

Triplet Superconductivity in a Ferromagnetic Vortex

Mikhail S. Kalenkov,¹ Andrei D. Zaikin,^{2,1} and Victor T. Petrushov³

¹*I.E. Tamm Department of Theoretical Physics, P.N. Lebedev Physical Institute, 119991 Moscow, Russia*

²*Institute of Nanotechnology, Karlsruhe Institute of Technology (KIT), 76021 Karlsruhe, Germany*

³*Department of Physics, Royal Holloway, University of London, Egham, Surrey TW20 0EX, United Kingdom*

(Received 10 May 2011; published 16 August 2011)

We argue that triplet superconductivity can be conveniently realized in hybrid superconductor-ferromagnet (S - F) structures with a ferromagnetic vortex. We demonstrate that due to proximity-induced long-range triplet pairing such S - F - S junctions can sustain appreciable supercurrent which can be directly measured in experiments. Depending on the contact geometry either zero- or π -junction regime can be realized in the system under consideration.

DOI: 10.1103/PhysRevLett.107.087003

PACS numbers: 74.45.+c, 74.50.+r, 75.70.Kw

A normal metal (N) sandwiched between two superconductors (S) can become superconducting as a result of penetration of Cooper pairs from the superconducting electrodes. The range of penetration is set by the so-called thermal length ξ_T which can easily reach several micrometers at sufficiently low temperatures [1–3]. The situation changes drastically if the normal metal is replaced by a ferromagnet (F). The quantum mechanical exchange interaction on the F side then destroys conventional spin-singlet Cooper pairs within a few nanometers (the so-called paramagnetic effect) [4]. Experiments to determine actual superconducting penetration depths in ferromagnets intensified more than a decade ago, when techniques were developed to fabricate hybrid nanoscale S - F structures with well controlled geometries. Several groups [5–9] reported an unexpectedly strong influence of superconductors that stimulated new theoretical efforts in a search for a sustainable superconductivity that is compatible with the exchange interaction. During the last decade several theoretical mechanisms were suggested [10–16], some of which were successfully realized experimentally [17–22]. A recent comprehensive review of the status of the field was given in Ref. [23].

Common to all mechanisms of long-range proximity effect in ferromagnets is the generation of triplet superconductivity within highly inhomogeneous ferromagnetic regions adjacent to superconductors. The systems studied up to date include intrinsically inhomogeneous ferromagnets [17], half-metallic ferromagnets with spin-active F - S interfaces [18,19], and engineered multilayers consisting of magnetic and nonmagnetic materials [20,21].

In this Letter we address a different situation of proximity-induced long-range triplet pairing in ferromagnets with magnetic vortex structure. Magnetic vortices are stable in systems intermediate between very small, 10 nm scale magnets, which behave as single giant spins, and macroscopic magnets with dimensions exceeding $\sim 1 \mu\text{m}$. The magnetic structure in such mesoscopic magnets is the result of a competition between exchange, anisotropy,

and dipolar energies and depends strongly on their shape. The latter property allows magnetic nanoengineering using modern nanolithography [24] opening possibility of investigating S - F - S structures with differing magnetic structures. Recently mesoscopic magnetic structures have attracted a lot of attention due to their remarkable transport properties [25–27], as well as the prospect of technological applications for magnetic storage of information of unprecedented density [28]. Below we will demonstrate that mesoscopic ferromagnetic structures can turn superconducting if attached to superconducting electrodes.

