Calculation of the Transition from Pairing Vibrational to Pairing Rotational Regimes between Magic Nuclei ^{100}Sn and ^{132}Sn via Two-Nucleon Transfer Reactions

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Absolute values of two-particle transfer cross sections along the Sn-isotopic chain are calculated. They agree with measurements within errors and without free parameters. Within this scenario, the predictions concerning the absolute value of the two-particle transfer cross sections associated with the excitation of the pairing vibrational spectrum expected around the recently discovered closed shell nucleus $^{132}_{50}Sn_{82}$ and the very exotic nucleus $^{100}_{50}Sn_{50}$ can be considered quantitative, opening new perspectives in the study of pairing in nuclei.

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A basic classification of nuclear species can be made in terms of their position with respect to magic numbers [1]. Nuclei far away from both N and Z magic numbers are deformed and superfluid. Single closed shell nuclei are spherical and superfluid. Doubly closed shell nuclei are both spherical and normal (see, e.g., [2], and references therein). Around closed shells collective modes display a typical vibrational spectrum, while away from closed shells a rotational one is displayed. How spontaneous symmetry breaking (deformation), emerges from condensation of vibrational quanta is a major unsolved problem in quantum mechanics at large and in nuclear structure, in particular. The corresponding quest in this last case has been hampered by the simultaneous presence of deformations in both 3D (as a rule quadrupole) and in gauge (superfluidity) spaces.

The systematic study, with the help of two-nucleon transfer reactions, of the ''full'' Sn-isotopic chain (i.e., of a single closed shell isotopic chain), from the doubly magic nucleus $^{100}_{50}Sn_{50}$ to the doubly magic nucleus $^{132}_{50}Sn_{82}$, could shed light on how nuclear superfluidity (deformation in gauge space) evolves from (pairing vibrational) phonon condensation, without the further complication of surface deformations. Of notice, however, is that this competition between quadrupole and pairing degrees of freedom is likely to reappear around the most exotic $N = Z$ doubly closed shell system, arguably, ever to be studied, namely $^{100}_{50}Sn_{50}$. Low-energy 0^+ states arising from $4p$-$4h$, α-like excitations are likely to be present in the spectrum of ^{100}Sn, leading to a coexistence phenomenon well known in the case of, e.g., ^{16}O, ^{40}Ca, etc. (see, for example, [3] and references therein). To what extent this could be a consequence of a possible competition of $T = 1$ and $T = 0$ (see, e.g., [4] and references therein) pairing correlations resulting eventually in an α-vibration [5] mode, is a fascinating subject which the experimental test of the predictions presented below could also help to clarify.

Customarily, the fingerprint of shell closure in nuclei is associated with a sharp, step-function-like distinction between occupied and empty single-particle states in correspondence with magic numbers (for a recent example, see [6]). Away from closed shell, medium-heavy nuclei become, as a rule, superfluid, the distinction between occupied and empty states being blurred within a 2–3 MeV energy interval centered around the Fermi energy. This phenomenon is clearly captured by the Bogoliubov-Valatin quasiparticle transformation $\alpha^\dagger_\nu = U_\nu a^\dagger_\nu - V_\nu a_{\bar{\nu}}$ It provides the rotation in Hilbert space of creation and annihilation fermion operators $a^\dagger$, a which diagonalizes the mean field pairing Hamiltonian $V_p = -G\alpha_0(P^\dagger + P) - G\alpha_0^2$ in the state, $|BCS\rangle = \prod_{\nu>0}(U_\nu + V_\nu a^\dagger_\nu a^\dagger_{\bar{\nu}})|0\rangle$, where U_ν and V_ν are the BCS occupation numbers. The state $|BCS\rangle$ is a wave packet in the number of pairs. A consequence of this fact is that the pair creation and annihilation operators $P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}$, $P = \sum_{\nu>0} a_{\bar{\nu}} a_\nu$, display a finite average value in it (condensed Cooper pair field), $\alpha_0 = e^{i\phi}\alpha'_0 = \langle BCS|P^\dagger|BCS\rangle = \langle BCS|P|BCS\rangle$, where $\alpha'_0 = \sum_{\nu>0} U_\nu V_\nu$ (the pairing gap Δ being $G\alpha_0$, where G is the strength of the pairing force, i.e., $H_p = -GP^\dagger P$). The $|BCS\rangle$ state thus defines a privileged orientation in gauge space, ϕ being the gauge angle between the laboratory and the intrinsic, body-fixed frame of reference. The associated emergent property corresponds to generalized rigidity with respect to pair transfer. Taking into account the correlations among quasiparticles associated with fluctuations in α_0 (gauge angle) induced by the field $(P^+ - P)$ leads to symmetry restoration. That is, to pairing rotations (Fig. 1, [7,8]; see also [9], Sec. 6.6 and Appendix I).

