Nonzero θ_{13} for Neutrino Mixing in the Context of A_4 Symmetry

Ernest Ma and Daniel Wegman

Department of Physics and Astronomy, University of California, Riverside, California 92521, USA

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In the original 2004 paper which first derived tribimaximal mixing in the context of A_4, i.e., the non-Abelian finite symmetry group of the tetrahedron, as its simplest application, it was also pointed out how $\theta_{13} \neq 0$ may be accommodated. On the strength of the new T2K result that $0.03(0.04) \leq \sin^2 2\theta_{13} \leq 0.28(0.34)$ for $\delta_{CP} = 0$ and normal (inverted) neutrino mass hierarchy, we perform a more detailed analysis of how this original idea may be realized in the context of A_4.

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Neutrino oscillations require nonzero neutrino masses as well as nonzero neutrino mixing angles. The current combined world data imply [1]

$$7.05 \times 10^{-5}\ \text{eV}^2 \leq \Delta m^2_{21} \leq 8.34 \times 10^{-5}\ \text{eV}^2, \tag{1}$$

$$2.07 \times 10^{-3}\ \text{eV}^2 \leq \Delta m^2_{32} \leq 2.75 \times 10^{-3}\ \text{eV}^2, \tag{2}$$

$$0.36 \leq \sin^2\theta_{23} \leq 0.67, \qquad 0.25 \leq \sin^2\theta_{12} \leq 0.37, \tag{3}$$

$$\sin^2\theta_{13} \leq 0.035 (90\%\text{C.L.}). \tag{4}$$

However, the T2K Collaboration recently announced that a new measurement [2] has yielded a nonzero θ_{13} at 90% confidence level, i.e.,

$$0.03(0.04) \leq \sin^2 2\theta_{13} \leq 0.28(0.34) \tag{5}$$

for $\delta_{CP} = 0$ and normal (inverted) neutrino mass hierarchy.

For several years now, the mixing matrix $U_{l\nu}$ linking the charged leptons (e, μ, τ) to the neutrino mass eigenstates (ν_1, ν_2, ν_3) has often been assumed to be of tribimaximal form [3], i.e.,

$$U_{\text{TB}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}, \tag{6}$$

which predicts $\theta_{13} = 0$. This is particularly appealing because it was derived in 2004 [4] from the simple application of the symmetry group A_4, first used for understanding maximal $\nu_\mu - \nu_\tau$ mixing in 2001 [5]. However, even in that original 2004 paper [4], the possibility of $\theta_{13} \neq 0$ was already anticipated. Although the new T2K result [2] is only 2.5σ away from zero, it is the most solid experimental indication to date of this possibility. Here we offer a more detailed analysis of how $\theta_{13} \neq 0$ may be realized in the context of A_4.

As is well known, A_4 is the group of the even permutation of 4 objects. It is also the symmetry of the perfect three-dimensional tetrahedron [6]. It has 12 elements and 4 irreducible representations: $\underline{1}$, $\underline{1}'$, $\underline{1}''$, $\underline{3}$, with the multiplication rule

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}. \tag{7}$$

The first step in understanding neutrino mixing is to show that A_4 allows the charged-lepton mass matrix to be diagonalized by the Cabibbo-Wolfenstein matrix [7,8]

$$U_{\text{CW}} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{8}$$

where $\omega = e^{2\pi i/3} = -1/2 + i\sqrt{3}/2$, with three independent eigenvalues, i.e., m_e, m_μ, m_τ. This has been achieved in two ways. One is the original proposal of 2001 [5]. The other was discovered later in 2006 [9]. In the former, the lepton assignments are $L_i = (\nu_i, l_i) \sim \underline{3}$, $l^c_1 \sim \underline{1}$, $l^c_2 \sim \underline{1}'$, $l^c_3 \sim \underline{1}''$, with 3 Higgs doublets $\Phi_i = (\phi^0_i, \phi^-_i) \sim \underline{3}$. In the latter, they are $L_i = (\nu_i, l_i) \sim \underline{3}$, $l^c_i \sim \underline{3}$, with 4 Higgs doublets $\Phi_i = (\phi^0_i, \phi^-_i) \sim \underline{3}$, $\Phi_0 \sim \underline{1}$. Assuming $v_1 = v_2 = v_3$ for the vacuum expectation values of Φ_i, which correspond to a Z_3 residual symmetry (lepton triality) [10–13], the seemingly impossible result of a diagonal charged-lepton matrix is always obtained from U_{CW} of Eq. (8), independent of the values of m_e, m_μ, m_τ. This is a highly nontrivial result, which motivates how the otherwise arbitrary 3×3 neutrino mass should be organized. It argues strongly for an underlying non-Abelian symmetry with a three-dimensional irreducible representation, the smallest of which is A_4.