The model and quasiclassical formalism.—We will consider a ferromagnetic film of thickness d located in the xy plane with magnetization forming a vortex [29]. This film is partially covered by two superconducting electrodes thus forming an S - F - S contact as it is shown in Fig. 1. Our main goal is to analyze superconducting correlations that penetrate into a ferromagnetic vortex from the electrodes. In order to accomplish this goal we will employ the quasiclassical Usadel equations [2,30] for energy-integrated

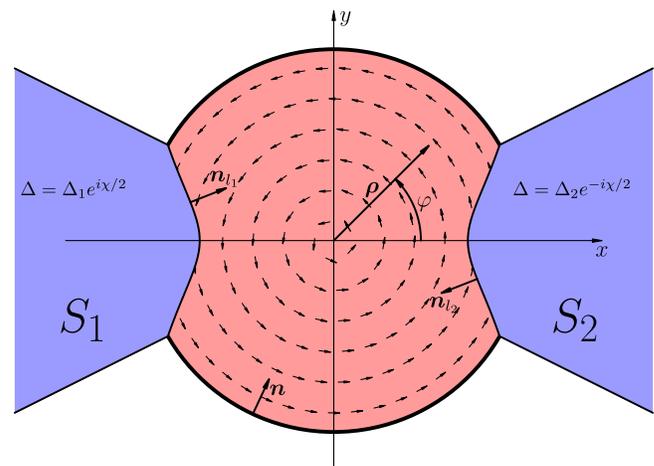


FIG. 1 (color online). S - F - S junction formed by two superconducting electrodes connected via ferromagnetic vortex.

matrix Matsubara-Green functions \check{G} . E.g., in the ferromagnet with diffusion constant D these equations read

$$iD\nabla(\check{G}\nabla\check{G}) = [\check{\Omega}, \check{G}], \quad \check{G}^2 = 1, \quad (1)$$

where

$$\check{G} = \begin{pmatrix} \hat{G} & \hat{F} \\ \hat{F}^+ & \hat{G}^+ \end{pmatrix}, \quad (2)$$

$$\check{\Omega} = \begin{pmatrix} i\omega_n \hat{1} - \hat{\sigma}\mathbf{h} & \Delta \hat{1} \\ -\Delta^* \hat{1} & -i\omega_n \hat{1} + \hat{\sigma}\mathbf{h} \end{pmatrix}$$

are 4×4 matrices in Nambu and spin spaces. Their commutator in Eq. (1) and below is denoted by square brackets. Accordingly, \hat{G} , \hat{F} , \hat{F}^+ and \hat{G}^+ are 2×2 matrices in the spin space, $\omega_n = \pi T(2n + 1)$ is the Matsubara frequency, \mathbf{h} is the exchange field in the ferromagnet and $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ represents the Pauli matrices in the spin space. The same Eqs. (1) and (2) hold also for superconducting electrodes, one should only replace D by the diffusion constant in the corresponding electrode. The superconducting order parameter Δ equals to zero in the ferromagnet, while in two superconducting terminals it is respectively $\Delta = \Delta_1 \exp(i\chi/2)$ and $\Delta = \Delta_2 \exp(-i\chi/2)$ with real $\Delta_{1,2}$ and χ being the superconducting phase difference across our S - F - S junction.

Eqs. (1) should be supplemented by appropriate boundary conditions at each of the two S - F interfaces which account for electron transfer across these interfaces. In what follows we will assume that there exist tunnel barriers at both S - F interfaces with the corresponding tunneling resistances per unit area $r_{1,2}$. In the tunneling limit it suffices to employ Kupriyanov-Lukichev boundary conditions [31] at each S - F -interface. E.g., at the interface between the first superconducting electrode ($z > 0$) and the ferromagnet ($z < 0$) these boundary conditions read

$$2r_1 \sigma \check{G}_F \partial_z \check{G}_F = [\check{G}_F, \check{G}_{S_1}], \quad (3)$$

where \hat{G}_F and \hat{G}_{S_1} are, respectively, the Green functions at the F - and S -sides of the first interface and σ is the Drude conductivity of a ferromagnet. Analogous boundary conditions hold for the second S - F interface.