Taking into account the fluctuations in α'_0 induced by the field $(P^+ + P)$ leads to two quasiparticlelike states called

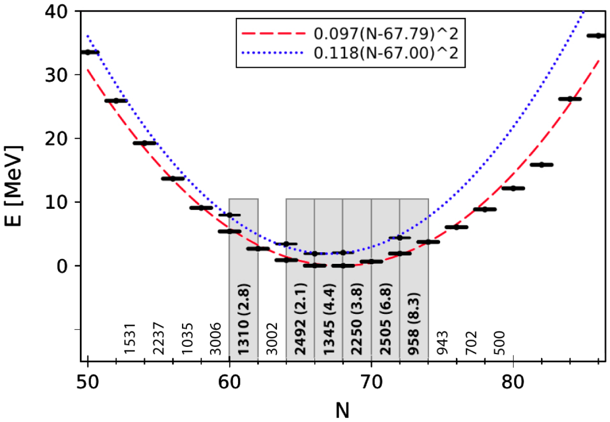

FIG. 1 (color online). Experimental energies of the $J^\pi = 0^+$ states of the Sn isotopes (ground state and pairing vibration), populated in (p, t) reactions. The heavy drawn horizontal lines represent the values of the expression $E = -B(^{A}_{50}\mathrm{Sn}_N) + E_{\mathrm{exc}} + 8.124A$ MeV $+46.33$ MeV, where $B(^{A}_{50}\mathrm{Sn}_N)$ is the binding energy of the Sn isotopes of mass number A, and E_{exc} is the weighted [with $\sigma(0_i^+)$] average energy of the excited 0^+ states below 3 MeV. The dashed and dotted lines represent the parabolas given in the insets, corresponding to the ground state and to the (average) excited state-based pairing rotational bands. The absolute values of the $g.s. \rightarrow g.s.$ integrated cross sections (in μb units) are given (perpendicular) to the abscissa, as a function of N. In the shaded areas we report the experimental values [11–15], while the remaining values correspond to theoretical predictions integrated in the range $0° \leq \theta \leq 80°$. For the first group (experimental), we also report (in parenthesis), the relative (in %) (p, t), pairing vibrational, cross sections $\sum_i \sigma(g.s. \rightarrow 0_i^+)$ normalized with respect to the ground state cross sections.

pairing vibrations, lying on top of twice the pairing gap [7,8]. In the superfluid case these vibrations are little collective (see Fig. 1, pairing rotational bands based on pairing vibrational excitations displaying a few percent cross section as compared to the $g.s. \rightarrow g.s.$ transitions). These fluctuations are, of course, already present in the normal ground state of closed shell nuclei in which case, due to the possibility of distinguishing between occupied and empty states, they are quite collective [8]. In other words, in closed shell nuclei ($\alpha_0 = 0$), the quantity

$$\sigma = \langle(\alpha - \alpha_0)^2\rangle^{1/2} = [(\langle 0|P^\dagger P|0\rangle + \langle 0|PP^\dagger|0\rangle)/2]^{1/2}$$
$$= \left[\sum_i [|\langle i(A_0 - 2)|P|0\rangle|^2 + |\langle i(A_0 + 2)|P^\dagger|0\rangle|^2]/2\right]^{1/2}, \quad (1)$$

(where $|0\rangle = |g.s.(A_0)\rangle$) displays finite values, of the order of $E_{\mathrm{corr}}/G \approx 1$ MeV$/G \approx 5$–10, E_{corr} being the average correlation energy of, e.g., two neutrons (two neutron holes) outside (in) the closed shell system of mass number A_0. Of notice that the marked deviations observed around $N = 50$ and $N = 82$ in the energies of the Sn-ground states as compared with the parabolic fitting (see Fig. 1), are associated with the fact that in these cases we have to deal with pair vibrations of a normal nucleus and not with pairing rotations of a superfluid system. As emerges from the above narrative, one can posit that two-nucleon transfer plays a special role in the study of pairing in nuclei. Let us further elaborate on this point. The understanding of pairing correlations, that is the detailed knowledge of the wave function describing the relative motion of the correlated, strongly overlapping nuclear Cooper pairs, present in both normal and superfluid nuclei, constitutes an essential ingredient of an accurate description of the nuclear structure. As can be seen from (1), this wave function is the form factor which enters in the calculation of the two-nucleon transfer cross sections (for details, we refer to [10]). Thus, two-nucleon transfer reactions can be viewed as the specific probe of nuclear pairing correlations.

