Local Realism of Macroscopic Correlations

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We identify conditions under which correlations resulting from quantum measurements performed on macroscopic systems (systems composed of a number of particles of the order of the Avogadro number) can be described by local realism. We argue that the emergence of local realism at the macroscopic level is caused by an interplay between the monogamous nature of quantum correlations and the fact that macroscopic measurements do not reveal properties of individual particles.

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Nonlocality is one of the most striking properties of quantum mechanics. Two distant observers, each holding half of an entangled quantum state and performing appropriate measurements, share correlations which are nonlocal: in 1964, John Bell used astonishingly simple reasoning in the form of an algebraic inequality (Bell inequality) to demonstrate that the predictions of quantum theory cannot be explained by a model consisting of local variables (local realism) [1]. His findings have been confirmed in numerous experiments in which various loopholes, resulting from experimental deficiencies, were closed individually [2]. Although there is still no conclusive experiment that simultaneously closes all the loopholes, most scientists think that on the microscopic scale the world is *not* local realistic.

In contrast, the macroscopic world we experience is described by classical physics, which is a special case of a local realistic theory. Note that, local realism does not rule out entanglement which is a manifestation of quantum weirdness [3]. An interesting question to ask is how a local realistic macroscopic world emerges from the microscopic scale, on which level it cannot be described by local realism. A number of resolutions to this question have been suggested. The more radical ones, the so-called collapse models [4], predict that quantum mechanics fails for sufficiently complex systems. Another approach is to look for classicality as a limit of quantum phenomena. The decoherence programme derives the lack of superposition of the pointer state of the measuring apparatus from an inevitable interaction between the quantum system and its environment [5]. Unlike these previous approaches, which have sought to explain why quantum properties such as entanglement are not present in macroscopic systems, we instead ask why, given limited experimental capabilities, we have never found correlations in a macroscopic system which cannot be explained by a local realistic theory. For a related, but different approach to this question; see [6].

The purpose of this Letter is to consider the correlations that we can reasonably measure on macroscopic samples consisting of spins numbering on the order of the Avogadro number (10^{23} particles). With existing experimental capabilities [7], it is impossible to independently, coherently operate on every individual particle and there might even be fundamental reasons why such manipulations would never be possible [8]. We show that if the number of measured particles is large enough, a local realistic description emerges, regardless of the quantum state of the whole system, even if this quantum state is highly entangled (note that highly entangled states between macroscopic systems have been prepared in the lab, for instance [9]). The roots of the local realistic behavior are the monogamous nature of quantum correlations [10,11] and the fact that macroscopic measurements do not reveal properties of individual particles. We note that this local realistic macroscopic limit is more general than classical physics itself and it is an interesting problem to identify classical physics within a set of local realistic theories.

We first show that quantum mechanical predictions for macroscopic correlations are described by effective states which admit explicit local realistic models, similar to those of Ref. [12] for any number of measurement settings smaller than the number of particles in the sample. Furthermore, in some cases the effective states satisfy an even stronger condition derived in Ref. [13]. Finally, we prove that all macroscopic correlations of the physically important rotationally invariant states (such as thermal states of Heisenberg spin Hamiltonians or ground states of high temperature superconductors) have a local and realistic description regardless of the number of measurement settings.

Macroscopic measurements.—Consider a macroscopic sample composed of many microscopic spins. This is often a good approximation to systems such as magnetic materials and metals [14]. The simplest observables to measure are different directions of magnetization in macroscopic parts of the system. The outcome of a magnetization measurement does not reveal information about the spin projections of individual particles; there are many configurations of individual spin projections that give the same total magnetization. The situation is therefore analogous to statistical mechanics, where one macrostate is realized by the averaging over an enormous number of microstates. Moreover, in a realistic experiment such as those performed in [7], where a macroscopic region of the sample is measured, one obtains directly the average value of magnetization with vanishingly small fluctuations without many repetitions of the experiment. The detailed probability distribution that determines the average value is, for all practical purposes, inaccessible.

