Dynamics of Social Group Competition: Modeling the Decline of Religious Affiliation

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When social groups compete for members, the resulting dynamics may be understandable with mathematical models. We demonstrate that a simple ordinary differential equation (ODE) model is a good fit for religious shift by comparing it to a new international data set tracking religious nonaffiliation. We then generalize the model to include the possibility of nontrivial social interaction networks and examine the limiting case of a continuous system. Analytical and numerical predictions of this generalized system, which is robust to polarizing perturbations, match those of the original ODE model and justify its agreement with real-world data. The resulting predictions highlight possible causes of social shift and suggest future lines of research in both physics and sociology.

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The tools of statistical mechanics and nonlinear dynamics have been used successfully not just to analyze physical systems, but also models of social phenomena ranging from language choice [1] to political party affiliation [2] to war [3] and peace [4]. Models of binary choice dynamics have been of particular interest. In this work, we focus on social systems composed of two mutually exclusive groups in competition for members [5–11]. We compile and analyze a new data set quantifying the declining rates of religious affiliation in a variety of regions worldwide and present a theory to explain this trend.

People claiming no religious affiliation constitute the fastest growing religious minority in many countries throughout the world [12]. Americans without religious affiliation comprise the only religious group growing in all 50 states; in 2008 those claiming no religion rose to 15% nationwide, with a maximum in Vermont at 34% [13]. In the Netherlands nearly half the population is religiously unaffiliated. Here we use a minimal model of competition for members between social groups to explain historical census data on the growth of religious nonaffiliation in 85 regions around the world. According to the model, a single parameter quantifying the perceived utility of adhering to a religion determines whether the unaffiliated group will grow in a society. The model predicts that for societies in which the perceived utility of not adhering is greater than the utility of adhering, religion will be driven toward extinction.

Model.—We begin by idealizing a society as partitioned into two mutually exclusive social groups, X and Y, the unaffiliated and those who adhere to any religion. We assume the attractiveness of a group increases with the number of members, which is consistent with research on social conformity [14–17]. We further assume that attractiveness also increases with the perceived utility of the group, a quantity independent of group size encompassing many factors including the social, economic, political, and security benefits derived from membership as well as spiritual or moral consonance with a group. Then a simple model of the dynamics of conversion is given by [1]

$$\frac{dx}{dt} = yP_{yx}(x, u_x) - xP_{xy}(x, u_x), \qquad (1)$$

where $P_{yx}(x, u_x)$ is the probability, per unit of time, that an individual converts from Y to X, x is the fraction of the population in group X at time t, $0 \leq u_x \leq 1$ is a measure of X's perceived utility, and y and u_y are complementary fractions to x and u_x. We require $P_{xy}(x, u_x) = P_{yx}(1 - x, 1 - u_x)$ to obtain symmetry under exchange of x and y and $P_{yx}(x, 0) = 0$ because no one will switch to a group with no utility. Moreover, since the change in the dynamics of Eq. (1) is small for small values of $P_{yx}(0, u_x)$, and data presented in this Letter are consistent with negligible probability for the birth of a new social group, for simplicity we set $P_{yx}(0, u_x) = 0$ (see Sec. S9 in the Supplemental Material [18]). The assumptions regarding the attractiveness of a social group also imply that P_{yx} is smooth and monotonically increasing in both arguments. Under these assumptions, for generic $P_{yx}(x, u_x)$ Eq. (1) has at most three fixed points, with alternating stability (see Sec. S2 in [18]).

Equation (1) provides a general theoretical framework that can be applied to a wide variety of physical and social systems. Appropriate choices of the function P_{yx} produce well-known physical models, e.g., the Ising model, with $P_{yx} \propto e^{-\Delta E_i/k_B T} H(\Delta E_i) + H(-\Delta E_i)$, where ΔE_i is the difference in configuration energies $E_i = -J\sum_j G_{ij} s_i s_j$ for

$s_i = \pm 1$, with H the Heaviside function and G_{ij} the coupling matrix, or the Kuramoto model with $P_{yx} = \omega - kx$, where $x = \langle \sin(\theta(\xi') - \theta(\xi)) \rangle_{\xi'}$. Here, in the context of social group competition, we choose a functional form for the transition probabilities consistent with the minimal assumptions of the model: $P_{yx}(x, u_x) = cx^a u_x$, where c and a are constants that scale time and determine the relative importance of x and u_x in attracting converts, respectively. If $a > 1$ there are three fixed points, one each at $x = 0$ and $x = 1$, which are stable, and one at $0 < x < 1$, which is unstable. For $a < 1$ the stability of these fixed points is reversed. For $a = 1$, there are only two fixed points, with opposite stability.

