Precise Determination of the $f_0(600)$ and $f_0(980)$ Pole Parameters from a Dispersive Data Analysis

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We use our latest dispersive analysis of $\pi\pi$ scattering data and the very recent $K_{\ell 4}$ experimental results to obtain the mass, width, and couplings of the two lightest scalar-isoscalar resonances. These parameters are defined from their associated poles in the complex plane. The analytic continuation to the complex plane is made in a model-independent way by means of once- and twice-subtracted dispersion relations for the partial waves, without any other theoretical assumption. We find the $f_0(600)$ pole at $(457^{+14}_{-13}) - i(279^{+11}_{-7})$ MeV and that of the $f_0(980)$ at $(996 \pm 7) - i(25^{+10}_{-6})$ MeV, whereas their respective couplings to two pions are $3.59^{+0.11}_{-0.13}$ and 2.3 ± 0.2 GeV.

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The $f_0(600)$ or sigma and $f_0(980)$ resonances are of great interest in several fields of physics. First, the two-pion exchange in the scalar-isoscalar channel, $I = 0$, $J = 0$, where these resonances appear, plays a key role in nuclear physics, where the nucleon-nucleon attractive interaction has been long [1] modeled by the exchange of a "sigma" resonance. Second, this channel is also relevant for the QCD non-Abelian nature, since it is where the lightest glueball is expected to appear. However, the glueball identification is complicated by its possible mixing into different states, like the $f_0(600)$, $f_0(980)$, and heavier f_0 resonances, which may be $\bar{q}q$ mesons, tetraquarks, molecules, or most likely a mixture of them all. Actually, most of the controversy around these resonances comes from the identification of scalar multiplets—see the Review of Particle Physics (PDG) "Note on Scalar Mesons" [2]. Third, the $f_0(600)$, being the lightest hadronic resonance with vacuum quantum numbers, plays a relevant role in many models of QCD spontaneous chiral symmetry breaking. Furthermore, this state is of interest in order to understand why, despite being so light and strongly coupled to pions, it plays such a small role, if any, in the saturation [3] of the low energy constants of chiral perturbation theory (ChPT). Moreover, the position of this pole could be setting the limit of applicability of the chiral expansion. Finally, this state is of interest for electroweak physics due to its many similarities—but even more by its many differences—with the Higgs mechanism now under scrutiny at the LHC.

Still, the properties of these resonances are the subject of an intense debate. Let us recall that the σ was listed in the PDG as "not well established" until 1974, removed in 1976, and listed back in 1996. This was due to its width being comparable to its mass, so that it barely propagates and becomes a broad enhancement in the traditional, and often contradictory, $\pi\pi$ scattering analyses, extracted from $\pi N \to \pi\pi N$ experiments, using different models affected by large systematic uncertainties. After 2000, these resonances have been observed in decays of heavier mesons, with well defined initial states and very different systematics from $\pi\pi$ scattering, which led the PDG to consider, in 2002, the $f_0(600)$ as "well established" but keeping until today a too conservative estimate of "mass: 400–1200 MeV" and "width: 600–1000 MeV." For the $f_0(980)$ the situation is not much better, with an estimated width "from 40 to 100 MeV." However, not all the uncertainty comes from experiment. The shape of these resonances varies from process to process, and that is why their masses and widths are quoted from their process-independent pole positions, defined as $\sqrt{s_{\rm pole}} \sim M - i\Gamma/2$. But many models do not implement rigorous analytic continuations and lead to incorrect determinations when poles are deep in the complex plane or close to threshold cuts, as happens with the $f_0(600)$ and the $f_0(980)$, respectively. Actually, this is one of the main causes of the huge PDG uncertainties [2].

This model dependence can be avoided by using dispersive techniques, which follow from causality and crossing and provide integral relations and a rigorous analytic continuation of the amplitude in terms of its imaginary part in the physical region, which can be obtained from data. For example, dispersion relations combined with ChPT determine the σ pole at $440 - i245$ MeV [4] or $(470 \pm 50) - i(260 \pm 25)$ MeV [5]. We focus here on dispersive analyses, but other approaches yield similar values [6,7]—see Table I and Ref. [14] for a review and references.

