a constant, possesses a scaling symmetry under the transformation $\mathbf{r} \rightarrow \lambda\mathbf{r}$. Associated with this scale invariance is an underlying symmetry SO(2,1) in dynamics of corresponding harmonically trapped Bose gases [10,11]. Such dynamical symmetry, discovered by Pitaevskii and Rosch [10], dictates a universal frequency belonging with breathing modes. Alternatively, deviations from this universality provide sensitive measurements of quantum many-body effects that break the PR symmetry. An example has been given by a pure 2D quantum Bose gas in Ref. [12].

We now extend the above analysis of scaling property and PR symmetry to quasi-2D. To this end, let us consider typical schemes to produce quasi-2D Bose gases that include a 1D optical lattice $V_{\text{opt}} = sE_R\sin^2(q_Bz)$. The lattice period is fixed by $q_B = \pi/d$ with $d$ being the lattice spacing, $s$ is a dimensionless factor labeled by the intensity of a laser beam, and $E_R = \hbar^2 q_B^2/2m$ is the recoil energy with $\hbar q_B$ being the Bragg momentum. The quasi-2D regime is reached when the energetic restriction $4t/\mu \ll 1$ is fulfilled, $t$ being the tunneling rate and $\mu$ the chemical potential. In this regime, tunneling is negligibly small compared to effects of tight confinement [7] and the system [1] can be effectively modeled by

$$H_{Q2D} = \sum_j \frac{p_{jx}^2 + p_{jy}^2}{2m} + g_{Q2D}\sum_{j<k}\delta^2(\mathbf{r}_{jk}). \quad (1)$$

 

Here, axial degrees of freedom are taken into account via a kinematically renormalized coupling constant $g_{Q2D}$ that reads [6,7]

$$g_{Q2D} = \frac{2\sqrt{2\pi}\hbar^2}{m} \frac{1}{a_{2D}/a_{3D} + 1/\sqrt{2\pi}\log[B/\pi^2 n_{2D} a_{2D}^2]}, \tag{2}$$

with $B = 0.915$ [7] and $n_{2D} = nd$ being the surface density. Here, the identification $a_{2D} = \sigma$ [7] has been made, with $\sigma$ characterizing the axial extension of local wave function determined from $d/\sigma \simeq \pi s^{1/4} \exp(-1/4\sqrt{s})$ [13]. The existence of two length scales in the coupling constant is immediately evident from Eq. (2). The consequences are (a) the appearance of a log density-dependent correction as a manifestation of 2D effects in scattering, its strength being measured by the ratio $a_{3D}/a_{2D}$, and (b) the identification of two distinctive scattering regimes: the 3D-scattering regime $a_{3D}/a_{2D} \ll 1$ with weak 2D effects and the 2D-scattering one $a_{3D}/a_{2D} \geq 1$ where kinematic modification of the coupling constant is important.

At this point, it is instructive to recall most experiments on 2D quantum gases to date [14], where a small ratio $a_{3D}/a_{2D} \leq 0.05$ is usually pursued. In this case, the density-dependent part consists of a 2% contribution to $g_{Q2D}$ and is typically ignored. Accordingly, the 2D coupling constant was frequently evaluated in the experiment according to $g_{Q2D} = \sqrt{8\pi}\hbar^2 a_{3D}/m a_{2D}$. Such an approximation is also widely used in theoretical studies [8].

Nevertheless, we stress that in ignoring the 2D effects in $g_{Q2D}$, contained in higher orders of $a_{3D}/a_{2D}$, one may also lose interesting physics associated with these fine structures in the coupling constant. Fundamentally, the scale invariance of $H_{Q2D}$ in Eq. (1) is immediately violated for even a 2% density dependence in $g_{Q2D}$. The corresponding PR symmetry thus inevitably breaks in the presence of the 2D effect, as small as it is, and breathing modes deviate from universality. An illustration is useful by using the equations of motion for the corresponding excitation operator $F_B = \sum_i (x_i^2 + y_i^2)$,

$$\frac{d^2 F_B}{dt^2} + 4\omega_\perp^2 F_B = \frac{4}{m}\left[E - \sum_{j<k}\{\mathbf{r}_j \cdot \nabla g_{Q2D} + \text{H.c.}\}\delta^2(\mathbf{r}_{jk})\right], \tag{3}$$

with $E$ being the total energy of the system. Equation (3) is equally valid classically as it is quantum mechanically [10,11]. Compared to Ref. [10], the frequency shift from the universal value of $\omega_B = 2\omega_\perp$ is evident with the addition of the density-dependent correction of the coupling constant. Moreover, as the 2D character in scattering becomes increasingly pronounced for $a_{3D}/a_{2D} \geq 1$, significant modifications in collective excitations are expected.