Long-range triplet pairing in a ferromagnetic vortex.— The presence of tunnel barriers at both S - F interfaces effectively implies weak electron tunneling regime in which case the proximity effect remains small and it suffices to linearize Usadel equations in the ferromagnet as

$$D\nabla^2 \hat{F} - 2\omega_n \hat{F} - i\{\hat{F}, \mathbf{h}(\mathbf{r})\hat{\sigma}\} = 0. \quad (4)$$

In Eq. (4) we restrict Matsubara frequencies to be positive $\omega_n > 0$ and denoted the anticommutator by curly brackets. A similar equation holds for the function \hat{F}^+ .

In general magnetization patterns in thin ferromagnetic films depend on the film geometry and are influenced by the following trade-off. On one hand, magnetostatic energy minimum is reached provided the film magnetization

remains in-plane. On the other hand, in some regions, such as, e.g., vortex cores, local magnetization can go out-of-plane in order to minimize the exchange energy. As the magnetic core radius typically remains small as compared to the superconducting coherence length, in the following we will assume that magnetization lies in-plane everywhere in the ferromagnet; see Fig. 1. In sufficiently thin films the exchange field \mathbf{h} depends only on in-plane coordinates (x, y) and can be represented as $\mathbf{h} = (h \cos\theta, h \sin\theta)$ where $\theta = \theta(x, y)$. In this case the spin structure of the anomalous Green function \hat{F} inside the ferromagnet can be chosen in the following form

$$\hat{F} = F_0 + \hat{\sigma}\mathbf{m}F_h + \hat{\sigma}[e_z, \mathbf{m}]F_t, \quad (5)$$

where F_0 describes the singlet pairing component, while F_h and F_t correspond to two different triplet components. In Eq. (5) we also introduced in-plane and normal to the plane unity vectors $\mathbf{m} = \mathbf{h}/h$ and e_z . Combining Eqs. (5) and (4) we arrive at the following equations:

$$D\nabla^2 F_0 - 2\omega_n F_0 = 2ihF_h, \quad (6)$$

$$\mathcal{D}F_h = DF_t \nabla_\rho^2 \theta + 2D(\nabla_\rho F_t, \nabla_\rho \theta) + 2ihF_0, \quad (7)$$

$$\mathcal{D}F_t = -DF_h \nabla_\rho^2 \theta - 2D(\nabla_\rho F_h, \nabla_\rho \theta), \quad (8)$$

where we defined the differential operator

$$\mathcal{D} = D\nabla^2 - D(\nabla_\rho \theta)^2 - 2\omega_n \quad (9)$$

and distinguished ∇ and ∇_ρ as, respectively, 3D and 2D (in-plane) gradient operators.

Note that Eqs. (6) and (7) contain the exchange field h thus providing the characteristic length scale both for F_0 and F_h of order $\xi_h \sim \sqrt{D/h}$. At the same time, Eq. (8) does not contain the h term and, hence, typical variations of F_t occur on a much longer length scale $\xi_T \sim \sqrt{D/T} \gg \xi_h$. This observation illustrates the difference between the two triplet components F_h and F_t and constitutes the essence of the long-range proximity effect in S - F - S structures: while the components F_0 and F_h decay already in the vicinity of an S - F interface, the triplet component F_t survives deep inside the ferromagnet provided the temperature remains sufficiently low.

Before turning to the solution of Eqs. (6)–(8) let us perform some further simplifications. Firstly, we will neglect both magnetic anisotropy and stray field effects. In this case outside the magnetic vortex core the function θ obeys the equation $\nabla_\rho^2 \theta = 0$, which allows to drop the first terms in the right-hand side of Eqs. (7) and (8). Secondly, we will assume the ferromagnetic film to be sufficiently thin $d \lesssim \xi_T$, in which case the dependence of the long-range triplet component F_t on the coordinate z can be neglected. Integrating Eq. (8) over z , we obtain

$$\mathcal{D}_\rho F_t = -2D(\nabla_\rho \bar{F}_h, \nabla_\rho \theta), \quad \bar{F}_h = \frac{1}{d} \int_{-d}^0 F_h dz, \quad (10)$$

where \mathcal{D}_ρ is defined by Eq. (9) with $\nabla^2 \rightarrow \nabla_\rho^2$.

