Generation and Amplification of Magnetic Islands by Drift Interchange Turbulence

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(Received 21 December 2010; published 23 August 2011)

We investigate the multiscale nonlinear dynamics of a linearly stable or unstable tearing mode with small-scale interchange turbulence using 2D MHD numerical simulations. For a stable tearing mode, the nonlinear beating of the fastest growing small-scale interchange modes drives a magnetic island with an enhanced growth rate to a saturated size that is proportional to the turbulence generated anomalous diffusion. For a linearly unstable tearing mode the island saturation size scales inversely as one-fourth power of the linear tearing growth rate in accordance with weak turbulence theory predictions. Turbulence is also seen to introduce significant modifications in the flow patterns surrounding the magnetic island.

DOI: 10.1103/PhysRevLett.107.095003 PACS numbers: 52.55.Tn, 52.35.Ra, 52.55.Fa

Plasmas in nature as well as in laboratory devices often harbor large scale magnetic structures that play an important role in the global dynamics of the system. Some well known examples of such structures are solar flares or coronal loops in the sun and magnetic islands in a tokamak plasma [1]. Quite frequently these structures coexist with fine scale micro-structures associated with turbulent fluctuations arising from various microinstabilities in the system. While the occurrence of these disparate scales of disturbances can be broadly attributed to the macrosopic and microscopic stability properties of the system, their origins and dynamics are often not independent of each other and can be the result of a complex multiscale interaction process. Thus a large scale magnetic island may result not only from a macroscopic tearing instability but may also be driven by small-scale microturbulence. Likewise a growing magnetic island may significantly influence the character and evolution of ambient turbulent fluctuations in the system and thereby alter its global transport properties. The mutual interaction between microturbulence and macro modes in a plasma continues to be an active and challenging area of research with important applications to several astrophysical and laboratory phenomena. One such fundamental problem involves the excitation and dynamical behavior of a tearing mode due to the presence of a background of microscopic turbulence. Early analytic attempts to investigate this important question used ad-hoc modeling of turbulence effects through anomalous transport coefficients [2] to study the threshold behavior and modified growth rates of tearing modes. In [3] a stochastic turbulence noise source was introduced in a Rutherford model to demonstrate the possibility of triggering a small size "seed" island. A multiscale numerical simulation study in [4] showed that microscopic fluctuations due to resistive drift wave turbulence could enhance the growth of a tearing mode to a rate much faster than its linear one. Conversely, it was shown in [5] that the reconnection rate may be reduced in the presence of a turbulent flow due to a transfer of energy from the MHD instability into shorter wavelength modes. In [6], a critical island width was identified such that below (above) it turbulence enhanced (healed) the magnetic island. Numerical simulation studies in [7,8] have addressed the problem of multiscale interactions, taking into account the nonlinear modifications of the equilibrium due to the interaction of the profiles, zonal flows and MHD instabilities with the turbulence. However, several basic questions still remain unresolved, e.g., the mechanism and conditions determining the excitation of a magnetic island by microturbulence and the nature and role of flows surrounding the saturated island structure. Our present study is motivated by these questions and is aimed at a detailed investigation of these points through numerical simulation studies of the interaction between tearing modes and drift interchange turbulence under a variety of conditions. Interchange type turbulence is generic in a tokamak plasma where they can arise in the core region through instabilities of the ion temperature gradient (ITG) mode or the trapped electron mode (TEM) [9]. They can also be found in the tokamak edge region under high β conditions when they tend to predominate over electrostatic drift modes. Our principal findings are that the generation and enhanced rate of growth of the magnetic island is predominantly due to the nonlinear beating of the fastest growing small-scale interchange modes and happens irrespective of whether the tearing mode is linearly stable or unstable. For a linearly stable tearing mode the nonlinear saturation size of the island has a linear dependence on the magnitude of the "anomalous" diffusion created by the turbulence. For a linearly unstable tearing mode the saturated island width scales inversely as the one-fourth power of the linear growth rate of the tearing mode. Another finding is that the large scale asymptotic flow structure outside the saturated island has a dipole pattern and differs significantly from the conventional quadrupole patterns seen in tearing mode islands that develop in the absence of turbulence. Finally, we also see evidence of the magnetic island modifying the nature of the turbulence through symmetry breaking mechanisms.