Macroscopic correlations.—We now divide a system of many spins into several regions $X = A, B, \ldots, K$ and study correlations between (generalized) magnetization observables of each region (see Fig. 1). The spins can be of arbitrary local Hilbert space dimension, we only require that within a region they are all the same dimension, d_X (qudits), and there are N_X of them, enumerated by $x = 1, \ldots, N_X$. Within each region, we consider S_X sets of measurement operators (POVMs) $E_{i,j}^{X,x}$. Each set, indexed by i satisfies a completeness relation over the possible outcomes j (of arbitrary number) such that $\sum_j E_{i,j}^{X,x} = \mathbb{1}_{d_X}$. These can be used to specify the operator of generalized magnetization in region X as

$$\mathcal{M}_i^X = \sum_{x=1}^{N_X} \sum_j f(j|i) E_{i,j}^{X,x}, \tag{1}$$

the usual magnetization being the case of $f(j|i) = j$. We assume, as a reflection of the macroscopic nature of the measurements, that POVM elements are the same for all particles within a region, and denote such elements as $E_{i,j}^X$ where the particle index is skipped. Because of this assumption, the correlations between macroscopic

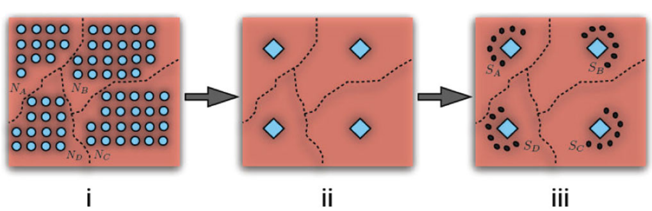

FIG. 1 (color online). The three stages of the LHV strategy outlined in the text. (i) A system with four regions ($X = A, B, C, D$) of N_X particles (shown as circles), measured with S_X different settings. As explained in the text, an effective state ρ_{eff} for these measurements can be found and is shown in (ii) where the squares depict effective particles. (iii) denotes the symmetric extension of ρ_{eff} to a state σ_N of S_X particles (depicted by circles surrounding squares) in each region. In (iii), all the S_X measurements in each region X commute, implying the existence of the joint probability distribution and therefore, an LHV model for the state in (i).

measurements in a state ρ, $\mathbb{E}_{\vec{i}} = \mathrm{Tr}(\rho \mathcal{M}_{i_A}^A \otimes \ldots \otimes \mathcal{M}_{i_K}^K)$, are described by an effective positive semidefinite operator of only K spins

$$\rho_{\mathrm{eff}} = \frac{1}{N_A \ldots N_K} \sum_{a \in A \ldots k \in K} \rho_{ab\ldots k}, \tag{2}$$

where $\rho_{ab\ldots k}$ is the reduced density matrix of the original state ρ on just spins a to k, one from each region. The formula for the correlations now reads

$$\mathbb{E}_{\vec{i}} = \left(\prod_{X=A}^{K} N_X \right) \sum_{\vec{j}} \left(\prod_{X=A}^{K} f(j_X|i_X) \right) P(\vec{j}|\vec{i}), \tag{3}$$

where $\vec{j} = (j_A, \ldots, j_K)$ is a vector of measurement outcomes and $P(\vec{j}|\vec{i}) = \mathrm{Tr}(\rho_{\mathrm{eff}} E_{i_A,j_A}^A \otimes \ldots \otimes E_{i_K,j_K}^K)$ gives the probability to obtain these outcomes if the effective state is measured with settings $\vec{i} = (i_A, \ldots, i_K)$.

Local realistic models.—If the states $\rho_{ab\ldots k}$ in Eq. (2) were arbitrary, then $P(\vec{j}|\vec{i})$ could violate a Bell inequality. However, these states must be compatible with a global state of the system which imposes a trade-off between their correlations; i.e., any failure to violate a Bell inequality is a consequence of the monogamy of correlations in the system. We show that $P(\vec{j}|\vec{i})$ admits a local hidden variable (LHV) model if the number of settings in a region does not exceed the number of particles, i.e. $S_X \leq N_X$. It then follows from Eq. (3) that also the set of correlations $\mathbb{E}_{\vec{i}}$ obtained from the state ρ admits a model. The crucial technical insight is that the effective state ρ_{eff} has a symmetric extension and therefore an extended version of the results of Ref. [12] apply to it. Generally speaking, a K-qudit state σ_K admits an N-qudit symmetric extension σ_N if all K-qudit reduced operators of σ_N are given by σ_K. In our case, the symmetric extension of ρ_{eff} can always be obtained from the initial state ρ by making it permutationally invariant