Equation (10) accounts for diffusion of the long-range triplet component F_t across the ferromagnet with nonuniform in-plane magnetization. It demonstrates that nonzero F_t is generated in the parts of the ferromagnet where both $\nabla_{\rho}\theta$ and $\nabla_{\rho}\bar{F}_h$ differ from zero. The condition $\nabla_{\rho}\theta \neq 0$ obviously holds everywhere in the ferromagnetic plane since the magnetization remains nonuniform there. As for the averaged component \bar{F}_h , it vanishes together with its gradient at distances exceeding $\sim\xi_h$ from S - F interfaces. In the immediate vicinity of such interfaces \bar{F}_h is nonzero, but its in-plane gradient remains small because in the main approximation it only depends on the absolute value of the exchange field h cf. Eq. (7). The gradient $\nabla_{\rho}\bar{F}_h$ becomes appreciable only in the region of the ferromagnet just below the edge of the superconducting film where \bar{F}_h changes abruptly. With this in mind we arrive at the following result for the long-range triplet component

$$F_t(\boldsymbol{\rho}) = \frac{iD^2}{h\sigma d} \sum_{k=1,2} \frac{F_{S_k}}{r_k} \int_{l_k} P_{\omega_n}^{\rho,\rho'}(\nabla_{\rho'}\theta(\boldsymbol{\rho}'), \mathbf{n}_{l_k}(\boldsymbol{\rho}')) dl_k, \quad (11)$$

which holds inside the ferromagnetic film. Here, F_{S_k} is anomalous Green function in the bulk of the k -th superconductor and \mathbf{n}_{l_k} is the outer unity vector normal to the superconducting plane S_k (see Fig. 1). Integration contours l_k in Eq. (11) are lines in the xy plane corresponding to the edge of the superconductor S_k and in the ferromagnet kernel $P_{\omega_n}^{\rho,\rho'}$ obeys the equation

$$\mathcal{D}_{\rho} P_{\omega_n}^{\rho,\rho'} = \delta(\boldsymbol{\rho} - \boldsymbol{\rho}'), \quad (12)$$

with boundary conditions $\partial P_{\omega_n}^{\rho,\rho'}/\partial \mathbf{n} = 0$. Also note that Eq. (11) can easily be generalized to the case $\nabla_{\rho}^2\theta \neq 0$.

Triplet pairing and Josephson effect.—As triplet pairing amplitude can survive deep in the ferromagnet, at sufficiently low temperatures our S - F - S junction can sustain appreciable supercurrent which is converted from singlet to triplet and back in the vicinity of S - F interfaces. In order to evaluate this supercurrent we will employ the standard expression for the current density

$$\mathbf{j} = \frac{\pi\sigma T}{2e} \text{Im} \sum_{\omega_n > 0} \text{Sp}[\hat{F}\nabla\hat{F}^+ - \hat{F}^+\nabla\hat{F}], \quad (13)$$

where the trace is taken over the spin degree of freedom. Combining Eqs. (5), (11), and (13) we recover the sinusoidal current-phase relation $I(\chi) = I_c \sin\chi$ with

$$I_c = \frac{2\pi TD^3}{eh^2\sigma dr_1 r_2} \sum_{\omega_n > 0} \frac{\Delta_1 \Delta_2}{\sqrt{(\omega_n^2 + \Delta_1^2)(\omega_n^2 + \Delta_2^2)}} \\ \times \int_{l_1, l_2} P_{\omega_n}^{\rho_1, \rho_2}(\nabla_{\rho} \theta(\boldsymbol{\rho}_1), \mathbf{n}_{l_1}(\boldsymbol{\rho}_1)) \\ \times (\nabla_{\rho} \theta(\boldsymbol{\rho}_2), \mathbf{n}_{l_2}(\boldsymbol{\rho}_2)) dl_1 dl_2 \quad (14)$$

Note that in the course of our derivation we always assumed the proximity effect to be sufficiently weak. This assumption is satisfied under the condition

$$\frac{1}{r_{1,2}\sigma} \sqrt{\frac{D}{h}} \ll \begin{cases} 1, & d \geq \xi_h, \\ d\sqrt{h/D}, & d \leq \xi_h. \end{cases} \quad (15)$$

Equation (14) together with its validity condition (15) represents the central result of our analysis which fully determines the Josephson critical current of an S - F - S junction with a ferromagnetic vortex. Actually this result applies not only to vortex configurations but also to a broader class of nonuniform magnetization patterns.