Our simulations are carried out on a minimalist two-dimensional plasma model based on the two fluid Braginskii equations in the drift approximation [10,11] with cold ions and isothermal electrons. The model includes magnetic curvature and electron diamagnetic effects [12]:

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + \{\phi,\nabla_\perp^2\phi\} = \{\psi,\nabla_\perp^2\psi\} - \kappa_1\frac{\partial p}{\partial y} + \mu\nabla_\perp^4\phi, \quad (1)$$

$$\frac{\partial}{\partial t}\psi = \{\psi, \phi - p\} - v_\star\frac{\partial\psi}{\partial y} + \eta\nabla_\perp^2(\psi - \psi_0), \quad (2)$$

$$\frac{\partial}{\partial t}p + \{\phi, p\} = -v_\star\left((1-\kappa_2)\frac{\partial\phi}{\partial y} + \kappa_2\frac{\partial p}{\partial y}\right) + \hat{\rho}^2\{\psi,\nabla_\perp^2\psi\} + \chi_\perp\nabla_\perp^2 p, \quad (3)$$

where the dynamical field quantities are the electrostatic potential ϕ, the electron pressure p and the total magnetic flux ψ (with $\psi_0 = \psi_0(x)$ denoting its equilibrium profile). The equilibrium consists of a constant pressure gradient and a magnetic field given by the Harris current sheet model [1,13], namely, $\mathbf{B_0}(x) = \tanh(\frac{x - L_x/2}{a})\hat{\mathbf{y}}$, where L_x is the box size in the x (radial) direction and a determines the width of the profile. Eqs. (1)–(3) are normalized using the characteristic Alfven speed v_A, the magnetic shear length $L_\perp$ and the Alfven time $\tau_A = L_\perp/v_A$. Curvature effects are included through κ_i ($i = 1, 2$) parameters and are responsible for the interchange instabilities.

The impact of interchange turbulence on the formation of a magnetic island is investigated by means of linear and nonlinear simulation of Eqs. (1)–(3). A semispectral code is used including a 2/3 dealiazing rule in the y (poloidal) direction, a resolution of 256 grid points in the x (radial) direction and 64 poloidal modes. The computational box size is $L_x = 2\pi$ and $L_y = 5\pi$. The perturbed fields are periodic in the y (poloidal direction) and are set to zero at the radial boundaries. The Fourier decomposition of the fields is typically defined as $\psi(x, y, t) = \sum_{m\in Z}\psi_m(x, t) \times \exp(ik_m y)$ with $k_m = 2\pi m/L_y$. The parity (odd or even symmetry in the spatial coordinate) of the eigenfunctions $\psi_m(x, t)$, $\phi_m(x, t)$, $p_m(x, t)$ provides a distinct marker of identification of a given mode m and helps in pinpointing the instability mechanism generating it. The resistive interchange mode m has (odd, even, even) parities with respect to $x \in [-L_x/2, L_x/2]$, for (ψ_m, ϕ_m, p_m), respectively, and (even, odd, odd) parities for tearing modes. In this study we have fixed $\hat{\rho} = 0.04$, $v_\star = 10^{-2}$, $\kappa_2 = 0.36$ and the dissipative parameters $(\mu, \chi_\perp, \eta)$ are taken to be equal to 10^{-4}. As shown in [12], in order to overcome the stabilizing effect of large scale magnetic field with respect to interchange modes, a large value of the curvature parameter is required and we set $\kappa_1 = 5$. We next categorize the most unstable interchange mode number by $m_\star$ and its growth rate by $\gamma_{m_\star}$. Typically $m_\star \gg 1$. In this work the modes $m \geq 2$ are stable with respect to tearing instability. The nature (parity) of the $m = 1$ mode depends on the competition between the interchange and tearing instabilities. The stiffness of the magnetic equilibrium profile modifies the growth rate of both interchange and tearing modes. As seen in [12], the smaller Δ' is, the larger are $m_\star$ and $\gamma_{m_\star}$ (e.g., for $\Delta' = -0.15$, the spectrum of the unstable modes is quite broad and ranges from $m = 1$ to $m = 29$ with $m_\star = 15$ and for $\Delta' = 1.16$ gives a spectrum from $m = 2$ to $m = 13$ with $m_\star = 6$). The value of the $m = 1$ tearing stability index Δ' can be seen as a measure of the stiffness. For the Harris equilibrium the value of Δ' for a given mode number can be easily varied by changing the parameter a while holding the box size to be constant.