In keeping with this fact, it is not surprising (see discussion below in connection with Fig. 2) that the same level of agreement between theoretical predictions and experimental findings is achieved by using U_ν, V_ν occupation numbers resulting from the BCS diagonalization of a pairing Hamiltonian H_p (see above) with constant matrix elements G, than with those resulting from state of the art shell model calculations, which diagonalize an effective interaction derived from the CD-Bonn NN potential, renormalizing its short range repulsion by means of a $v_{\mathrm{low}\text{-}k}$ approach (see, e.g., [11–15] and references therein). In other words, the properties associated with a coherent state like $|\mathrm{BCS}\rangle$, and thus with the members of the associated pairing rotational band (see Fig. 1), can be quite accurately described, also by means of a rather simple Hamiltonian, like the BCS one (i.e., $H_p = -GP^\dagger P$).

From the above narrative, and from the definition of the (nuclear) correlation length $\xi = \hbar v_F/2\delta$, where $\delta = \Delta = G\alpha_0'$ in the case of superfluid nuclei, and $\delta = E_{\mathrm{corr}}$, in the case of closed shell nuclei, Cooper pair partners are paired over distances considerably larger than nuclear dimensions ($\xi \approx 20$–30 fm). This estimate, together with the fact that $(g.s.) \rightarrow (g.s.)$ two-particle transfer cross section can be written as

$$d\sigma(g.s. \rightarrow g.s.)/d\Omega \sim \begin{cases} \alpha_0^2 & (\alpha_0 \neq 0), \\ \sigma^2 & (\alpha_0 = 0), \end{cases} \quad (2)$$

implies that Cooper pair partners can remain correlated across regions of the system for which the short range pairing interaction vanishes, a result first realized in connection with the Josephson effect [16].

Consequently, and with an exception being made for the closing of single-particle channels due to Q-value effects (see below), one expects that successive transfer induced by the single-particle mean field $U(r) = \int d^3r' \rho(r') V_{\mathrm{np}}(|\vec{r} - \vec{r}'|)$ will be dominant over simultaneous transfer, let alone over transfer induced by the pairing interaction. In all the calculations presented in the present Letter, the proton-neutron interaction $\mathbf{V_{np}}$ appearing in $U(r)$ has been parametrized according to [17], its strength adjusted so as to reproduce the intermediate channel deuteron binding energy.

With the help of a Saxon-Wood potential ([18] p. 239) and a $-GP^{\dagger}P$ pairing interaction single-particle levels were calculated and BCS occupation parameters $U_\nu V_\nu$ were obtained adjusting G so as to reproduce the (four point) odd-even mass difference for the nuclei of mass number A and $A-2$. Making use of the associated two-particle transfer spectroscopic amplitudes $B_\nu = (j_\nu + 1/2)^{1/2} U_\nu(A-2) V_\nu(A)$ [10], and standard optical parameters [11–15], the absolute differential cross sections associated with the reactions ${}^{A}\mathrm{Sn}(p,t)^{A-2}\mathrm{Sn}(g.s.)$ ($102 \leq A \leq 130$) were calculated. In all cases, successive, simultaneous, and nonorthogonality contributions (postrepresentation) to the cross section were considered (see [19–21] and references therein, see also [22]). We display in Fig. 2 the theoretical predictions in comparison with the experimental data for all of the six mass numbers ($A = 124$, 122, 120, 118, 116, and 112) for which observations have been carried out ([11–15]). Theory provides, without any free parameters, an account of the absolute value of all measured differential cross sections within limits well below the (estimated) 15% (systematic) experimental error, and almost within statistical errors.