$$\sigma_N = \frac{1}{N_A! \ldots N_K!} \sum_{\Pi_A \ldots \Pi_K} (\Pi_A \otimes \ldots \otimes \Pi_K) \times \rho(\Pi_A^\dagger \otimes \ldots \otimes \Pi_K^\dagger), \tag{4}$$

where the normalization factor counts the number of permutations Π_X of particles within each region X. Clearly, all the K-qudit reduced operators of σ_N are the same, and are given by ρ_{eff}. Note that σ_N is a state of $\sum_X N_X$ qudits which we call "virtual particles."

Now we are in a position to present the proof of existence of an LHV model based on the symmetric extension of ρ_{eff}. We use the well known result of [15], which states that if there exists a joint probability distribution for the outcomes of measurements, there is a LHV model for these measurements. When $S_X = N_X$ for all regions X, the symmetric extension replaces the effective state ρ_{eff} by the state σ_N of $\sum_X S_X$ virtual particles. Therefore, instead of measuring S_X noncommuting observables on the Xth particle of ρ_{eff} we measure all S_X observables, one on each virtual particle of σ_N. This procedure reproduces all the statistics that would be obtained if measurements were performed on ρ_{eff} itself. Since the measurements performed on different virtual particles commute, the joint probability distribution for the observables measured on ρ_{eff} exists. In this approach, the local hidden variables are given by the sequence of measurement outcomes for different settings in each region. Each such hidden variable occurs with the probability given by the corresponding joint probability distribution. This procedure to show the existence of an LHV model for ρ_{eff} is illustrated in Fig. 1.

The result readily extends in two ways. Firstly, observe that in the S_X measurement settings, any two can be set equal to each other, and the result still holds. Thus, in fact, the result holds provided all $S_X \leq N_X$. Secondly, we can examine many-body observables. Magnetization is just a one-body observable because the POVM elements in Eq. (1) involve only one particle. More generally, M-body observables contain POVMs for M particles in each region. This has the knock-on effect of simply rescaling the limiting number of Bell measurements to N_X/M, assuming this is an integer. So, a system of say 10^{23} particles divided into 10^7 partitions, and involving 10^7-body observables would still require at least 10^9 measurement settings to possibly measure some violation of a Bell inequality, which we consider highly infeasible.

This no-go theorem gives a very strong bound on the degree of control we would need over large systems for there to possibly be a violation of a Bell inequality. Indeed, it is tight, at least for simple cases. Clearly, if $N_A = N_B = 1$ and $S_A = S_B = 2$, one violates the CHSH inequality. We also verified that for $N_A = 1$, $N_B = 2$ and $S_A = 3$, $S_B = 3$ there exist three-qubit states for which the Collins-Gisin inequality is violated [16].

Constraints on symmetric extensions.—We now restrict our attention to two measurement settings per region and an effective state of many qubits. We show that in this scenario, if a state has a symmetric extension to an adequate number of qubits, not only do its correlations admit a LHV model, but they also satisfy the sufficient condition for the existence of a LHV model derived in Ref. [13]; i.e., they are a proper subset of all possible LHV models. It was shown there that a sufficient condition for a LHV model for a two-setting Bell experiment reads

$$\vec{T}\cdot\vec{T} \equiv \sum_{l_A,\ldots,l_K=\{x,y\}} T^2_{l_A\ldots l_K} \leq 1, \tag{5}$$

for any choice of orthogonal local coordinate directions $\vec{x}$ and $\vec{y}$ (and hence for any pair of local settings), where $T_\Theta = \mathrm{Tr}(\rho\Theta)$ denotes the correlation functions for a given ρ and some tensor product of Pauli operators, Θ.

