Quantum Metrology with Entangled Coherent States

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We present an improved phase estimation scheme employing entangled coherent states and demonstrate that these states give the smallest variance in the phase parameter in comparison to NOON, "bat," and "optimal" states under perfect and lossy conditions. As these advantages emerge for very modest particle numbers, the optical version of entangled coherent state metrology is achievable with current technology.

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As full quantum computing based on very large quantum resources remains on the technological horizon for now, there is significant current interest in quantum technologies that offer genuine quantum advantage with much more modest quantum resources. Quantum metrology is one field where such technologies could emerge. Nonclassical states of light can offer enhanced imaging or spatial resolution, nonclassical states of mechanical systems could offer enhanced displacement resolution, nonclassical states of spins could enable enhanced field resolution, and entangled atoms could provide the ultimate accuracy for clocks. Since it became known that optical quantum states can beat the classical diffraction or shot-noise limit [1], in recent years quantum metrology has been widely investigated in partnership with the rapidly developing field of quantum information [2]. For example, the precision limits of quantum phase measurements are given by the Cramer-Rao lower limit bounded by quantum Fisher information [3]. In the ideal quantum information version of metrology, a maximally entangled state is viewed as the best resource for quantum metrology; i.e., the optimal phase uncertainty of the NOON state reaches to the Heisenberg limit and it is thus considered for many applications (e.g., Bell's inequality tests, quantum communication, and quantum computing) [4]. However, current quantum technologies have a long way to go to the manipulation of many-qubit entanglement for these applications, and of course all realistic quantum technologies will be subject to loss and decoherence. Therefore, quantum metrology utilizing very modest entangled resources and with robustness against loss could be accessible for these applications in the near future [5], revealing a fundamental difference between classical and quantum physics in both theory and practice.

A major research question in quantum metrology is how to implement NOON states with large particle numbers (called high NOON states). Many successful demonstrations have shown the potential for quantum-enhanced metrology using small NOON states [6–8]. However, it remains a challenge to obtain a practical high NOON state in linear (or even nonlinear) optics. Even if high NOON states become achievable, a critical consideration is that these states are extremely fragile to particle loss because the resultant mixed state loses phase information rapidly. Thus other quantum states have been studied for improved robustness against particle loss [9,10]. Further recent developments have shown the potential advantages of nonlinearities [11] and the importance of the query complexity for quantum metrology [12], concluding that the same phase operation is required for the appropriate resource count in different states.

In this Letter, we report that a superposition of macroscopic coherent states shows a noticeable improved sensitivity for phase estimation when compared to that for NOON states, in the region of very modest photon or particle numbers. Taking into account the same average particle number, an entangled coherent state (ECS) outperforms the phase enhancement achieved by NOON states both in the lossless, weak, moderate and high loss regimes. This advantage is also maintained over other well-known quantum states used in metrology such as bat [10], and uncorrelated states. So even though simple coherent states $|\alpha\rangle$ are known as the most "classical-like" quantum states [13], superpositions thereof are very useful and robust for quantum metrology [14]. This phenomenon can be understood as follows. For pure states, the ECS can be understood as a superposition of NOON states with different photon numbers; thus, the larger photon-number NOON states make a contribution to a better sensitivity than the average photon-number NOON state in the ECS. For mixed states, the resultant state given by photon loss does not depend on the number of particles lost but its loss rate—thus, this state still contains some phase information even in the large loss rate. In order to demonstrate this perhaps surprising phenomenon, we suggest an implementation scheme using both photon-number and parity measurements for modest ECSs which should be feasible with current optical technology (see Fig. 1) [15].

We choose to compare the phase uncertainty of various quantum states with and without loss, using the widely accepted approach of quantum Fisher information [3]. The interferometric setup generally consists of four steps.

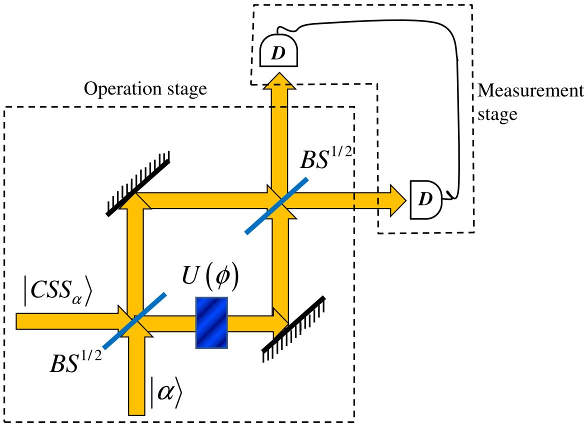

FIG. 1 (color online). Schematic illustration of an interferometric setup for the pure ECS. Two input states ($|\mathrm{CSS}_\alpha\rangle$ and $|\alpha\rangle$) are applied to the first BS and become the ECS. After a phase shifter $U(\phi)$ in a mode, the parity measurement is performed at the measurement stage.