Let us now assume that our ferromagnetic film has the form of a disk with radius R and vortexlike magnetization pattern with vortex core located in the disc center. Then the function θ equals to $\varphi + \pi/2$ for clockwise or $\varphi - \pi/2$ for counterclockwise magnetization, where φ is the azimuthal angle (see Fig. 1). Eq. $\nabla_{\rho}^2\theta = 0$ is fulfilled in this case. Remarkably, Eq. (14) yields exactly the same result for quite different magnetization patterns: vortexlike ($\theta = \varphi \pm \pi/2$), antivortexlike ($\theta = -\varphi$ or $\theta = -\varphi + \pi$) and hedgehoglike ($\theta = \varphi$ or $\theta = \varphi + \pi$) states. This property holds since the function $\nabla_{\rho}\theta$ remains the same (up to a sign) for all these magnetization patterns. Note, however, that for the last two patterns stray magnetic field is not confined to the disc center and may influence superconductivity in the electrodes.

For illustration, typical spatial profile of the long-range superconducting triplet component F_t induced by one superconducting electrode in the ferromagnet with a vortex is schematically depicted in Fig. 2. As it was expected, F_t is most efficiently generated close to the edge of a superconductor where the scalar product $|(\nabla_{\rho}\theta(\boldsymbol{\rho}), \mathbf{n}_l(\boldsymbol{\rho}))|$ reaches its maximum values. Provided the proximity effect remains weak, the total value of F_t is given by a superposition of independent contributions from two superconducting electrodes; cf. also Eq. (11).

The sign of the long-range triplet component F_t in the ferromagnet is controlled by the (coordinate dependent) scalar product $(\nabla_{\rho}\theta(\boldsymbol{\rho}), \mathbf{n}_l(\boldsymbol{\rho}))$ along the edge of the

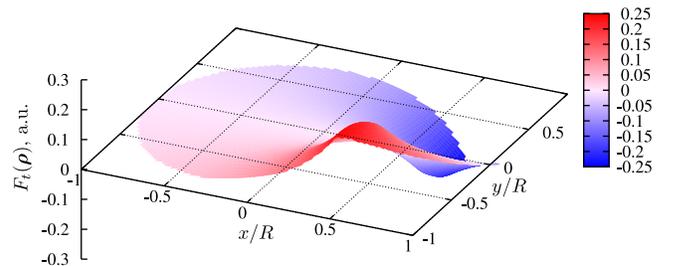


FIG. 2 (color online). Typical spatial distribution of the long-range superconducting triplet component F_t induced in the ferromagnetic disk with vortexlike magnetization by one superconducting electrode ($x > R/2$, $z > 0$) with real Δ .

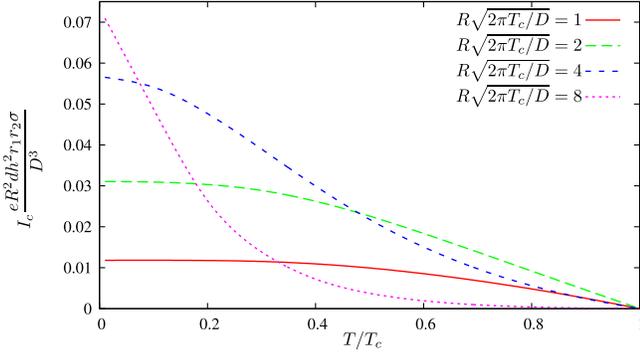


FIG. 3 (color online). I_c versus temperature (normalized by the critical temperature T_c) in S - F - S junctions containing a ferromagnetic vortex at different values of R . The edges of superconducting electrodes (contours $l_{1,2}$) are chosen to coincide with straight lines $y = \pm R/2$.