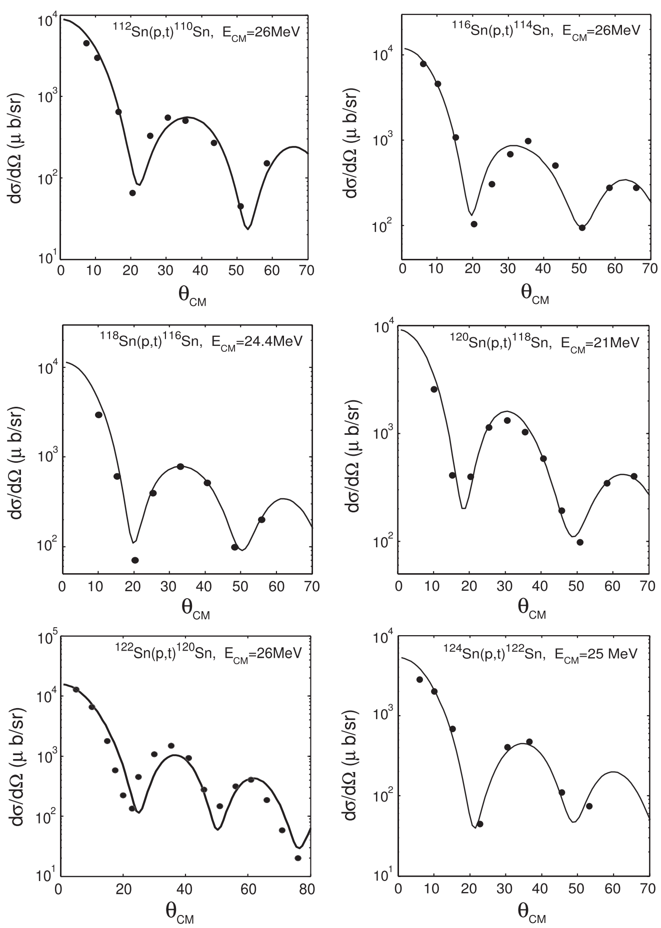

FIG. 2. Absolute calculated cross section predictions in comparison with the experimental results of [11–15].

In keeping with the results displayed in this figure, we present below predictions concerning the expected pairing vibrational spectrum of the closed shell nuclei ${}^{100}_{50}\mathrm{Sn}_{50}$, ${}^{132}_{50}\mathrm{Sn}_{82}$, and the associated absolute differential cross sections. Within the harmonic approximation [8,10], the two-phonon pairing vibrational 0^+ states of ^{132}Sn and ^{100}Sn are predicted at an excitation energy of 6.6 and 7.1 MeV, respectively [see Figs. 3 (I) and (II)]. At variance with the superfluid (pairing rotational) case (see Fig. 1), these excited 0^+ state are predicted to be populated with a cross section comparable to or larger than that associated with the $g.s. \to g.s.$ transition, a direct consequence of the clear distinction which can be operated between occupied ($V^2 \approx 1$, $U^2 \approx 0$) and empty ($V^2 \approx 0$, $U^2 \approx 1$) states

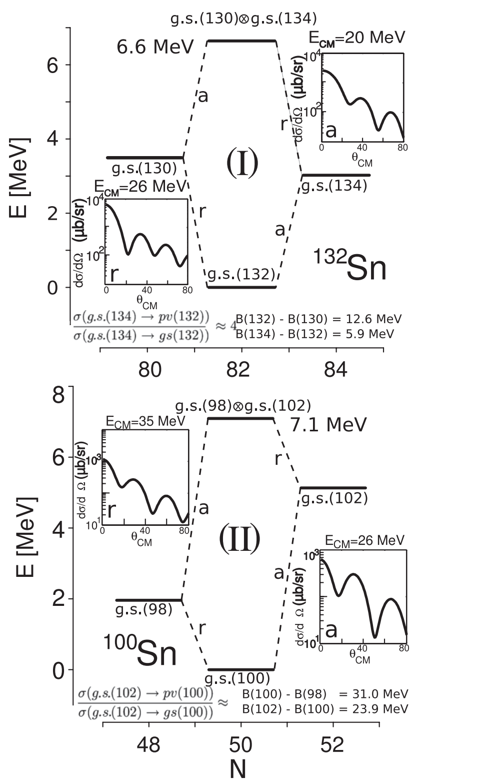

FIG. 3. (I) The solid bold lines represent the values of the expression $E = B(^{132}\mathrm{Sn}) - B(^{A}\mathrm{Sn}_N) - 4.75(82 - N)$ MeV (see also caption to Fig. 1) corresponding to the pair addition (a), pair removal (r), and two-phonon pairing vibrational state ($E = 6.6$ MeV) of ^{132}Sn. The absolute differential cross sections associated with the reaction ${}^{134}\mathrm{Sn}(p,t){}^{132}\mathrm{Sn}(g.s.)$ (pair addition, a) and ${}^{132}\mathrm{Sn}(p,t){}^{130}\mathrm{Sn}(g.s.)$ (pair removal, r) at $E_{\mathrm{c.m.}} = 20$ MeV and 26 MeV, respectively, are also displayed. At the bottom we show the ratio of cross section associated with the reactions ${}^{134}\mathrm{Sn}(p,t){}^{132}\mathrm{Sn}(\mathrm{pv}; 6.6\ \mathrm{MeV})$ and ${}^{134}\mathrm{Sn}(p,t){}^{132}\mathrm{Sn}(g.s.)$, i.e., the relative cross section of the ^{132}Sn, 0^+ two-phonon pairing vibrational state. (II) The same as above but for ^{100}Sn. In this case $E = B(^{100}\mathrm{Sn}) - B(^{A}\mathrm{Sn}_N) - 14.5(50 - N)$ MeV.