Next, we calculate analytically the frequency shift in breathing modes of a quasi-2D Bose gas, along the crossover from the 3D-scattering to 2D-scattering regime. To begin with, we generalize the hydrodynamic equations [8] to quasi-2D Bose gases in a harmonic trap $V_{ho}(\mathbf{r}) = \frac{1}{2} m(\omega_\perp^2 r_\perp^2 + \omega_z^2 z^2)$ within the local density approximation [15],

$$\frac{\partial n}{\partial t} + \nabla \cdot [n(\mathbf{r})v] = 0, \tag{4}$$

$$\hbar \frac{\partial v_i}{\partial t} + \frac{1}{m_i}\nabla_i[\mu_{Q2D}(n) + V_{ho}(\mathbf{r})] = 0, \tag{5}$$

with $i = x$, $y$, and $z$. Here $m_i$ is the mass $m$ for $i = x$, $y$ and effective mass $m^*$ for $i = z$, respectively. Equations (4) and (5) are justified by sufficiently weak tunneling which is nevertheless nonnegligible to ensure full coherence of the order parameter between different wells [8], and by assuming the Thomas-Fermi (TF) limit [15]. The 3D density $n(\mathbf{r})$ is determined from $\mu_0 = \mu_{Q2D}[n(\mathbf{r})] + V_{ho}(\mathbf{r})$, where $\mu_0$ is the ground state value of the chemical potential, fixed by the proper normalization of $n(\mathbf{r})$. The local equation of state $\mu_{Q2D}$ is at the core of hydrodynamic analysis, which can be derived from the density derivative of the ground state energy corresponding to Eq. (1).

*Ground state of a quasi-2D Bose gas.*—To this end, we first adopt from Ref. [16] a general expression for the ground state energy of dilute Bose gases in the presence of a 1D optical lattice,

$$\frac{E_g}{V} = \frac{1}{2}\tilde{g}_e n^2\left[1 + \frac{m\tilde{g}_e}{2\pi^2\hbar^2 d} F(2t/\tilde{g}_e n)\right], \tag{6}$$

with $\tilde{g}_e = g_{Q2D} d$ and the function

$$F(x) = \frac{(x+1)}{2}[(3x+1)\arctan(1/\sqrt{x}) - 3\sqrt{x}] + \frac{\pi}{2}\log[(2x+1+2\sqrt{x^2+x})/x] - \pi\text{arcsinh}(\sqrt{x}) + 2\int_0^{\sqrt{x}} \frac{\tan^{-1}(z)}{z} dz \tag{7}$$

arises from beyond-mean-field contributions due to quantum fluctuations. Equation (6), derived within the tight-binding model and Bogoliubov approximation, is valid all the way from the anisotropic 3D regime to the opposite pure 2D one [16]. Proceeding from Eq. (6), we restrict ourselves within the quasi-2D regime and hence take the limit $x = 2t/n\tilde{g}_e \ll 1$ where $F(x) = \pi/4 - \pi/2 \log x$ is asymptotically approached. The result then reads

$$\frac{E_g}{V} = \frac{1}{2}\tilde{g}_e n^2\left[1 + A\tilde{g}_e\left(\frac{\pi}{4} - \frac{\pi}{2}\log[2t/\tilde{g}_e n]\right)\right], \tag{8}$$

with $A = m/2\pi^2\hbar^2 d$. Hereafter, we shall derive the $\mu_{Q2D}$ and find breathing modes by solving Eqs. (4) and (5) in the limit $\sqrt{m/m^*}\omega_z/\omega_\perp \ll 1$ for $a_{3D}/a_{2D} \ll 1$ and $a_{3D}/a_{2D} \geq 1$, respectively.