We now consider the neutrino mass matrix in the original A_4 basis. Let there be 6 heavy Higgs triplets [14]:

$$\xi_1 \sim \underline{1}, \qquad \xi_2 \sim \underline{1}', \qquad \xi_3 \sim \underline{1}'', \qquad \xi_i \sim \underline{3}(i = 4, 5, 6), \tag{9}$$

where $\xi_i = (\xi^{++}_i, \xi^+_i, \xi^0_i)$. Then,

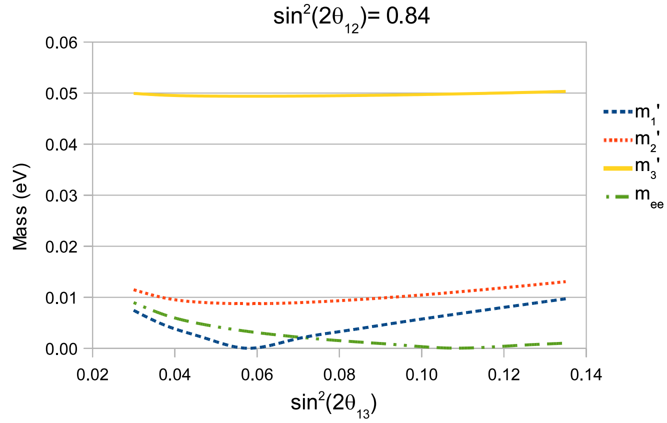

FIG. 1 (color online). Physical neutrino masses $|m'_{1,2,3}|$ and the effective m_{ee} for neutrinoless double beta decay of this model in the range $0.03 \leq \sin^2 2\theta_{13} \leq 0.135$ for $\sin^2 2\theta_{23} = 1$ and $\sin^2 2\theta_{12} = 0.84$.

$$\mathcal{M}_\nu = \begin{pmatrix} a+b+c & f & e \\ f & a+\omega b+\omega^2 c & d \\ e & d & a+\omega^2 b+\omega c \end{pmatrix}, \tag{10}$$

where a comes from $\langle \xi_1^0 \rangle$, b from $\langle \xi_2^0 \rangle$, c from $\langle \xi_3^0 \rangle$, d from $\langle \xi_4^0 \rangle$, e from $\langle \xi_5^0 \rangle$, f from $\langle \xi_6^0 \rangle$. As it stands, there is of course no prediction at all. For a pattern to emerge, the way A_4 breaks into its subgroups must be considered. For $b = c$ and $e = f = 0$, which breaks A_4 to Z_2, the neutrino mass matrix, written in the basis where the charged-lepton mass matrix is diagonal, is given by

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = U_{\rm CW}^\dagger \mathcal{M}_\nu U_{\rm CW}^* = \begin{pmatrix} a+(2d/3) & b-(d/3) & b-(d/3) \\ b-(d/3) & b+(2d/3) & a-(d/3) \\ b-(d/3) & a-(d/3) & b+(2d/3) \end{pmatrix}, \tag{11}$$

which is indeed diagonalized by $U_{\rm TB}$ of Eq. (6), with eigenvalues $m_1 = a - b + d$, $m_2 = a + 2b$, and $m_3 = -a + b + d$. It has been shown [15] how this pattern is obtained from A_4 alone with the help of lepton number.

Deviations from tribimaximal mixing may be obtained for $b \neq c$. This will allow ν_1 to mix with ν_3 and θ_{13} becomes nonzero. However, this same mixing will move θ_{12} to a larger value [4] so that $\tan^2\theta_{12} > 0.5$ which is not favored by current data. To allow $\tan^2\theta_{12} < 0.5$, it was proposed [4] that $e = -f \neq 0$ in Eq. (10). This is maintained by an assumed residual symmetry of the $\xi\Phi\Phi$ soft terms of the Higgs potential under which $\xi_5 \leftrightarrow -\xi_6$ and $\Phi_2 \leftrightarrow \Phi_3$. As a result, the neutrino mass matrix under $U_{\rm TB}$ is no longer diagonal, but is given by [4]