To investigate how the small-scale interchange modes affect the formation of a magnetic island, we carry out a range of linear and nonlinear simulations with different values of Δ'. Figure 1 shows the linear growth rate of the modes $m = m_\star$ and $m = 1$ as a function of the parameter Δ'. The colors distinguish the nature (parity) of the modes. The mode $m_\star(\Delta') \in [6, 17]$ is linearly unstable and has an interchange parity.

Linearly, the $m = 1$ mode develops an interchange parity as soon as $|\Delta'|$ is of the order of unity or smaller (Δ' is not large enough to let the tearing mode grow in place of the linear $m = 1$ interchange mode). The nonlinear simulations show, that, starting from a linear situation where the $m = 1$ mode is driven by interchange instability without the formation of a magnetic island, a nonlinear beating of

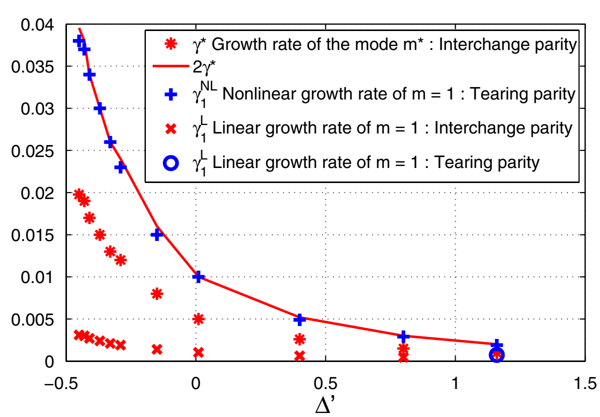

FIG. 1 (color online). Growth rates of the modes $m_\star$ and $m = 1$: blue points correspond to a mode with tearing parity and red points correspond to a mode with interchange parity.