The first is the preparation step where the input state $|\psi_K^{\rm in}\rangle_{12}$ is prepared in modes 1 and 2. Then, a unitary operation U in mode 2 is applied, given by

$$U(\phi, k) = e^{i\phi(a_2^\dagger a_2)^k} \tag{1}$$

for phase ϕ, order parameter of nonlinearity k, and creation operator $a_i^\dagger$ in mode i. In this Letter we assume $k = 1$, implying that the operation $U(\phi, 1)$ is a conventional phase shifter $U(\phi)$ (although future studies will extend this to other k values). The outcome state is called $|\psi_K^{\rm out}\rangle_{12} = [\mathbb{1} \otimes U(\phi)]|\psi_K^{\rm in}\rangle_{12}$. For the case of particle loss, we add two variable beam splitters (BSs) with loss modes 3 and 4 located after the phase operation. After the BSs, the mixed state ρ_{12}^K (given by tracing out the loss modes 3 and 4) is finally measured for the estimation of phase uncertainty. A change of transmission rate T in the BSs characterizes the robustness of phase estimation for the input state against the loss. The phase optimization given by the quantum Cramér-Rao bound [3] for the outcome states $|\psi_K^{\rm out}\rangle$ is described by

$$\delta\phi_K \geq \frac{1}{\sqrt{\mu F_K^Q}}, \tag{2}$$

where $\mu = 1$ for a single-shot experiment [12]. For a pure state, quantum Fisher information is given by

$$F_K^Q = 4[\langle\psi_K'|\psi_K'\rangle - |\langle\psi_K'|\psi_K^{\rm out}\rangle|^2] \tag{3}$$

for $|\psi_K'\rangle = \partial|\psi_K^{out}\rangle/\partial\phi$ [10,16]. If the outcome state is the mixed state ρ_{12}^K, the quantum Fisher information is given by

$$F_K^Q = \sum_{i,j} \frac{2}{\lambda_i + \lambda_j} |\langle\lambda_i|(\partial\rho_{12}^K(\phi)/\partial\phi)|\lambda_j\rangle|^2, \tag{4}$$

where λ_i ($|\lambda_i\rangle$) are the eigenvalues (eigenvectors) of ρ_{12}^K.

Here we focus on three important input states as $|\psi_K^{\rm in}\rangle$ ($K = N, B, C$) corresponding to NOON $|\psi_N^{\rm in}\rangle$, bat $|\psi_B^{\rm in}\rangle$ [10,17], and ECS [18] given by

$$|\psi_{C_\alpha}^{\rm in}\rangle_{12} = e^{-(|\alpha|^2)/2}\mathcal{N}_\alpha \sum_{n=0}^{\infty} \frac{\alpha^n}{n!}[(a_1^\dagger)^n + (a_2^\dagger)^n]|0\rangle_1|0\rangle_2$$
$$= \mathcal{N}_\alpha[|\alpha\rangle_1|0\rangle_2 + |0\rangle_1|\alpha\rangle_2], \tag{5}$$

where $|0\rangle_i$ and $|\alpha\rangle_i$ are, respectively, Fock vacuum and coherent states in spatial mode i and $[\mathcal{N}_\alpha = 1/\sqrt{2(1 + e^{-|\alpha|^2})}]$ [13]. Note that $|\psi_{C_\alpha}^{\rm in}\rangle$ can be understood as a superposition of NOON states [18] (a related explanation is given in [7]) and the phase operation is imprinted in the outcome state

$$|\psi_{C_\alpha}^{\rm out}\rangle_{12} = \mathcal{N}_\alpha[|\alpha\rangle_1|0\rangle_2 + |0\rangle_1|\alpha e^{i\phi}\rangle_2]. \tag{6}$$

Considering first the situation with no loss, the optimal phase estimation of the pure states is analytically soluble. For the NOON and bat states, it is equal to $\delta\phi_N \geq 1/N$ and $\delta\phi_B \geq 1/\sqrt{N(N/2+1)}$, respectively, and for the ECS

$$\delta\phi_C \geq \frac{1}{2\alpha\mathcal{N}_\alpha\sqrt{1 + [1 - (\mathcal{N}_\alpha)^2]\alpha^2}}. \tag{7}$$