TABLE I. Absolute differential cross sections associated with the reaction ^{132}Sn$(p, t)^{130}$Sn$(g.s.)$ at four c.m. bombarding energies integrated over the range $0° \leq \theta_{\text{c.m.}} \leq 80°$. Successive, simultaneous, nonorthogonality, simultaneous+(nonorthogonality), and total cross sections are displayed.

	σ (μb)			
	5.11 MeV	6.1 MeV	10.07 MeV	15.04 MeV
Total	1.29×10^{-17}	3.77×10^{-8}	39.02	750.2
Successive	9.48×10^{-20}	1.14×10^{-8}	44.44	863.8
Simultaneous	1.18×10^{-18}	8.07×10^{-9}	10.9	156.7
Nonorthogonal	2.17×10^{-17}	7.17×10^{-8}	22.68	233.5
Nonorthogonal + simultaneous	1.31×10^{-17}	3.34×10^{-8}	3.18	17.4
Pairing	1.01×10^{-19}	6.86×10^{-10}	0.97	14.04

around closed shell systems. The eventual experimental control of these predictions would observe, quite probably, deviations from harmonicity and associated breaking of two-particle transfer strength, deviations which could likely be correlated with medium polarization effects, that is, with state dependent interweaving of single-particle and collective (density, surface, spin) modes.

While the bombarding conditions used in the calculations which are at the basis of Fig. 3 are similar to those encountered in connection with the experimental data shown in Fig. 2 [11–15], the inverse kinematic techniques required when dealing with ^{132}Sn will pose severe limitations to such (optimal) choices. It turns out that such limitations, in particular, in the case of ^{132}Sn, may contain the key for a qualitative advance in the understanding of the two-nucleon transfer reaction mechanism at large, similar, with all required caveats, to that which took place in the understanding of pair tunneling phenomena in connection with the Josephson effect (see, e.g., [23], and references therein).

This is in keeping with the fact that there exists an important (Q-value, kinematiclike) difference between the pairing coupling scheme expected around and away from closed shells. In fact, while the binding energy of open shell superfluid isotopes is a smoothly varying function of mass number, the situation is quite different around closed shell (A_0). Both the A_0 system, as well as the ($A_0 \pm 2$) nuclei are well bound, the associated Q value being, as a rule, rather unfavorable for single-particle transfer, let alone for twice such a process (successive).

This is seen from the (bombarding) energy dependence of the absolute cross sections associated with the reaction ^{132}Sn$(p, t)^{130}$Sn (Table I). It is of notice that at center-of-mass (c.m.) bombarding energy below 4.1 MeV no (real) two-particle transfer can take place. One needs to reach c.m. bombarding energies of the order of 10 MeV, to obtain values of the absolute cross section which are barely observable, as a result of the cancellation taking place between simultaneous and nonorthogonality contributions and of the (hindered) Q-value dependence of successive transfer. By properly tuning the bombarding conditions, one can reduce the role successive transfer plays in the process, and thus change the shape and absolute value of the two-particle transfer differential cross section.

One can conclude that the eventual experimental test of the predicted absolute differential two-particle transfer cross sections all the way up to ^{132}Sn and down to ^{100}Sn can help at clarifying important features of the mechanism which is at the basis of the normal-superfluid nuclear phase transition, from the pairing vibrational spectrum expected around the two closed shell systems ($N = 132$ and $N = 50$), to the rotational regime observed to be followed by the Sn isotopes with values of N away from the two magic numbers $N = 82$ and $N = 50$. The competition between shape coexistence 0^+ and pairing vibrational states expected in the very exotic $^{100}_{50}$Sn$_{50}$ which likely manifests itself, e.g., in a marked anharmonicity of the two-phonon pairing vibrational state, and in an associated breaking of the two-particle transfer strength, could be instrumental to learning about the relative importance of α-like vibrations and thus, eventually of the competition between $T = 1$ and the elusive $T = 0$ pairing correlations. Last but not least, applying the same theoretical tools to the pairing vibrational coupling scheme expected around the closed shell nucleus $^{132}_{50}$Sn$_{82}$, likely opens the possibility of shedding light, by accurately tuning the bombarding conditions, on the reaction mechanism which is at the basis of two-nucleon transfer reactions.

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