*3D-scattering regime.*—Defined by $a_{3D}/a_{2D} \ll 1$, this regime features a 3D character in scattering, with small modifications from the 2D effect. Hence, by linearizing Eq. (2) with respect to $a_{3D}/a_{2D}$, we obtain for $\tilde{g}_e = g_{Q2D}d$ in Eq. (8) as

$$\tilde{g}_e = \tilde{g}\left[1 - \frac{1}{\sqrt{2\pi}}\frac{a_{3D}}{a_{2D}}\log(B/\pi^2 n_{2D} a_{2D}^2)\right], \quad (9)$$

where $\tilde{g} = 4\pi\hbar^2 a_{3D} d/\sqrt{2\pi} m a_{2D}$ is the familiar 3D lattice renormalized coupling constant and the log density-dependent term presents the leading 2D correction. The $\mu_{Q2D}$ is then readily derived from Eq. (8), yielding

$$\mu_{Q2D} = \tilde{g}n\left[1 + \frac{1}{\sqrt{2\pi}}\frac{a_{3D}}{a_{2D}}\left(\frac{1}{2} - \log\left[\frac{B}{\pi^2 n_{2D} a_{2D}^2}\right]\right)\right]. \quad (10)$$

Here, we have ignored effects of quantum fluctuations in Eq. (8) that give higher order corrections with respect to the pure 3D value $\tilde{g}n$. Finally, rewriting $\mu_{Q2D} = \tilde{g}n[1 + k_{2D}(n)]$, we identify $k_{2D}(n) = a_{3D}/(\sqrt{2\pi}a_{2D}) \times [1/2 - \log(B/\pi^2 n_{2D} a_{2D}^2)]$ as the first correction to the 3D mean-field equation of state arising from 2D effects.

From Eq. (10), one solves the equation for the 3D ground state density by iteration and finds $n(\mathbf{r}) = n_{\rm TF} - a_{3D}/(\sqrt{2\pi}a_{2D})[1/2 - \log(B/\pi^2 d n_{\rm TF} a_{2D}^2)]n_{\rm TF}$, with $n_{\rm TF}(\mathbf{r}) = [\mu_0 - V_{\rm ext}(\mathbf{r})]/\tilde{g}$ being the 3D TF density. The 2D effects are then transferred to the 3D stationary shape of a cloud. Linearizing Eqs. (4) and (5) with substitutions of $n(\mathbf{r})$ and Eq. (10) gives

$$m\omega^2\delta n + \tilde{\nabla}\cdot[\tilde{g}n_{\rm TF}\tilde{\nabla}\delta n] = -\tilde{\nabla}^2\left[\tilde{g}n_{\rm TF}^2\frac{\partial k_{2D}}{\partial n_{\rm TF}}\delta n\right], \quad (11)$$

with $\tilde{\nabla} \equiv [\nabla_\perp, \nabla_z\sqrt{m/m^*}]$. Note that the axial harmonic trapping frequency has been renormalized by the effect of lattice $\tilde{\omega}_z = \sqrt{m/m^*}\omega_z$. Equation (11) in the absence of $k_{2D}$ recovers the usual 3D hydrodynamic equation in the presence of a 1D optical lattice [8]. Against this background, the perturbation from the right of Eq. (11) leads to a fractional frequency shift

$$\frac{\delta\omega}{\omega} = -\frac{\tilde{g}}{2m\omega^2}\frac{\int d^3\mathbf{r}\tilde{\nabla}^2\delta n^*(n_{\rm TF}^2\frac{\partial k_{2D}}{\partial n_{\rm TF}}\delta n)}{\int d^3\mathbf{r}\delta n^*\delta n}, \quad (12)$$

where $\omega$ is the 3D mean-field result of collective frequencies.

We are interested in transverse breathing modes in a very elongated trap ($\tilde{\omega}_z/\omega_\perp \ll 1$). Substitutions of $\delta n(\mathbf{r}) \sim r_\perp^2 - \mu_0/m\omega_\perp^2$ and $\omega_B = 2\omega_\perp$ into Eq. (12) yield

$$\frac{\delta\omega}{\omega_B} = \frac{1}{4\sqrt{2\pi}}\frac{a_{3D}}{a_{2D}}. \quad (13)$$

For typical experiments to date [17], $a_{3D} = 5.31$ nm and the lattice period $d = 297.3$ nm. The frequency shift in Eq. (13) can then be reached $\sim 0.48\%$ for $s = 4$. As small as it is, given an accuracy of $\sim 0.3\%$ in measuring collective frequencies within current facilities [18], this 2D correction presents a visible effect in experiments [19].