superconductor [see Eq. (11)] which can take both positive and negative values (cf. Figure 2). Accordingly, the sign of the critical current (14) can be either positive or negative, i.e., both zero- and π -junction states can be realized in our S - F - S structure. By implementing proper asymmetry in S - F contacts (e.g., by “cutting out” their parts) one can suppress positive (negative) part of F_t penetrating from the left (right) superconducting electrode. The product of two long-range triplet components turns negative in this case, hence, implying the π -junction regime.

We further consider a symmetric situation, set $\Delta_{1,2} = |\Delta|$ and assume that the relevant Thouless energy $\varepsilon_{\text{Th}} \sim D/(2R)^2$ remains smaller than the superconducting gap $|\Delta|$. Then in the limit $T \ll \varepsilon_{\text{Th}}$ from Eq. (14) we find

$$I_c \sim D^2 \varepsilon_{\text{Th}} / (edh^2 r_1 r_2 \sigma), \quad (16)$$

while for $\varepsilon_{\text{Th}} \ll T \ll |\Delta|$ the Josephson current follows the standard exponential dependence on temperature

$$I_c \sim \frac{TD^2}{edh^2 r_1 r_2 \sigma} \exp(-L\sqrt{2\pi T/D}), \quad (17)$$

where L is an effective distance between the two S - F contacts ($L = 2R$ for small area contacts). For illustration the Josephson critical current I_c is plotted in Fig. 3 as a function of temperature for different values of R .

Our result for I_c in S - F - S systems turns out to be by the factor $\sim \varepsilon_{\text{Th}}^2/h^2$ smaller than that for conventional diffusive S - N - S junctions with identical geometry cf., e.g., [3,31]. The critical current of our S - F - S structure can further be increased by a proper choice of the system parameters. For a simple estimate let us employ Eq. (16) at the border of its applicability range (15). Then for $T \ll \varepsilon_{\text{Th}}$ and $d \gtrsim \xi_h$ we obtain

$$I_c \sim D^2 \sigma / (eR^2 dh) \sim (\xi_h/d)^2 \varepsilon_{\text{Th}} / (eR_N), \quad (18)$$

where R_N is the normal state resistance of the ferromagnetic film between two superconducting electrodes. This

estimate is also supported by our independent calculation (not presented here) which yields contributions to $I_c \propto 1/h$ in higher orders in barrier transmissions. Equation (18) demonstrates that for $d \gtrsim \xi_h$ one can expect to reach values of I_c only by the factor $\sim \xi_h^2/d^2$ smaller than the absolute maximum $I_c \sim \varepsilon_{\text{Th}}/eR_N$ achieved for S - N - S junctions [32]. Actually, the latter maximum value can also be reached, but only for extremely thin films $d \lesssim \xi_h$ [cf. Eqs (16) and (15)] with large values of R_N .

In summary, we demonstrated that long-range triplet superconductivity can coexist with a ferromagnetic vortex and evaluated the supercurrent across S - F - S junctions containing such vortex. For properly chosen system parameters the effect is well in the measurable range and can be directly tested in future experiments.

This work was supported in part by DFG, by RFBR under grant 09-02-00886 and by British EPSRC grant EP/F01689/1.