Consider a K-qubit state, ρ_K, with symmetric extension ρ_N involving $N = \sum_{X=1}^{K} N_X$ qubits. We first prove that condition (5) is satisfied if $N_X \geq 2^{X-1}$. To this end, pick up one particle in A, two in $B, \ldots, 2^{K-1}$ in region K and draw a graph of a binary tree in which vertices are the particles and different levels correspond to different values of K; i.e., we have one particle on level A connected to two particles on level B, each of which is connected to two different particles on level C and so on. The existence of a symmetric extension ρ_N implies, in particular, that every of its reduced K-qubit density operators $\rho_{ab\ldots k}$ corresponding to a path from the root to the leaves of the tree is the same as ρ_K, so $T^{(AB\ldots K)}_{l_A l_B\ldots l_K} = T^{(ab\ldots k)}_{l_A l_B\ldots l_K}$, and

$$T^{(AB\ldots K)}_{l_A l_B\ldots l_K} = \frac{1}{2^{K-1}}\sum_k T^{(ab\ldots k)}_{l_A l_B\ldots l_K}, \tag{6}$$

where the normalization factor counts the reduced K-qubit density operators of the paths. Note that once K's qubit, k, is specified, the path, and hence all the other $K-1$ qubits, is uniquely specified.

That we satisfy (5) uses the monogamy result of [17], where it was shown that any set of correlation functions T_{Θ_i}, with $\Theta_i^2 = \mathbb{1}$ and for which all Θ_i pairwise anticommute, satisfies the bound $\sum_i T^2_{\Theta_i} \leq 1$. Our approach is to subdivide the set of correlations in $\vec{T}$ between a set of mutually orthogonal vectors $\vec{W}(y)$ for $y \in \{0,1\}^{K-1}$,

$$\vec{T} = \frac{1}{2^{K-1}}\sum_{y\in\{0,1\}^{K-1}} \vec{W}(y), \tag{7}$$

such that all correlations in each vector come from pairwise anticommuting Pauli observables, and hence the length of each will be bounded by 1, ultimately bounding the length of $\vec{T}$ by 1. The set of 2^K correlation functions in a given vector $\vec{W}(y)$ is given by the rows of a matrix which divides into K column blocks, X, each specified by a matrix

$$\mathbb{1}_{X-1}\otimes\begin{pmatrix}1+y_{X-1}\\2-y_{X-1}\end{pmatrix}\otimes\underline{1}_{K-X}$$

where $\mathbb{1}_{X-1}$ is the $2^{X-1}\times 2^{X-1}$ identity matrix, and $\underline{1}_{K-X}$ is a column vector of 2^{K-X} ones. The elements of the matrix, i, translate into using a Pauli matrix σ_i. An illustrative example is provided in Table I.

It is now easy to see that the operators of the rows mutually anticommute (for general K, this is easily proved by induction). Different values of y just correspond to permutations of elements within the block columns, and do not alter the anticommutation properties. According to Ref. [17], each of the 2^{K-1} vectors $\vec{W}(y)$ has length no greater than 1, and therefore Eq. (5) is satisfied.

In the above derivation, the number of particles per region is exponential in the number of regions and the most densely populated region contains 2^{K-1} particles. As a consequence, for us to prove that the condition (5) holds, the maximal number of regions into which N qubits can be divided is $K = O(\log N)$. This bound can be improved such that the highest population per region is given

TABLE I. In the case of $K = 3$, a set of eight mutually anti-commuting operators $\vec{W}(0,0)$ is constructed using the listed Pauli operators.