Generically, the main difficulty lies in the calculation of the left cut integral, which in Refs. [4,5] was just approximated. This left cut is due to crossing symmetry and can be incorporated rigorously in a set of infinite coupled equations written long ago by Roy [15] (see also [16] for

TABLE I. Other recent determinations of the σ pole and coupling, using analyticity. Results come from Roy equations and ChPT [8], conformal fits to $K_{\ell 4}$ decays and averaged $\pi\pi$ data around 800–900 MeV with only statistical [9] or also systematic [10] uncertainties, the chiral unitary approach [11] (only statistical error), a K matrix with a form factor shape [12], and ChPT + elastic dispersion relations (two loops [13]).

Reference	$\sqrt{s_\sigma}$ (MeV)	$\lvert g_{\sigma\pi\pi}\rvert$ (GeV)
[8]	$(441^{+16}_{-8}) - i(272^{+9}_{-12.5})$	$3.31^{+0.35}_{-0.15}$
[9]	$(474 \pm 6) - i(254 \pm 4)$	3.58 ± 0.03
[10]	$(463 \pm 6^{+31}_{-17}) - i(254 \pm 6^{+33}_{-34})$	...
[11]	$(443 \pm 2) - i(216 \pm 4)$	2.97 ± 0.04
[12]	$(452 \pm 12) - i(260 \pm 15)$	2.65 ± 0.10
[13] (fit D)	$453 - i271$	3.5

applications and references). Recently, Roy equations have been used to study low energy $\pi\pi$ scattering [17], sometimes combined with ChPT [18], or also to test ChPT [19], as well as to solve old data ambiguities [20]. Most recently [8], the $f_0(600)$ and $f_0(980)$ poles were shown to lie within the applicability region of Roy equations. Since data were not reliable and to improve accuracy, Roy equations were supplemented by ChPT predictions in Ref. [8], to yield $\sqrt{s_\sigma} = (441^{+16}_{-8}) - i(272^{+9}_{-19.5})$ MeV, without using data below 800 MeV on S and P waves. In that work, an $f_0(980)$ pole is also found at $\sqrt{s} = 1001 - i14$ MeV. Note that, generically, $\pi\pi$ scattering data around 900 MeV tend to produce a narrower $f_0(980)$ [7,8,11] than that seen in production processes or the PDG estimate. In Table II, we list some other recent determinations of the $f_0(980)$ parameters.

Our aim in this work is to provide a precise and model-independent simultaneous determination of the $f_0(600)$ and $f_0(980)$ parameters from data alone, profiting from two relevant results developed over the past half year: on the one hand, the final analysis of $K_{\ell 4}$ decays by the NA48/2 Collaboration [26], which provides reliable and precise $\pi\pi$ scattering phases below the mass of the kaon and, on the other hand, a set of Roy-like equations—called García-Martín–Kamiński–Peláez–Yndurain (GKPY) equations and developed by our group [27]—which is much more stringent in the resonant region than standard Roy equations. The reason is that, in order to avoid divergences, dispersion relations are weighted at low energy with "subtractions," but then amplitudes are determined only up to a polynomial, whose coefficients depend on threshold parameters. Since Roy equations have two subtractions, they have an s polynomial term multiplied by the isospin-2 scalar scattering length, whose large uncertainty thus grows markedly in the $f_0(600)$ and $f_0(980)$ region. In contrast, the GKPY equations have just one subtraction, and their output, even without using ChPT predictions at all, provides [27] a very precise description of $\pi\pi$ scattering data, discarding a long-standing conflict concerning the inelasticity—and to a lesser extent the phase shift—right above the $f_0(980)$ region.

TABLE II. Recent determinations of $f_0(980)$ parameters. For Ref. [21] our estimate covers the six models considered there. The last three poles come from scattering matrices and the rest from production experiments.