the most unstable interchange modes, $m_\star$ and $m_{\star\pm1}$, gives rise to the formation of an $m = 1$ island with an enhanced growth rate. As an example, for $\Delta' = -0.45$, the most unstable mode is $m_\star = 17$ with $\gamma_{m_\star} \approx 0.0198$. The sideband growth rates are of nearly equal magnitude, $\gamma_{m_\star \pm 1} \approx 0.0196$. As shown in Fig. 1, the nonlinear growth rate of the $m = 1$ mode is about 0.038 which is much larger than the linear growth rate and nearly the sum of the growth rates of the $m = m_\star$ interchange mode and its sideband. This trend of $\gamma_1^{\mathrm{NL}} \sim \gamma_{m_\star} + \gamma_{m_{\star\pm1}} \sim 2\gamma_{m_\star} > \gamma_1^L$ holds over the entire range of Δ' values shown in Fig. 1. The nonlinear beating of the interchange modes also leads to a change in the parity of the driven $m = 1$ mode. As a consequence a magnetic island is nonlinearly generated by the pumping of the interchange modes even when $\Delta' < 0$. Indeed, an important property of *all* the nonlinearities in Eqs. (1)–(3) is that, if initially, the system is driven by small-scale interchange modes I_{ss}, their mutual interactions can only drive tearing parity large scale fluctuations T_{ls}: $\{I_{\mathrm{ss}}, I_{\mathrm{ss}}\} \rightarrow T_{\mathrm{ls}}$. One might ask at this point whether a secondary tearing instability governs the generation of the nonlinear $m = 1$ magnetic island? A detailed analysis of our simulation results show that, during the nonlinear formation of the island, the mode beating process does not generate a local modification of the equilibrium profile. In other words, the nonlinear interactions do not lead to an alteration of the mode ψ_0 and indeed does not bring about an increase of Δ' and a consequent destabilization of the tearing mode. Figure 2 shows the time evolution of the kinetic energy of the modes (a) $m = 0$, 1, and (b) $m = m_\star$, $m_\star + 1$ for a nonlinear simulation run with $\Delta' = -0.45$. Different regimes are observed. For $0 \leq t/\tau_A \leq 300$, the dynamics is purely linear. The modes possess the interchange parity and grow at the linear interchange growth rates. Then, for $300 \leq t/\tau_A \leq 750$, we enter the accelerating phase where the growth rate of mode $m = 1$ is $\sim 2\gamma_{m_\star}$. More precisely, the mode $m = 0$ is generated with a tearing parity thanks to a beating of the mode $m_\star$ with itself which gives $\gamma_0 \sim 2\gamma_{m_\star}$. So as soon as $2\gamma_\star > \gamma_1$, the tearing instability being no more efficient, the pumping mechanism from small scales to large scale governs the formation of the magnetic island. It should be mentioned that a similar enhancement of the island growth was first observed in [4] where an unstable tearing mode was evolved in the presence of drift wave turbulence.

Then, from $t/\tau_A > 750$, the dynamics becomes fully nonlinear. The island continues to grow slowly and finally reaches the saturation regime asymptotically. At the interchange scales the system saturates energetically where the large scale modes $m = 0$ and $m = 1$ dominate (Fig. 2). The interchange mode $m = m_\star$ starts to lose its parity and tends to get a tearing parity. A cascade directly from the large tearing scale to the small scales becomes dominant. Indeed, the nonlinear properties of Eqs. (1)–(3) show that the mutual nonlinear interaction of large scale tearing modes T_{ls} can drive only tearing parity small-scale fluctuations T_{ss}: $\{T_{\mathrm{ls}}, T_{\mathrm{ls}}\} \rightarrow T_{\mathrm{ss}}$. In fact, an accurate analysis of the energy transfer processes shows that the tearing parity is transmitted to the small scales by a beating of the mode $m = 1$ with the other modes ($m = 2$, $m = 3, \ldots$) mainly through the Maxwell stress. This mechanism changes the nature of the turbulence and together with the Ohmic dissipation balances the pumping of the small-scales energy by the magnetic island.