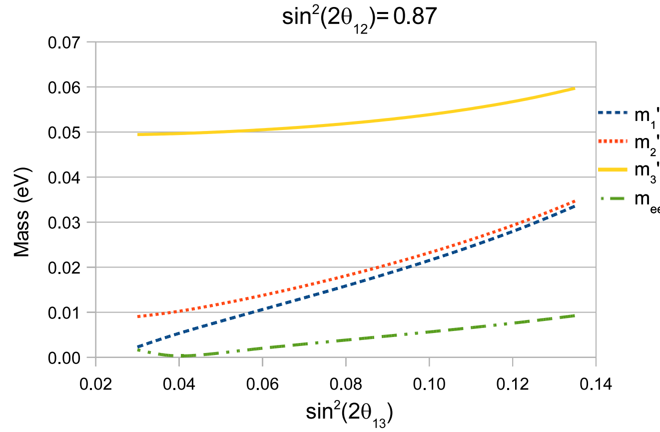

FIG. 2 (color online). Physical neutrino masses $|m'_{1,2,3}|$ and the effective m_{ee} for neutrinoless double beta decay of this model in the range $0.03 \leq \sin^2 2\theta_{13} \leq 0.135$ for $\sin^2 2\theta_{23} = 1$ and $\sin^2 2\theta_{12} = 0.87$.

$$\mathcal{M}_\nu^{(1,2,3)} = \begin{pmatrix} m_1 & 0 & m_4 \\ 0 & m_2 & m_5 \\ m_4 & m_5 & m_3 \end{pmatrix}, \tag{12}$$

where $m_1 = a - (b+c)/2 + d$, $m_2 = a + b + c$, $m_3 = -a + (b+c)/2 + d$, $m_4 = \sqrt{3}/2(c - b)$ and $m_5 = -i\sqrt{2}e$. If $m_4 = 0$, then ν_2 mixes with ν_3 and it can be shown that the allowed range of θ_{23} from Eq. (3) implies $\sin^2 2\theta_{13} \leq 0.04$ which lies on the outer edge of the allowed region of Eq. (5). In the following, we consider both m_4 and m_5 to be nonzero and study various numerical solutions to the T2K data.

The atmospheric neutrino mixing is assumed to be maximal, i.e., $\sin^2 2\theta_{23} = 1$, which is also the assumption of T2K in obtaining their new result. The solar neutrino mixing is taken to be $\sin^2 2\theta_{12} = 0.87 \pm 0.3$ [1]. We also use $\Delta m^2_{32} = 2.40 \times 10^{-3}$ eV^2 which is the value used

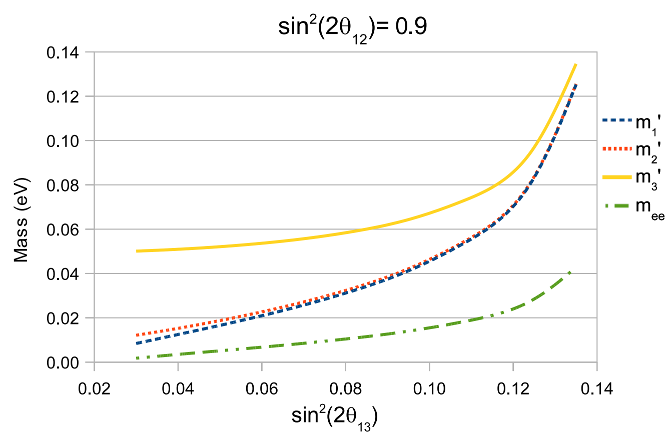

FIG. 3 (color online). Physical neutrino masses $|m'_{1,2,3}|$ and the effective m_{ee} for neutrinoless double beta decay of this model in the range $0.03 \leq \sin^2 2\theta_{13} \leq 0.135$ for $\sin^2 2\theta_{23} = 1$ and $\sin^2 2\theta_{12} = 0.90$.