Reference	$\sqrt{s_{f_0(980)}}$ (MeV)	$\lvert g_{f_0\pi\pi}\rvert$ (GeV)
[22]	$(978 \pm 12) - i(28 \pm 15)$	2.25 ± 0.20
[21]	$(988 \pm 10 \pm 6) - i(27 \pm 6 \pm 5)$	2.2 ± 0.2
[23]	$(977 \pm 5) - i(22 \pm 2)$	1.5 ± 0.2
[24]	$(965 \pm 10) - i(26 \pm 11)$	2.3 ± 0.2
[11]	$(986 \pm 3) - i(11 \pm 4)$	1.1 ± 0.2
[12]	$(981 \pm 34) - i(18 \pm 11)$	1.17 ± 0.26
[25]	$999 - i21$	1.88

If we now use these GKPY dispersion relations to continue analytically that amplitude, we find

$$\sqrt{s_\sigma} = (457^{+14}_{-13}) - i(279^{+11}_{-7})\ \text{MeV}, \quad (1)$$

$$\sqrt{s_{f_0(980)}} = (996 \pm 7) - i(25^{+10}_{-6})\ \text{MeV}. \quad (2)$$

Let us describe next the whole approach in detail and provide determinations for other quantities of interest, like their couplings and the $\rho(770)$ parameters, as well as other checks of our calculations from Roy equations.

Ours is what is traditionally called an "energy-dependent" analysis of $\pi\pi$ scattering and $K_{\ell 4}$ decay data [28,29]—in particular, the latest results from NA48/2 [26]. Our procedure, described in a series of works [27,30], was to obtain as a first step a simple set of *unconstrained* fits to these data (UFD) for each partial wave separately up to 1420 MeV and Regge fits above that energy. Next we obtained *constrained* fits to data (CFD) by varying the UFD parameters in order to satisfy within uncertainties two crossing sum rules, a complete set of forward dispersion relations as well as Roy and GKPY equations, while simultaneously describing the data. The details for all CFD waves can be found in Ref. [27], but since we are now interested in the scalar-isoscalar partial wave $t_0^{(0)}$, we show in Fig. 1 the resulting $\delta_0^{(0)}$ phase shift. It should be noticed that the CFD result is indistinguishable to the eye from the UFD, except in the 900–1000 MeV region, which we also show in detail and is essential for the determination of the $f_0(980)$ parameters. Note that both the UFD and CFD describe the data in that region, but the GKPY dispersion relations require the CFD phase to lie somewhat higher than the UFD one. This is relevant since it yields a wider $f_0(980)$, correcting the above-mentioned tendency to obtain a too narrow $f_0(980)$ from unconstrained fits to $\pi\pi$ scattering data alone. In the inner top panel, we show the good description of the latest NA48/2 data on $K_{\ell 4}$ decays, which are responsible for the small uncertainties in our input parametrization and constrain our subtraction constants. As seen in Fig. 1, the inelasticity $\eta_0^{(0)}$ shows a