In Figs. 1(a)–1(c) we fit the model to historical census data from regions of Switzerland, Finland, and the Netherlands, three of 85 worldwide locations for which we compiled and analyzed data. The initial fraction unaffiliated x_0 and the perceived utility u_x were varied to optimize the fit to each data set, while c and a were taken to be global. A broad minimum in the error near $a = 1$ indicated that as a reasonable choice (see Sec. S4 in [18]). Figure 1(c) shows that, if the model is accurate, nearly 70% of the Netherlands will be nonaffiliated by midcentury. Figure 1(d) shows the totality of the data collected and a comparison to the prediction of Eq. (1) with $a = 1$, demonstrating the general agreement with our model. Time has been rescaled in each data set and the origin shifted so that they lie on top of one another. See Sec. S3 in [18] for more details.

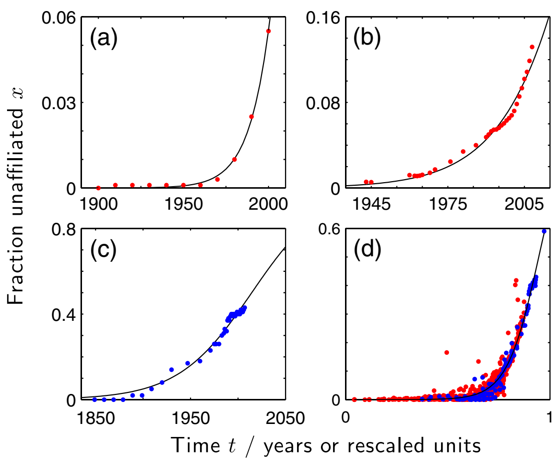

FIG. 1 (color online). Fraction of population religiously unaffiliated versus time for (a) Schwyz Canton in Switzerland, (b) the autonomous Aland islands region of Finland, (c) the Netherlands, and (d) all 85 data sets. Dots indicate data points from census surveys; light gray (red) dots correspond to regions within countries and dark gray (blue) dots to entire countries. Black lines indicate model fits. For (a)–(c), relative utilities for the religiously unaffiliated populations as determined by model fits were $u_x = 0.70, 0.63, 0.56$. In (c) we extend the model's projection to show the expected change in concavity around 2025. For (d) time has been rescaled so data sets lie on top of one another and the solution curve with $u_x = 0.65$. Representative data sets were chosen to show varied current rates of nonaffiliation (low, medium, and high).

The behavior of the model can be understood analytically for $a = 1$, in which case we have $dx/dt = cx(1 - x)(2u_x - 1)$, logistic growth. An analysis of the fixed points of this equation tells us that religion will disappear if its perceived utility is less than that of nonaffiliation, regardless of how large a fraction initially adheres to a religion (i.e., if $u_x > 0.5$). However, if a is less than but close to 1, a small social group can indefinitely coexist with a large social group. Even if $a \geq 1$ it is possible that society will reach such a state if model assumptions break down when the population is nearly all one group.

One might ask whether our model explains data better than a simple empirical curve. Logistic growth would be a reasonable null hypothesis for the observed data, but here we have provided a theoretical framework for expecting a more general growth law, Eq. (1), and have shown that data suggest logistic growth as a particular case of the general law. Our framework includes a rational mathematical foundation for the observed growth law.

Generalizations.—Thus far, we have implicitly assumed that society is highly interconnected, because the function P_{yx} depends on x, a variable measuring global participation in group X. One might imagine a more general model where P_{yx} instead depends on a measure of the local participation in group X among an individual's social peers [6,19]. To create such a generalization, we represent a social network by a binary adjacency matrix $\mathbf{A}$ ($A_{ij} = 1$ if i and j are socially linked) and we define the local mean religious affiliation among social peers of individual i as $x_i = \sum_{j=1}^N A_{ij} R_j / \sum_{j=1}^N A_{ij}$. Each individual's affiliation is tracked by the binary-valued vector $\mathbf{R}$, where $R_i = 1$ for unaffiliated and $R_i = 0$ for affiliated. Equation (1) then becomes

$$\frac{d\langle R_i \rangle}{dt} = (1 - \langle R_i \rangle) P_{yx}(x_i, u_x) - \langle R_i \rangle P_{yx}(1 - x_i, 1 - u_x). \tag{2}$$

We have used angled brackets to indicate that this equation holds only in the sense of ensemble average over many realizations, since this is a stochastic rather than deterministic system. In the all-to-all coupling limit, $\mathbf{A} = \mathbf{1}$, $x_i = \bar{x}$, and Eq. (2) reduces to Eq. (1).