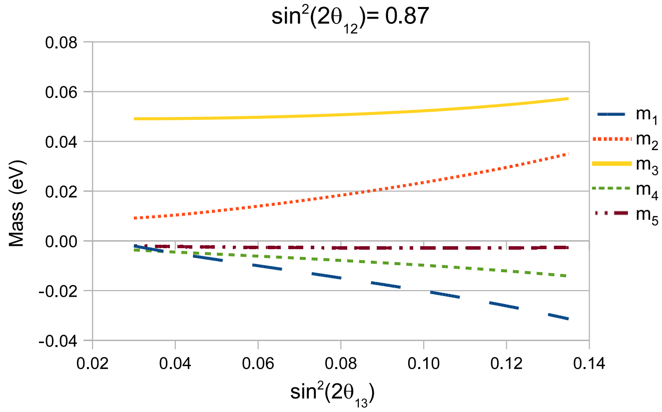

FIG. 4 (color online). The A_4 parameters $m_{1,2,3,4,5}$ of this model in the range $0.03 \leq \sin^2 2\theta_{13} \leq 0.135$ for $\sin^2 2\theta_{23} = 1$ and $\sin^2 2\theta_{12} = 0.87$.

by T2K, and $\Delta m^2_{21} = 7.65 \times 10^{-5}$ eV^2. For the central value of $\theta_{12} = 34.43°$, we have $\tan^2\theta_{12} = 0.47$ which is rather close to the tribimaximal prediction of 0.5. Using this and assuming the central value of $\sin\theta_{13} = 0.168$ ($\sin^2 2\theta_{13} = 0.11$), the zero entry of the neutrino mass matrix of Eq. (12) implies the condition

$$0.007\,655 m'_1 - 0.020\,990 m'_2 + 0.013\,342 m'_3 = 0, \quad (13)$$

where $m'_{1,2,3}$ are the mass eigenvalues of Eq. (12). Hence they are related to the measured Δm^2_{32} and Δm^2_{21} by

$$m'_2 = \pm\sqrt{m'^2_1 + \Delta m^2_{21}}, \quad (14)$$

$$m'_3 = \pm\sqrt{m'^2_1 + \Delta m^2_{21}/2 \pm \Delta m^2_{32}}. \quad (15)$$

There is only one solution to Eq. (13), i.e.

$$m'_1 = 0.0246 \text{ eV}, \qquad m'_2 = -0.0261 \text{ eV}, \qquad m'_3 = -0.0552 \text{ eV}, \quad (16)$$

which exhibits normal mass hierarchy. From this solution, we then obtain $m_{1,2,3,4,5}$ and the original A_4 parameters a, b, c, d, e. The ν_e mass observed in nuclear beta decay is given by $\sum_i |U_{ei}|^2 |m'_i| = 0.026$ eV. The effective mass m_{ee} for neutrinoless double beta decay is

$$m_{ee} = |a + (2/3)d| = |(2/3)m_1 + (1/3)m_2|. \quad (17)$$

We plot in Figs. 1–3 the solutions for $|m'_{1,2,3}|$ and m_{ee} as a function of $\sin^2 2\theta_{13}$ in the range 0.03 to 0.135 [corresponding to the upper bound given in Eq. (4)] for $\sin^2\theta_{23} = 1$ and the values $\sin^2 2\theta_{12} = 0.84$, 0.87, 0.90. Thus m_{ee} is predicted to be at most 0.04 eV. As for the ν_e mass in nuclear beta decay, it can be read off as approximately given by $(2|m'_1| + |m'_2|)/3$. We also plot in Fig. 4 the values of $m_{1,2,3,4,5}$ for $\sin^2 2\theta_{12} = 0.87$. This shows that m_4 and m_5, i.e., the parameters of A_4 which deviate from tribimaximal mixing, are indeed small. In terms of A_4 symmetry, the following breaking patterns are in effect: in the charged-lepton sector, A_4 breaks to Z_3 (which may be verified experimentally from Higgs-boson decay [13]); in the neutrino sector, A_4 breaks first to Z_2 (the tribimaximal limit), and then Z_2 is also broken with the pattern $b \neq c$ and $f = -e$, which may be maintained by a suitably chosen set of soft terms.

In conclusion, on the strength of the recent observation [2] of a nonzero θ_{13} for neutrino mixing, the original A_4 proposal of 2004 [4] is updated. We find that solutions are indeed possible with the most recent data but only in a normal hierarchy of neutrino masses, i.e. $|m'_1| < |m'_2| < |m'_3|$. We confirm that the parameters of A_4 which deviate from tribimaximal mixing, i.e. m_4 and m_5, are indeed small. We also make predictions on the effective m_{ee} in neutrinoless double beta decay.

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