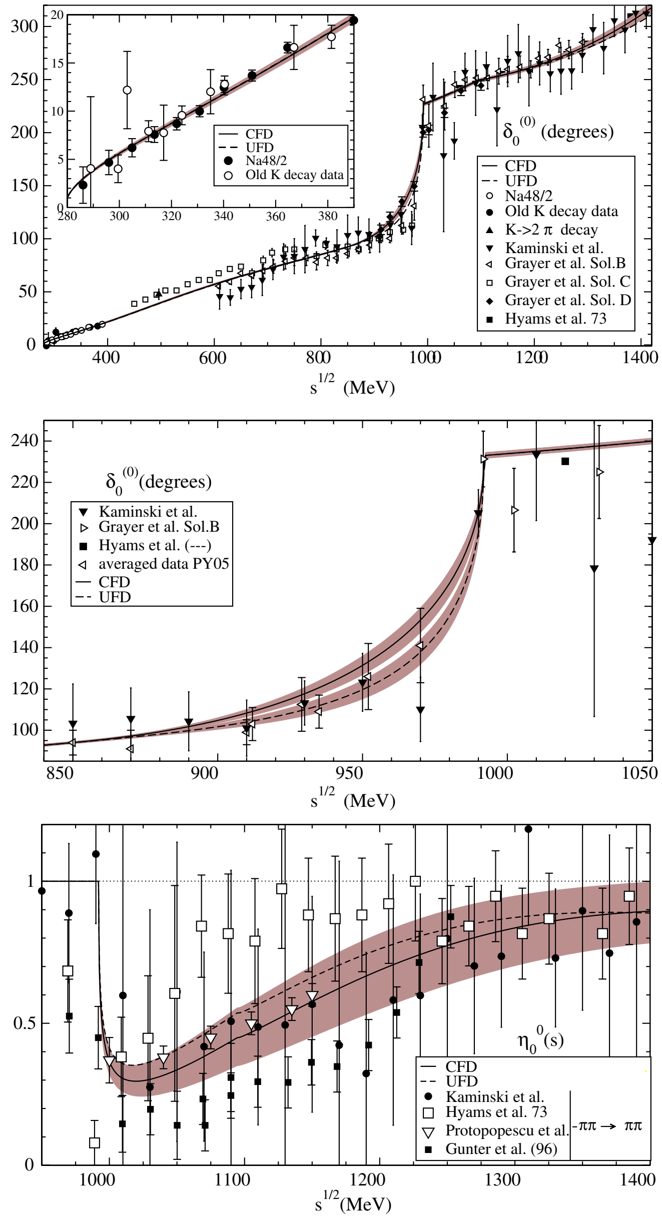

FIG. 1 (color online). S0 wave phase and inelasticity from UFD and CFD. Dark bands cover the uncertainties. The data come from Refs. [26,28].

"dip" structure above 1 GeV required by the GKPY equations [27], which disfavors the alternative "nondip" solution. Having this long-standing dip versus "no-dip" controversy [31] settled [27] is very relevant for a precise $f_0(980)$ determination.

The interest of this CFD parametrization is that, while describing the data, it satisfies within uncertainties Roy and GKPY relations up to their applicability range, namely, 1100 MeV, which includes the $f_0(980)$ region. In addition, the three forward dispersion relations are satisfied up to 1420 MeV. In Fig. 2, we show the fulfillment of the S0 wave Roy and GKPY equations and how, as explained above, the uncertainty in the Roy equation is much larger

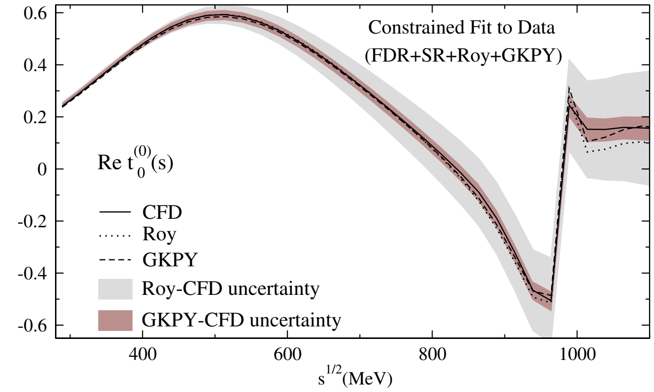

FIG. 2 (color online). Fulfillment of S0 wave Roy and GKPY equations. The CFD parametrization is the input to both the Roy and GKPY equations and is in remarkable agreement with their output. Note how the uncertainty in the Roy equation is much larger than that of the GKPY equation above roughly 500 MeV.

than for the GKPY equation in the resonance region. The latter will allow us now to obtain a precise determination of the $f_0(600)$ and $f_0(980)$ poles from data alone, i.e., without using ChPT predictions.

Hence, we now feed our CFD parameterizations as input for the GKPY and Roy equations, which provide a model-independent analytic continuation to the complex plane, and determine the position and residues of the second Riemann sheet poles. It has been shown [8] that the $f_0(600)$ and $f_0(980)$ poles lie well within the domain of validity of Roy equations, given by the constraint that the t values which are integrated to obtain the partial wave representation at a given s should be contained within a Lehmann-Martin ellipse. These are conditions on the analytic extension of the partial wave expansion, unrelated to the number of subtractions in the dispersion relation, and they equally apply to GKPY equations.