Taking into account equivalent resource counts for the states [19], we consider the same average photon number for mode 1 given by

$$\langle n_K\rangle = \langle\psi_K^{\rm in}|a_1^\dagger a_1|\psi_K^{\rm in}\rangle = \frac{N}{2} = \mathcal{N}_\alpha^2 \cdot |\alpha|^2. \tag{8}$$

Then, the phase uncertainty for the ECS can be compared with respect to N for the NOON and bat states as shown in Fig. 2. When N becomes large, $\delta\phi_C \approx \delta\phi_N$, which indicates that the ECS becomes approximately equivalent to the NOON state, being dominated by the NOON amplitude at $N = |\alpha|^2$. However, interestingly, $\delta\phi_N$ is signifi-

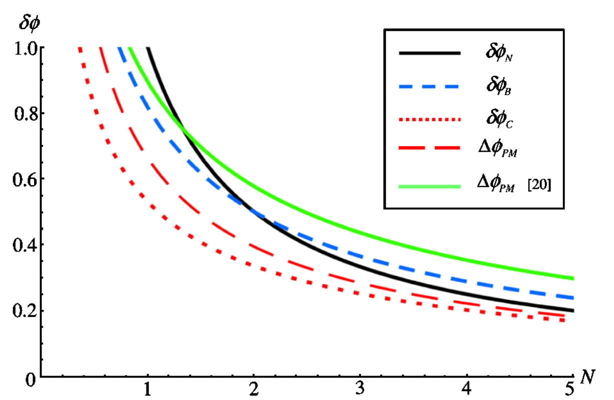

FIG. 2 (color online). The optimal phase estimations for NOON, bat, and ECSs with no particle loss are depicted in black solid, blue dashed, and red dotted lines ($\langle n\rangle = N/2 = \mathcal{N}_\alpha^2 \cdot |\alpha|^2$). Curves for NOON and bat states are shown as continuous for comparison, but are clearly only defined at the appropriate integers N according to Eq. (8). For small N, $\delta\phi_N$ is significantly bigger than $\delta\phi_C$ while $\delta\phi_N \approx \delta\phi_C$ for large N. The crossover between δ_N and δ_B at $N = 2$ indicates that the NOON and bat states are identical. The green solid and red long dashed lines show the phase estimation of the state given by Eq. (6) in Ref. [20] and $\Delta\phi_{\rm PM}$ above Eq. (10), respectively.

cantly bigger than $\delta\phi_C$ for small N because $|\psi_C^{\rm in}\rangle$ contains a superposition of NOON states including N values exceeding $|\alpha|^2$. Furthermore, for small α, the two terms in Eq. (5) are not orthogonal (and only tend to being so in the large α limit). The importance of nonorthogonality in a superposed single-mode state has been investigated as a quantum ruler [14]. These superposition properties enable an advantage for the coherent states at small $|\alpha|^2$. For a more detailed example, taking $N = 4$ for the NOON and bat states $\langle n_{N_4}\rangle = \langle n_{B_4}\rangle = 2$ and $\alpha = 2.0$ for the ECS (which gives a slightly lower resource count $\langle n_{C_2}\rangle = 1.964$), the values of the optimal phase estimation are equal to $\delta\phi_{N_4} = 0.25$, $\delta\phi_{B_4} \approx 0.289$, and $\delta\phi_{C_2} \approx 0.205$. This indicates that even with a slight resource disadvantage $\langle n_{C_2}\rangle < \langle n_{N_4}\rangle = \langle n_{B_4}\rangle$ there is still a phase estimation advantage $\delta\phi_{C_2} < \delta\phi_{N_4} < \delta\phi_{B_4}$ (see Fig. 2 at around $N = 4$). This is all very well in the zero loss regime; however, the more important question is on the robustness of the phase sensitivity enhancement in the realistic scenario of particle loss (for instance $\delta\phi_B < \delta\phi_N$ for large loss [10] indicating bat states are more sensitive in this regime). In order to obtain quantum Fisher information for a mixed state due to particle loss, calculation of eigenvalues and eigenvectors is required. From previous work [10], the optimal phase estimations for ρ_{12}^N (NOON) and ρ_{12}^B (bat) are already known when loss is included. Thus, we only need to focus on obtaining the phase estimation of the ECS $|\psi_{C_\alpha}^{\rm out}\rangle$. We can model such loss by two beam splitters with the same transmission coefficient T. The total state can be written by $|\Psi_{C_\alpha}\rangle_{1234} = {\rm BS}_{1,3}^T {\rm BS}_{2,4}^T |\psi_{C_\alpha}^{\rm out}\rangle_{12}|0\rangle_3|0\rangle_4$. Tracing out modes 3 and 4 we obtain the mixed state $\rho_{12}^{C_\alpha} = \sum_{n,m=0}^{\infty} P_{nm}\rho_{nm}$, where $P_{nm} = {}_{1234}\langle\Psi_{C_\alpha}|nm\rangle_{34} \times \langle nm|\Psi_{C_\alpha}\rangle_{1234}$ is the probability of detecting particles n in mode 3 and m in mode 4 (ρ_{nm} is the resultant density operator with P_{nm}). Because the case of particle loss in mode 3 (4) projects into state $|S_L\rangle_1 = |\alpha\sqrt{T}\rangle_1$ ($|S_R\rangle_2 = |\alpha\sqrt{T}e^{i\phi}\rangle_2$), the density operator can be written simply by $\rho_L = \rho_{n0} = |S_L\rangle_1\langle S_L| \otimes |0\rangle_2\langle 0|$ ($\rho_R = \rho_{0m} = |0\rangle_1 \times \langle 0| \otimes |S_R\rangle_2\langle S_R|$). Thus, the mixed state can be written in only two components given by