-
- [1] C. J. Lambert and R. Raimondi, *J. Phys. Condens. Matter* **10**, 901 (1998).
 - [2] W. Belzig *et al.*, *Superlattices Microstruct.* **25**, 1251 (1999).
 - [3] A. A. Golubov, M. Yu. Kupriyanov, and E. Il'ichev, *Rev. Mod. Phys.* **76**, 411 (2004).
 - [4] A. I. Buzdin, L. N. Bulaevskii, and S. V. Panyukov, *JETP Lett.* **35**, 178 (1982).
 - [5] V. T. Petrashov *et al.*, *JETP Lett.* **59**, 523 (1994).
 - [6] M. Giroud *et al.*, *Phys. Rev. B* **58**, R11 872 (1998).
 - [7] V. T. Petrashov *et al.*, *Phys. Rev. Lett.* **83**, 3281 (1999).
 - [8] J. Aumentado and V. Chandrasekhar, *Phys. Rev. B* **64**, 054505 (2001).
 - [9] P. Nugent, I. A. Sosnin, and V. T. Petrashov, *J. Phys. Condens. Matter* **16**, L509 (2004).
 - [10] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, *Phys. Rev. Lett.* **86**, 4096 (2001); *Rev. Mod. Phys.* **77**, 1321 (2005).
 - [11] A. Kadigrobov, R. I. Shekhter, and M. Jonson, *Europhys. Lett.* **54**, 394 (2001).
 - [12] M. Eschrig *et al.*, *Phys. Rev. Lett.* **90**, 137003 (2003); M. Eschrig and T. Löfwander, *Nature Phys.* **4**, 138 (2008).
 - [13] A. F. Volkov, F. S. Bergeret, and K. B. Efetov, *Phys. Rev. Lett.* **90**, 117006 (2003).
 - [14] Ya. V. Fominov, A. F. Volkov, and K. B. Efetov, *Phys. Rev. B* **75**, 104509 (2007).
 - [15] Y. Asano, Y. Tanaka, and A. A. Golubov, *Phys. Rev. Lett.* **98**, 107002 (2007).
 - [16] A. V. Galaktionov, M. S. Kalenkov, and A. D. Zaikin, *Phys. Rev. B* **77**, 094520 (2008).
 - [17] I. A. Sosnin *et al.*, *Phys. Rev. Lett.* **96**, 157002 (2006).
 - [18] R. S. Keizer *et al.*, *Nature (London)* **439**, 825 (2006).
 - [19] M. S. Anwar *et al.*, *Phys. Rev. B* **82**, 100501 (2010).
 - [20] J. W. A. Robinson, J. D. S. Witt, and M. G. Blamire, *Science* **329**, 59 (2010); J. W. A. Robinson *et al.*, *Phys. Rev. Lett.* **104**, 207001 (2010).
 - [21] T. S. Khaire *et al.*, *Phys. Rev. Lett.* **104**, 137002 (2010).
 - [22] J. Wang *et al.*, *Nature Phys.* **6**, 389 (2010).
 - [23] M. Eschrig, *Phys. Today* **64**, 43 (2011).

- [24] V.L. Mironov *et al.*, *Phys. Rev. B* **81**, 094436 (2010).
- [25] P. Bruno *et al.*, *Phys. Rev. Lett.* **93**, 096806 (2004).
- [26] A.A. Fraerman and O.G. Udalov, *Phys. Rev. B* **77**, 094401 (2008).
- [27] A. Neubauer *et al.*, *Phys. Rev. Lett.* **102**, 186602 (2009).
- [28] R.P. Cowburn *et al.*, *Phys. Rev. Lett.* **83**, 1042 (1999).
- [29] A somewhat similar configuration was also considered by M.A. Silaev, *Phys. Rev. B* **79**, 184505 (2009). The analysis of that paper does not address the supercurrent and does not allow us to reproduce the results derived here.
- [30] K.D. Usadel, *Phys. Rev. Lett.* **25**, 507 (1970).
- [31] M. Yu. Kuprianov and V.F. Lukichev, *Sov. Phys. JETP* **67**, 1163 (1988).
- [32] A.D. Zaikin and G.F. Zharkov, *Fiz. Nizk. Temp.* **7**, 375 (1981) [*Sov. J. Low Temp. Phys.* **7**, 184 (1981)]; P. Dubos *et al.*, *Phys. Rev. B* **63**, 064502 (2001).