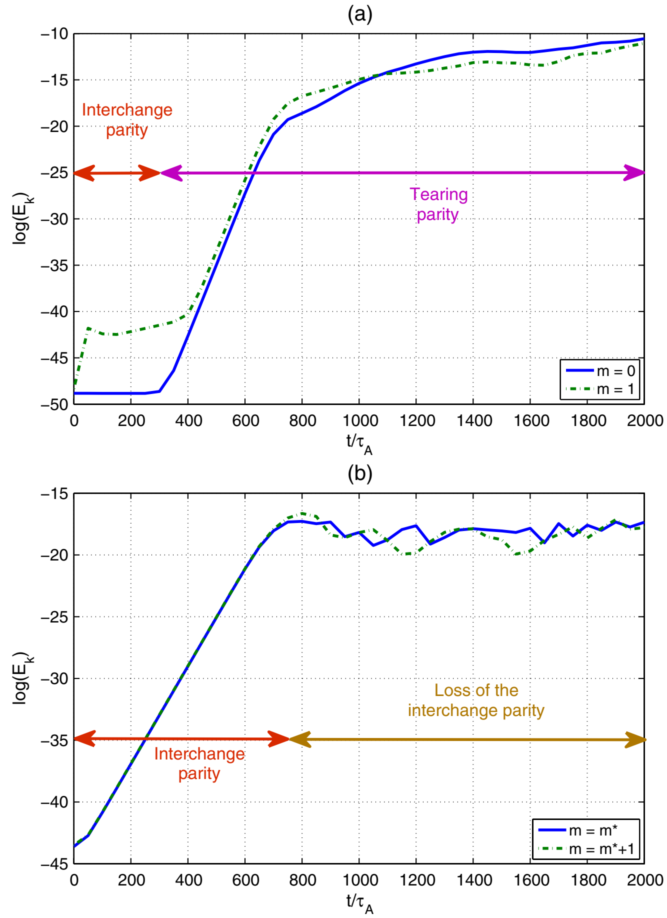

FIG. 2 (color online). $\Delta' = -0.45$. Time evolution of the kinetic energy of the modes: (a) $m = 0$, $m = 1$ (b) $m = m_\star$, $m = m_\star + 1$

The snapshots of the electrostatic potential ϕ and the magnetic flux ψ during the final nonlinear regime at $t = 5600\tau_A$ are presented in Fig. 3. They reveal that large scale tearing parity modes dominate and a magnetic island is present. The nonlinear structure of the observed mode in Fig. 3 is seen to be strongly affected by the presence of the small scales and is linked to their radial localization in the vicinity of the resonant surface of the energy source. The pattern of the electrostatic equipotential contours surrounding the magnetic island display a dipolar structure instead of the quadrupolar structure usually observed in the absence of the interchange turbulence. The corresponding velocity flow is therefore one that is going in and out in

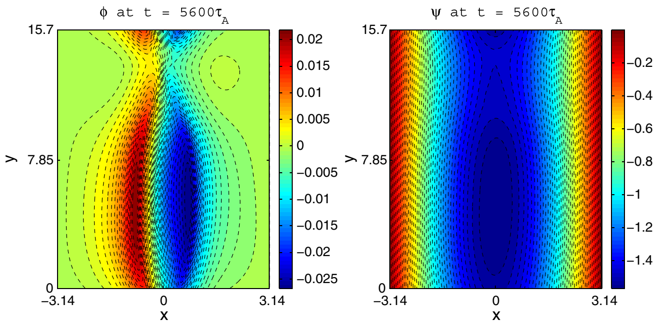

FIG. 3 (color online). $\Delta' = -0.45$. Snapshots of the electrostatic potential ϕ and the magnetic flux ψ at $t = 5600\tau_A$ during the fully nonlinear regime.

the vicinity of the x point. A qualitative understanding of this flow pattern can be obtained from an analysis of the energy transfer processes [12] which shows that the main nonlinear energy transfers from small scales to large scales are driven through the advection of the pressure by the flow in Eq. (3). Thus the nonlinear interaction of the small-scales interchange modes through the pressure advection is the energy source for the growth and the saturation of the island. The system therefore needs to adopt an appropriate structure to evacuate this energy and in fact the dipolar structure enhances the dissipation of the energy through Joule effect along the separatrices [14]. The effect of interchange small scales on the saturated island size is also an important question. For $\Delta' < 0$, a Rutherford analysis [1] that includes the curvature contribution in Eq. (1) shows the island saturation size to scale as: $w_{\rm sat} = C/(\psi_0''^2 \Delta')$ where $C = -6.17\kappa_1 v_\star$. Our numerical results show that in the fully nonlinear regime the saturated island size is nearly independent of Δ'. Such a behavior can be attributed to the fact that $p \neq p(\psi)$ and that the island is generated by small-scale turbulence properties and not by equilibrium parameters. The saturated island size is in fact observed (see Fig. 4) to be linearly dependent on the anomalous diffusion coefficient $D_a = \gamma_{m_\star}/m_\star^2$—a measure of the turbulence level obtained from mixing length arguments [15].