$$\rho_{12}^{C_\alpha} = P_{00}\rho_{00} + P_D\rho_D, \tag{9}$$

where the density operator for no particle loss is equal to $\rho_{00} = |S_{00}\rangle_{12}\langle S_{00}|$ $[|S_{00}\rangle = \mathcal{N}_{\alpha\sqrt{T}}(|S_L\rangle_1|0\rangle_2 + |0\rangle_1|S_R\rangle_2)]$ and the non-normalized mixed state is $\rho_D = \rho_L + \rho_R$. Note that the resultant state is a mixture of the ECS with $\alpha\sqrt{T}$ (for no loss) and the other mixed state ρ_D (for particle losses). The probability of no particle detection is $P_{00} = (e^{|\alpha|^2 T} + 1)/(e^{|\alpha|^2} + 1)$ and that of particle detections is $P_D = \sum_{n=1}^{\infty} P_{0n} = (\mathcal{N}_\alpha)^2(1 - e^{|\alpha^2|(T-1)})$.

To calculate quantum Fisher information for our state $\rho_{12}^{C_\alpha}$, we choose $\alpha = 2.0$ providing us the interesting regime for pure states and truncate the Fock basis at $n = 15$ (corresponding to a maximum error of approximately 10^{-5}). The mixed state in Eq. (9) is then approximately equal to $\tilde{\rho}_{12}^{C_2} = P_{00}\tilde{\rho}_{00} + (\sum_{n=1}^{15} P_{0n})\tilde{\rho}_D$. Using eigenvalues and eigenvectors of the truncated density matrix $\tilde{\rho}_{12}^{C_2}$, we obtain the optimal phase estimation of $\tilde{\rho}_{12}^{C_2}$ which we depict in Fig. 3. The optimal phase estimation of the entangled coherent state clearly improves on that of NOON, bat, and uncorrelated states under conditions of loss, for essentially the whole range of T. For $T \approx 1$, the value of the entangled coherent state follows that of the NOON state because $|\tilde{S}_{00}\rangle_{12}$ is the dominant factor of $\tilde{\rho}_{12}^{C_2}$ with large probability (see the inset in Fig. 3). However, it merges to that of the uncorrelated state at $T \ll 1$ because $\tilde{\rho}_D$ makes a major contribution in $\tilde{\rho}_{12}^{C_2}$ and is slightly better than the uncorrelated state, $\frac{1}{4}(|1\rangle_1|0\rangle_2 + e^{i\phi}|0\rangle_1|1\rangle_2)^{\otimes 4}$, due to phase coherence in the projected state given by the particle detection (see the state ρ_D). For large N, $\delta\phi_C$ approaches $\delta\phi_N$ due to $|\psi_C^{\rm in}\rangle \approx |\psi_N^{\rm in}\rangle$. We further remark on comparison with so-called "optimal states" [16]. Because of the concavity of Fisher information, the engineering of optimal input states for a known lossy rate has been considered [16]. These states effectively provide a smooth interpolation between NOON at high T and uncorrelated at low T, and so ECSs also offer advantage over these states.