X	1	2		3			
$n\backslash x$	1	1	2	1	2	3	4
1	σ_x	σ_x	$\mathbb{1}$	σ_x	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$
2	σ_x	σ_x	$\mathbb{1}$	σ_y	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$
3	σ_x	σ_y	$\mathbb{1}$	$\mathbb{1}$	σ_x	$\mathbb{1}$	$\mathbb{1}$
4	σ_x	σ_y	$\mathbb{1}$	$\mathbb{1}$	σ_y	$\mathbb{1}$	$\mathbb{1}$
5	σ_y	$\mathbb{1}$	σ_x	$\mathbb{1}$	$\mathbb{1}$	σ_x	$\mathbb{1}$
6	σ_y	$\mathbb{1}$	σ_x	$\mathbb{1}$	$\mathbb{1}$	σ_y	$\mathbb{1}$
7	σ_y	$\mathbb{1}$	σ_y	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$	σ_x
8	σ_y	$\mathbb{1}$	σ_y	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$	σ_y

by $\lceil \frac{2^{K-2}}{K-1} \rceil$, using certain modifications of the presented algorithm (see [18] for further details). Moreover, as we did not prove that our algorithm is optimal, perhaps a stronger bound can be derived. It is also interesting to explore the relationship between the sufficient condition (5) and the existence of a symmetric extension for the state.

Rotationally invariant systems.—Stronger results can be proved for restricted classes of state ρ, such as N qubit rotationally invariant states, $\rho = (U \otimes \ldots \otimes U)\rho(U^\dagger \otimes \ldots \otimes U^\dagger)$, for all single qubit unitaries U. This is a wide class of physically important states, including thermal states of the Heisenberg model. We show that macroscopic correlations measured on such states admit a LHV model no matter how many measurement settings are allowed.

Any reduced density matrix ρ_{ij} obtained from a rotationally invariant density matrix ρ is also rotationally invariant, i.e., $\rho_{ij} = U \otimes U \rho_{ij} U^\dagger \otimes U^\dagger = V_{ij}|\psi_-\rangle \times \langle\psi_-|_{ij} + (1-V)\frac{\mathbb{1}_i \otimes \mathbb{1}_j}{4}$ [3]. Thus, any two-party effective state $\rho_{\rm eff}$ inherits the same property:

$$\rho_{\rm eff} = V|\psi_-\rangle\langle\psi_-|_{AB} + (1-V)\frac{\mathbb{1}_A \otimes \mathbb{1}_B}{4}, \tag{8}$$

where $-\frac{1}{3} \leq V \leq 1$. It was proven in Ref. [19] that for $-\frac{1}{3} \leq V \leq 0.66$ this state admits a LHV description for all sets of projective quantum measurements. The upper bound on this range can be extended to $2/3$ by invoking the results of [20] within the formalism presented in [19]. It is also known [21] that if $p \leq \frac{5}{12}$, there is no Bell inequality violation at all, even allowing for POVMs. From our prior description of $\rho_{\rm eff}$, we can say that

$$V = \frac{1}{N_A N_B}\sum_{ij} V_{ij} \tag{9}$$

and, from singlet monogamy [22], one can prove that

$$V \leq \frac{R_{ab} + 2}{3R_{ab}} \tag{10}$$

where $R_{ab} = \max(N_A, N_B)$. Thus, provided our sample contains more than two qubits and is divided into two regions, we can never violate a macroscopic Bell inequality (of any number of settings) composed of projective measurements. If N_A or $N_B \geq 8$, there are no Bell inequalities whatsoever that can be violated.

Conclusions.—We have studied the conditions under which one can sustain a local realistic description of correlations between measurements on macroscopic groupings of spins. For a large system of spins ($N \approx 10^{23}$) in an arbitrary quantum state, partitioned into $k \geq 2$ regions, each containing a number of spins of the order of $\frac{N}{k}$, provided the number of measurement directions in each region is smaller than the number of particles in the region, a LHV model that is consistent with the measurable observables exists. Even when such a model describes the correlations, entanglement can still be detected between the different regions using similar measurements. There are some physically important classes of states, such as rotationally invariant ones, which can never violate a Bell inequality formed from these correlations for any number of measurement settings. The monogamy of certain quantum correlations and the lack of individual addressability in macroscopic systems was seen to lie behind this manifestation of local realism. Our proof of the emergence of local realism in macroscopic systems was derived using the quantum formalism. It would be interesting to demonstrate this from the principle of no-signalling and a reasonable notion of macroscopic measurements. It will also be interesting to impose that two-body correlators can only be measured between spins which are neighbors on a lattice.

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