We also consider a further generalization to a system with real-valued rather than binary-valued group affiliation (so individual religiosity lies in a continuum between fully unaffiliated and fully affiliated); such a model can be constructed with the introduction of a spatial dimension. The spatial coordinate ξ will be allowed to vary from -1 to 1 with a normalized coupling kernel $G(\xi, \xi')$ determining the strength of social connection between spatial coordinates ξ and ξ'. The binary religious affiliation vector $\mathbf{R}$ from the previous network model is now reinterpreted as a continuous real-valued function $0 \leq R(\xi, t) \leq 1$ that varies spatially and temporally. Then the dynamics of R satisfy

$$\frac{\partial R}{\partial t} = (1 - R)P_{yx}(x, u_x) - RP_{yx}(1 - x, 1 - u_x) \quad (3)$$

in analogy with the discrete system. Here x again represents the local mean religious affiliation, $x(\xi, t) = \int_{-1}^{1} G(\xi, \xi')R(\xi', t)d\xi'$ (this time an integral over a coupling kernel rather than a sum over an adjacency matrix).

We may still recover the original model Eq. (1) by considering the special case of all-to-all coupling $G(\xi, \xi') = 1/2$ and spatially uniform $R(\xi, t) = R_0(t)$; then $x(\xi, t) = \frac{1}{2}\int_{-1}^{1} R(\xi', t)d\xi' = R_0(t)$, and Eq. (3) becomes

$$\frac{\partial R_0}{\partial t} = (1 - R_0)P_{yx}(R_0, u_x) - R_0 P_{yx}(1 - R_0, 1 - u_x), \quad (4)$$

which follows dynamics identical to Eq. (1).

Note that Eq. (2) represents a stochastic system with binary-valued vector $\mathbf{R}$, while Eq. (3) represents a deterministic system for real $R(\xi, t) \in [0, 1]$, but both limit to the same dynamics for large N if the adjacency matrix $\mathbf{A}$ and coupling kernel G are chosen analogously.

We can impose perturbations to both the coupling kernel (i.e., the social network structure) and the spatial distribution of $R(\xi, t)$ to examine the stability of this system and the robustness of our results for the all-to-all case. One very destabilizing example consists of perturbing the system towards two separate clusters. These clusters might represent a polarized society that consists of two social cliques in which members of each are more strongly connected to others in their clique than to members of the other clique. Mathematically, this can be written as $G(\xi, \xi') = \frac{1}{2} + \frac{1}{2}\delta \,\mathrm{sgn}\xi \,\mathrm{sgn}\xi'$, where δ is a small parameter ($\delta \ll 1$) that determines the amplitude of the perturbation. This kernel implies that individuals with the same sign of ξ are more strongly coupled to one another than they are to individuals with opposite-signed ξ.

The above perturbation alone is not sufficient to change the dynamics of the system—a uniform state $R(\xi, t_0) = R_0$ will still evolve according to the dynamics of the original system Eq. (1).

We add a further perturbation to the spatial distribution of religious affiliation by imposing $R(\xi, t_0) = R_0 + \epsilon \,\mathrm{sgn}\xi$, where ϵ is a small parameter. This should conspire with the perturbed coupling kernel to maximally destabilize the uniform state.

Surprisingly, an analysis of the resulting dynamics reveals that this perturbed system must ultimately tend to the same steady state as the unperturbed system with $\delta = \epsilon = 0$ [which follows the same dynamics as Eq. (1)]. Furthermore, the spatial perturbation must eventually decay exponentially, although an initial growth is possible (see Sec. S5 in the Supplemental Material [18]).

The implication of this analysis is that systems that are nearly all-to-all should behave very similarly to an all-to-all system. In the next section we describe a numerical experiment that tests this prediction.

Numerical experiment.—We design our experiment with the goal of controlling the perturbation from an all-to-all network through a single parameter. We construct a social network consisting of two all-to-all clusters initially disconnected from one another, and then add links between any two nodes in opposite clusters with probability p. Thus $p = 1$ corresponds to an all-to-all network that should simulate Eq. (1), while $p = 0$ leaves the network with two disconnected components. Small perturbations from all-to-all correspond to p near 1, and p can be related to the coupling kernel perturbation parameter δ described above as $p = (1 - \delta)/(1 + \delta)$ (assuming all links in the network have equal weight). The size of each cluster is determined by the initial condition x_0 as $N_X = x_0 N$, $N_Y = (1 - x_0)N$, where all members of cluster X initially have $R = 1$ and all members of cluster Y initially have $R = 0$.

Figure 2 compares the results of simulation of system Eq. (2) with varying perturbations off of all-to-all. The theoretical (all-to-all) separatrix between basins of attraction is a vertical line at $u_x = 1/2$. Even when $p = 0.01$, when in-group connections are 100 times more numerous than out-group connections, the steady states of the system and basins of attraction remain essentially unchanged.