It is worth noting that Eq. (12) applies to other low-lying collective excitations as well. Particularly, it follows that surface modes with $\tilde{\nabla}^2\delta n = 0$ are unperturbed by dimensional effects. Additionally, Eq. (12) is valid for various anisotropies of harmonic traps. For example, in a disklike geometry ($\sqrt{m/m^*}\omega_z/\omega_\perp \gg 1$), one finds $\delta\omega/\omega = m^* a_{3D}/(6\sqrt{2\pi} m a_{2D})$ for the lowest compression mode with the zeroth order dispersion $\omega = \sqrt{m/m^*}\sqrt{3}\omega_z$ and density oscillation $\delta n(\mathbf{r}) \sim z^2 - 2m^*\mu_0/3m\omega_z^2$.

*2D-scattering regime.*—In this regime, $a_{3D}/a_{2D} \geq 1$ and 2D effects determine the nature of scattering, resulting in strong log density dependence of $g_{Q2D}$ in Eq. (2). Meanwhile, effects of quantum fluctuations become increasingly important in transiting deep into the 2D-scattering regime [1]. Hence, by substituting the $g_{Q2D}$ in Eq. (2) into $\tilde{g}_e = g_{Q2D}d$ and fully accounting for effects of quantum fluctuations in Eq. (8), we obtain the equation of state to the second order in $\tilde{g}_e$,

$$\mu_{Q2D} = \tilde{g}_e n + \tilde{g}_e^2 n\frac{m}{4\pi\hbar^2 d}[1 - \log(2t/\tilde{g}_e n)]. \quad (14)$$

To find breathing modes, we solve Eqs. (4) and (5) via the Castin-Dum-Kagan-Surkov-Shlyapnikov (CDKSS) scaling ansatz [20,21]: $n = n_0(r_i/\lambda_i)/\Pi_j\lambda_j$, $v_i = (\dot{\lambda}_i/\lambda_i)r_i$ $(i = x, y, z)$. The scaling parameter $\lambda_i$ thus obeys the CDKSS equation

$$d^2\lambda_i/dt^2 + \Omega_i^2\lambda_i + (\Omega_i^2/\lambda_i)F(\lambda_x, \lambda_y, \lambda_z) = 0, \quad (15)$$

with $\Omega_{x(y)} = \omega_\perp$, $\Omega_z = \sqrt{m/m^*}\omega_z$, and

$$F = \frac{1}{m(\omega_i)^2\langle r_i^2\rangle_0}\frac{1}{\lambda_i}\int n_0(r_i/\lambda_i)r_i\frac{\partial\mu_{Q2D}}{\partial r_i}d\mathbf{r} \quad (16)$$

being independent of the particular coordinate $i$. Then, linearizing Eq. (15) and taking the limit $\sqrt{m/m^*}\omega_z/\omega_\perp \ll 1$, one finds the fractional shift for breathing modes of a quasi-2D Bose gas reading

$$\frac{\delta\omega}{\omega_B} = \left[1 + \frac{1}{\sqrt{2\pi}^3}\frac{1}{\frac{a_{3D}}{a_{2D}} + \frac{1}{\sqrt{2\pi}}\log[B\hbar\omega_0/\pi\mu]}\right]^{1/2} - 1, \quad (17)$$

where $B = 0.915$ and $\hbar\omega_0/\mu = 1/\pi n(0)a_{2D}^2$, with $n(0)$ being the density at the center of the harmonic trap. In typical experiments, the gas is deep in the quasi-2D regime for $\hbar\omega_0/\mu = 6$ [14]. Thus, at the onset of dimensional crossover in scattering defined by $a_{3D}/a_{2D} = 1$, the fractional shift is calculated as 5%, a significant rise in comparison with that in the 3D-scattering regime.

Equations (13) and (17) are key results of this Letter. Frequency shifts in both equations represent leading order corrections to breathing mode frequency that has its origin in broken PR symmetry by 2D effects in quasi-2D. Equation (13) shows that such effects in the 3D-scattering regime are indeed small and only cause consequences down to scales $\leq 0.5\%$. This explains, therefore, their invisibility in previous experiments [9,19] and also justifies the approximation of $g_{Q2D}$ by a constant in previous studies [8,14], provided $a_{3D}/a_{2D} \ll 1$ is satisfied and the physics concerned is on a scale larger than thatof dimensional effects. On the other hand, we point out that a precision of $\leq 0.3\%$ in measuring collective frequencies [18] has already been established, offering opportunities to probe many-body physics associated with fine structures in the coupling constant. In contrast, Eq. (17) shows a dimensional effect well in reach in experiments, giving an important implication that the physical nature of quasi-2D Bose gases for $a_{3D}/a_{2D} \geq 1$ may be profoundly influenced by 2D effects in scattering and subsequent regularization of the coupling constant.