Thus, in Table III, we show the $f_0(600)$, $f_0(980)$, and $\rho(770)$ poles resulting from the use of the CFD parametrization inside Roy or GKPY equations. We consider that our best results are those coming from GKPY equations, since their uncertainties are smaller, although, of course, both results are compatible.

Several remarks are in order. First, statistical uncertainties are calculated by using a Monte Carlo Gaussian sampling of the CFD parameters with 7000 samples distributed

TABLE III. Poles and residues from Roy and GKPY equations.

	$\sqrt{s_{\rm pole}}$ (MeV)	$\lvert g\rvert$
$f_0(600)^{\rm Roy}$	$(445 \pm 25) - i(278^{+22}_{-18})$	3.4 ± 0.5 GeV
$f_0(980)^{\rm Roy}$	$(1003^{+5}_{-27}) - i(21^{+10}_{-8})$	$2.5^{+0.2}_{-0.6}$ GeV
$\rho(770)^{\rm Roy}$	$(761^{+4}_{-3}) - i(71.7^{+1.9}_{-2.3})$	$5.95^{+0.12}_{-0.08}$
$f_0(600)^{\rm GKPY}$	$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	$3.59^{+0.11}_{-0.13}$ GeV
$f_0(980)^{\rm GKPY}$	$(996 \pm 7) - i(25^{+10}_{-6})$	2.3 ± 0.2 GeV
$\rho(770)^{\rm GKPY}$	$(763.7^{+1.7}_{-1.5}) - i(73.2^{+1.0}_{-1.1})$	$6.01^{+0.04}_{-0.07}$

within 3 standard deviations. A systematic uncertainty due to the different charged and neutral kaon masses is relevant for the $f_0(980)$ due to the existence of two $\bar{K}K$ thresholds separated by roughly 8 MeV, which we have treated as a single $\bar{K}K$ threshold at $\hat{m}_K = (m_{K^0} - m_{K^+})/2 \simeq 992$ MeV. In order to estimate this systematic uncertainty, we have refitted the UFD and CFD sets to the extreme cases of using m_{K^0} or m_{K^+} instead of $\hat{m}_K$. As could be expected, the only significant variation is for the $f_0(980)$—actually, only for its half-width, which changes by ± 4.4 MeV for GKPY equations and ± 5.6 MeV for Roy equations. The $f_0(600)$ changes by roughly 1 MeV, and the $\rho(770)$ barely notices the change—less than 0.1 MeV. The effect on residues is smaller than that of rounding the numbers. We have added all these uncertainties in quadrature to the statistical ones. Second, both the mass and width of the $f_0(600)$ are compatible with those in Ref. [8] within 1 standard deviation. Since we are not using ChPT and Ref. [8] did not use data below 800 MeV, this is a remarkable check of the agreement between ChPT and low energy data. Third, the $f_0(980)$ width is no longer so narrow—as happens in typical $\pi\pi$ scattering analyses—and we find $\Gamma = 50^{+20}_{-12}$ MeV, very compatible with results from production processes. The mass overlaps within 1 standard deviation with the PDG estimate. These results show that the effect of the too narrow $f_0(980)$ pole and the use of further theoretical input like ChPT do not affect significantly the resulting $f_0(600)$ parameters.

In Table III, we also provide for each resonance its coupling to two pions, defined from its pole residue as

$$g^2 = -16\pi \lim_{s\to s_{\rm pole}} (s - s_{\rm pole}) t_\ell(s)(2\ell + 1)/(2p)^{2\ell}, \quad (3)$$

where $p^2 = s/4 - m_\pi^2$. This residue is relevant for models of the spectroscopic nature of these particles, particularly for the $f_0(600)$ [32], which are beyond the pure data analysis scope of this work. Differences between previous values of these couplings can be seen in Tables I and II.

In summary, using a recently developed dispersive formalism, which is especially accurate in the resonance region, we have been able to determine, in a model-independent way, the $f_0(600)$ and $f_0(980)$ poles and couplings from data with no further theoretical input. We hope this work helps to clarify the somewhat controversial situation regarding the parameters of these resonances.

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