Having demonstrated that moderate-size ECSs offer advantage with phase estimation, we also need to consider how such states could be implemented in order to realize this advantage. In principle this is achievable with current technology. There are basically four stages. (1) Generation of coherent state superpositions (CSS) $|{\rm CSS}_\alpha\rangle = N_\alpha(|\alpha\rangle + |-\alpha\rangle)$ for $N_\alpha = 1/\sqrt{2(1 + e^{-2|\alpha|^2})}$. (2) Application of ${\rm BS}_{1,2}^{1/2}$ to a coherent state $|\alpha\rangle_1$ and the CSS $|{\rm CSS}_\alpha\rangle_2$, with a resultant state $|\psi_{C_{\alpha'}}^{\rm in}\rangle = \mathcal{N}_{\alpha'}(|\alpha'\rangle_1|0\rangle_2 + |0\rangle_1|\alpha'\rangle_2)$ where $\alpha' = \sqrt{2}\alpha$. Thus, the state $|{\rm CSS}_{\sqrt{2}}\rangle$ is required for the input state $|\psi_{C_2}^{\rm in}\rangle$. According to experimental reports

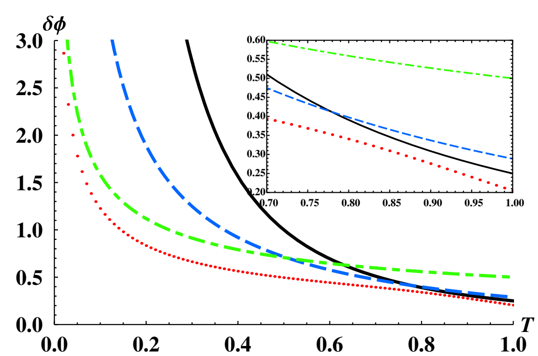

FIG. 3 (color online). The graphs show the phase uncertainty with respect to particle loss (T, transmission rate of the BSs) for four states ($N = 4$ and $\alpha = 2$). The legend of Fig. 2 is used here while the dash-dotted green line indicates uncorrelated states [10]. As shown in the magnified inset, the ECS curve starts from $\delta_{C_2} \approx 0.205$ at $T = 1$ and follows the NOON curve at $T \approx 1$.

[15], $|\mathrm{CSS}_\alpha\rangle$ with $\alpha \approx 1.5$ are already feasible in optics. (3) Application of a typical phase shifter on mode 2, with resultant state $|\psi^{\mathrm{out}}_{C_{\alpha'}}\rangle$. (4) Final measurement of the variance of the particle number in mode 2. This measurement scheme has already been demonstrated for mixed states with the help of the concavity of quantum Fisher information [16]. Alternatively, a final parity measurement [21] may be applicable, such as $(\Delta\phi_{\mathrm{PM}})^2 = (1 - \langle\Pi_2\rangle^2)/(\partial\langle\Pi_2\rangle/\partial\phi)^2$ given by the expectation value

$$\langle\Pi_2\rangle = \frac{2 + e^{-|\alpha|^2\cos\phi}(e^{-i|\alpha|^2\sin\phi} + e^{i|\alpha|^2\sin\phi})}{2 + 2e^{|\alpha|^2}} \tag{10}$$

for $\Pi_2 = e^{i\pi b_2^\dagger b_2}$. As shown in the red long-dashed line in Fig. 2, although the parity measurement on the pure ECS does not saturate the optimal phase estimation given by the quantum Fisher information for this state, it still beats the phase enhancement provided by the NOON state. However, for mixed states, the parity measurement advantage (over the optimal positive operator valued measure saturating the phase enhancement given by quantum Fisher information for NOON and bat) holds only in a narrow window ($0.995 \le T \le 1$) at $T \approx 1$, as the parity result diverges rapidly from the optimal ECS result with loss.

In summary, we have evaluated analytically and numerically the phase uncertainty of the ECS and shown that this state can outperform the phase enhancement limit given by NOON and other states possessing the same mean particle number, for the realistic scenarios of small particle number and loss. Clearly other superpositions of NOON states could be considered [18,22], but the ECS superposition is of specific interest due to its practical realizability. In current optical technology, it is already feasible to obtain a traveling CSS which is a key ingredient for the ECS. The photon-number [16] and parity measurement [21] approaches are likely to demonstrate an advantage over NOON and related states with current technology. Recent studies have investigated mixing squeezed and coherent states and nonlinearity of the phase operation [11,22]. Therefore, study of the effects of squeezing variables in the CSS and investigation of nonlinear effects in the phase operation form very interesting future research avenues, including imperfection studies [23].

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