In the case of the continuous deterministic system Eq. (3), the equivalent to Fig. 2 is extremely boring: numerically, the steady states of the perturbed system are indistinguishable from those of the unperturbed all-to-all system, regardless of the value of p (see Sec. S6 and Fig. S5 in [18]).

The only notable difference between the dynamics of the continuous networked system and the dynamics of the original all-to-all system Eq. (1) is a time delay d apparent before the onset of significant shift between groups (see Fig. 3). We were able to find an approximate expression for that time delay as $d \propto -\ln p/(2u_x - 1)$ (see Sec. S7, Figs. S6 and S7 [18]).

What we have shown by the generalization of the model to include network structure is surprising: even if

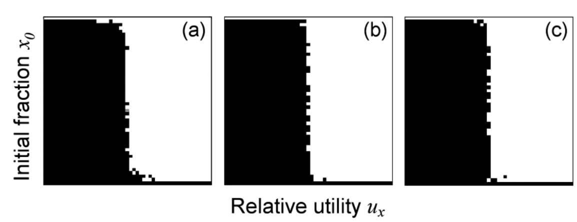

FIG. 2. Results of simulation of the discrete stochastic model Eq. (2) on a network with two initial clusters weakly coupled to one another. The ratio p of out-group coupling strength to in-group coupling strength is (a) $p = 0.01$, (b) $p = 0.40$, (c) $p = 0.80$ ($N = 500$). Steady states are nearly identical to the predictions of the all-to-all model Eq. (1).

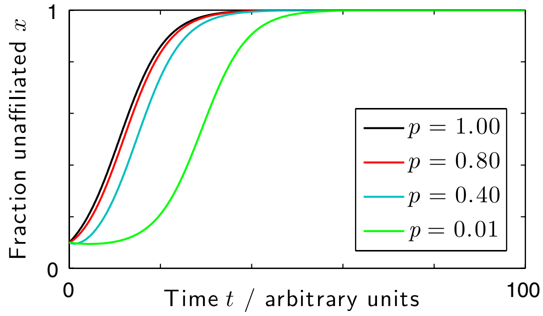

FIG. 3 (color online). Variation in the behavior of Eq. (3) with increasing perturbation off of all-to-all ($N = 500$, $x_0 = 0.1$, $u = 0.6$). Equivalent values of the perturbation parameter δ in order of decreasing p are $\delta = 0$, $\delta = 0.11$, $\delta = 0.43$, and $\delta = 0.98$.

conformity to a local majority influences group membership, the existence of some out-group connections is enough to drive one group to dominance and the other to extinction. In the language of Refs. [6,8,10], the population will reach the same consensus, despite the existence of individual cliques, as it would without cliques, with only the addition of a time delay.

In a modern secular society there are many opportunities for out-group connections to form due to the prevalence of socially integrated institutions—schools, workplaces, recreational clubs, etc. Our analysis shows that just a few out-group connections are sufficient to explain the good fit of Eq. (1) to data, even though Eq. (1) implicitly assumes all-to-all coupling.

*Conclusions.—*We have developed a general framework for modeling competitive systems. When applied to physical systems, appropriate choices of the function P_{yx} can produce a variety of well-known physical models, but we have focused on an application to competition between social groups and analyzed the behavior of the model under modest relevant assumptions. We found that a particular case of the solution fits census data on competition between religious and irreligious segments of modern secular societies in 85 regions around the world. The model indicates that in these societies the perceived utility of religious nonaffiliation is greater than that of adhering to a religion, and therefore predicts continued growth of nonaffiliation, tending toward the disappearance of religion. According to our calculations, the steady-state predictions should remain valid under small perturbations to the all-to-all network structure that the model assumes, and, in fact, the all-to-all analysis remains applicable to networks very different from all-to-all. Even an idealized highly polarized society with a two-clique network structure follows the dynamics of our all-to-all model closely, albeit with the introduction of a time delay. This perturbation analysis suggests why the simple all-to-all model fits data from societies that undoubtedly have more complex network structures.

The models we have presented, although greatly idealized, are significant in that they provide a new framework for the understanding of human behavior in competitive majority or minority social systems. We have shown good agreement with historical data, with the surprising result that the perceived utility of nonaffiliation is higher than the utility of religious affiliation in all the societies we examined. We recognize that the simplifications in our models may limit their applicability (see Sec. S8 [18]); nonetheless, our work suggests a line of research for social scientists: perhaps standard sociological methodology can be used to compare perceived utilities of affiliation and nonaffiliation in societies where nonaffiliation is growing.

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