For the case of an unstable tearing mode ($\Delta' > 0$), we find (see Fig. 5) that the saturated island size scales as $\sim|2\gamma_\star|^{-1/4}$. This is in agreement with the earlier theoretical prediction made in [4], for the case of an unstable tearing mode accelerated by the presence of drift wave turbulence, if we note that $\gamma_t \sim 2\gamma_\star$.

To summarize, we have studied the effect of small-scale interchange turbulence on a marginally stable or unstable tearing mode. The presence of the interchange turbulence has a major influence on the excitation and evolution mechanisms of a magnetic island. As soon as the growth of the interchange modes is fast enough (i.e., $2\gamma_\star > \gamma_1$), a magnetic island is formed at a large scale thanks to a nonlinear beating of the fastest growing interchange small scales even when the tearing mode is stable ($\Delta' < 0$). The nature of the nonlinear large scale flow patterns outside the island is also affected by the presence of the interchange turbulence, as evidenced by the dipolar structure of the electrostatic equipotential contours. The saturated seed island size is controlled by the power injected in the system through the interchange instability. Such an excitation and control mechanism can have important physical applications in laboratory and astrophysical contexts. For example, in long pulsed or steady state tokamaks, such as ITER, it is well known that the plasma pressure is ultimately limited by the onset of a subcritical (nonlinear) variety of tearing mode—the so-called neoclassical tearing mode (NTM) [16]. Being a subcritical instability, the excitation of an NTM needs a seed magnetic island to exist in the plasma and the origin of such a trigger is still an open problem. Our simulation results could prove useful in an experimental investigation of a possible such mechanism for seed island generation in a tokamak. Likewise the turbulence induced novel dipole flow patterns around the island may constitute distinct experimental signatures to

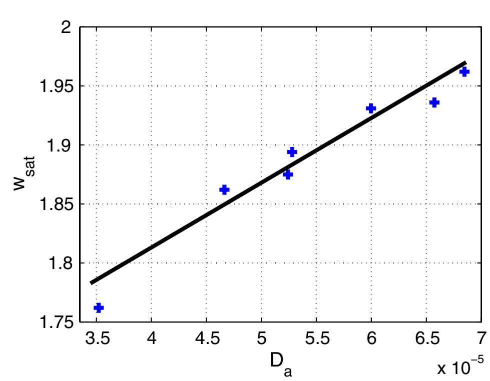

FIG. 4 (color online). Control of the saturated island size by the turbulence level (for different negative values of $\Delta' = \{-0.15, -0.29, -0.33, -0.37, -0.41, -0.43, -0.45\}$).

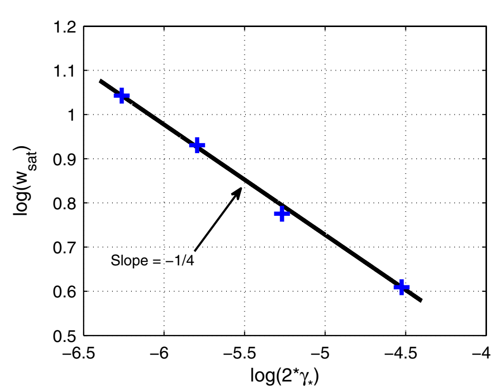

FIG. 5 (color online). Effect of interchange instability on the island size in the marginally unstable tearing mode case (for different positive values of $\Delta' = \{0.01, 0.40, 0.76, 1.16,\}$).

look for in multiscale interaction processes and may provide useful clues for understanding energy transfer mechanisms in such a scenario.

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