*Discussion.*—Careful comparisons with other influences have to be made, including finite size, nonlinearity, temperature and vortex, etc. Finite size effects originate from kinetic energy pressure typically ignored in the TF scheme [15]. Its consequence on transverse breathing modes can be analyzed via a sum rule approach [15] with $\omega = \sqrt{m_3/m_1}$ where $m_3 = (8\hbar^4/m^2)(E_{\mathrm{kin}\perp} + E_{\mathrm{ho}\perp} + E_{\mathrm{int}})$ and $m_1 = (2\hbar^2/m)N\langle F_B\rangle$ are, respectively, the cubic energy weighted and the energy weighted moments of the dynamic structure factor. With the viral identity $E_{\mathrm{kin}\perp} - E_{\mathrm{ho}\perp} + E_{\mathrm{int}} = 0$, one finds $\omega = 2\omega_\perp$ unaffected by the finite size effect within the Gross-Pitaevskii approximation.

Nonlinear effect arises from large amplitude oscillations that induce mode coupling and shift collective frequencies by $\delta\omega/\omega = A^2\delta$ [22]. Here, the amplitude $A$ can be tuned to below 10%. In our case, we find the coefficient $\delta = \frac{5}{2}\epsilon^2\frac{(q_- -2)(q_+ -4)(q_- -5)}{(4q_+ - q_-)(q_- - q_+)^2}[-1 + \frac{15}{4}\frac{\epsilon^2}{q_+^2}] - \frac{15}{16}\times\frac{1}{(q_- - q_+)^2}[-q_+ + 2\epsilon^2 q_+ - 9\epsilon^2 + 8]^2 - \frac{9}{4}\frac{(q_- -4)}{q_+(q_+ - q_-)} - \frac{3}{20}\times\frac{q_+ -3}{q_+ - q_-}[-10\epsilon^2 q_+ + 37\epsilon^2 + 11q_+ - 54]$ with $q_\pm = 2 + \frac{3}{2}\epsilon^2 \pm \frac{1}{2}\sqrt{9\epsilon^4 - 16\epsilon^2 + 16}$. The final shift is $\leq 10^{-4}\%$ for typical trap anisotropy $\epsilon = \sqrt{m/m^*}\omega_z/\omega_\perp \sim 0.05$, much smaller than dimensional effects. Additionally, thermal effects have also been observed to cause unusually small frequency shifts in transverse breathing modes of an elongated condensate [11,23].

The issue of vortex is closely related to operational anisotropy in excitation schemes. To excite transverse breathing modes, it is required to quench transverse harmonic trap frequencies in phase by $\delta\omega_{x(y)} = \delta\omega \ll \omega_\perp$. On the contrary, an out-of-phase operation with $\delta\omega_x = \delta\omega$ and $\delta\omega_y = \delta\omega'$ gives rise to an excitation operator $F = m\omega_\perp(\delta\omega + \delta\omega')/2 \times F_B + m\omega_\perp(\delta\omega - \delta\omega')/2\sum_i r_i^2(Y_{2,2} + Y_{2,-2})$ ($Y_{lm}$ being spherical harmonics) resulting in additional excitations of quadrupole modes. Such a situation may be further spoiled by the presence of a quantum vortex which splits the $m = \pm 2$ quadrupole modes by approximately $(\omega_+ - \omega_-)/2\omega_\perp = 7\omega_\perp(15\epsilon N\tilde{a}_{3D}/a_\perp)^{-2/5}$ [24]. For typical parameters $N = 2\times 10^6$, $\epsilon = 0.05$, and $a_{3D}/a_{2D} = 0.05$, we calculate the shift as $\sim 0.07\%$, again negligibly small in comparison.

In conclusion, we find PR symmetry breaks in a quasi-2D Bose gas for the arbitrary small presence of 2D effects in scattering. The results consist of a shift of breathing modes away from the PR-symmetry-dictated value. Such a shift rises markedly from 0.5% to 5% in transiting from the 3D-scattering ($a_{3D}/a_{2D} \ll 1$) to the 2D-scattering regime ($a_{3D}/a_{2D} \geq 1$), both observable within current experimental facilities. Observing this dimensional effect directly would present an important step in revealing the interplay between dimensionality and quantum fluctuations in quasi-2D.

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