Chiral Magnetic Wave at Finite Baryon Density and the Electric Quadrupole Moment of the Quark-Gluon Plasma

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The chiral magnetic wave is a gapless collective excitation of quark-gluon plasma in the presence of an external magnetic field that stems from the interplay of chiral magnetic and chiral separation effects; it is composed of the waves of the electric and chiral charge densities coupled by the axial anomaly. We consider a chiral magnetic wave at finite baryon density and find that it induces the electric quadrupole moment of the quark-gluon plasma produced in heavy ion collisions: the ''poles'' of the produced fireball (pointing outside of the reaction plane) acquire additional positive electric charge, and the ''equator'' acquires additional negative charge. We point out that this electric quadrupole deformation lifts the degeneracy between the elliptic flows of positive and negative pions leading to $v_2(\pi^+) < v_2(\pi^-)$, and estimate the magnitude of the effect.

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Introduction.—The axial anomaly has been found to induce the following two phenomena in the quark-gluon plasma subjected to an external magnetic field: the chiral magnetic effect (CME) and the chiral separation effect (CSE). The CME is the phenomenon of electric charge separation along the axis of the applied magnetic field in the presence of a fluctuating topological charge [1–5]. The CME in QCD coupled to electromagnetism assumes an asymmetry between the densities of left- and right-handed quarks, parametrized by an axial chemical potential μ_A. At finite μ_A, an external magnetic field induces the vector current $j_i = \bar{\psi}\gamma_i\psi$:

$$\boldsymbol{j}_V = \frac{N_c e}{2\pi^2}\mu_A \boldsymbol{B}; \quad (1)$$

in our present convention the electric current is $e j_V$.

Recently, the STAR [6,7] and PHENIX [8,9] Collaborations at the Relativistic Heavy Ion Collider at Brookhaven National Laboratory reported experimental observation of charge asymmetry fluctuations possibly providing evidence of the CME; this interpretation is still under intense discussion (see, e.g., [10,11] and references therein).

The CSE refers to the separation of chiral charge along the axis of the external magnetic field at finite density of vector charge (e.g., at finite baryon number density) [12,13]. The resulting axial current is given by

$$\boldsymbol{j}_A = \frac{N_c e}{2\pi^2}\mu_V \boldsymbol{B}, \quad (2)$$

where μ_V is the vector chemical potential. Both CME and CSE effects have been proved robust in holographic QCD models in a strong coupling regime [14–21] as well as in lattice QCD computations [22,23]. The effects also persist in relativistic hydrodynamics, as shown in Ref. [24].

Recently, two of us studied the properties of the chiral magnetic wave (CMW) [25] stemming from the coupling of the density waves of electric and chiral charge induced by the axial anomaly in the presence of an external magnetic field; a related idea has been also discussed in [26]. The CMW is a gapless collective excitation; its existence is a straightforward consequence of the relations of Eqs. (1) and (2). Indeed, consider a local fluctuation of electric charge density; according to Eq. (2) it induces a local fluctuation of the axial-vector current. This fluctuation of axial current in turn induces a local fluctuation of the axial chemical potential, and thus according to Eq. (1) a fluctuation of electric current. The resulting fluctuation of electric charge density completes the cycle leading to the CMW that combines the density waves of electric and chiral charges.

The plasma created in heavy ion collisions possesses a finite baryon density. The CSE [12,13,24] then implies the separation of chiral charge: the ''poles'' of the fireball acquire the chiral charges of opposite sign. The CME current at the opposite poles then according to Eq. (1) flows in opposite directions, as argued recently in [27]. In this Letter we will show that CMW induces a static quadrupole moment of the electric charge density.

Chiral magnetic wave.—The CMW is a long wavelength hydrodynamic mode of chiral charge densities; its propagation in space-time along the direction of magnetic field (denoted x_1 here) is described by the following equation [25]:

$$(\partial_0 \mp \partial_1 v_\chi - D_L\partial_1^2 - D_T\partial_T^2) j^0_{L,R} = 0, \quad (3)$$

where v_χ is the wave velocity and D_L (D_T) is the longitudinal (transverse) diffusion constant.

In the case of N_f quark flavors with electric charges q_f there will be N_f independent CMWs with the velocities and longitudinal diffusion constants determined by q_f, eB, and T. In this Letter we consider the propagation of u and d flavored CMWs, since there is no net density of strange quarks in the plasma. The full flavor symmetry $U(2)_f$ contains $U(1)_u \times U(1)_d$ which defines independent $U(1)$ flavor symmetries of u and d quarks. Considering the same triangle anomalies leading to CME and CSE that now involve each of these $U(1)$ symmetries, one obtains

$$\boldsymbol{j}^f_{V,A} = q_f \frac{N_c e}{2\pi^2} \mu^f_{A,V} \boldsymbol{B}, \tag{4}$$

where μ^f are chemical potentials of $U(1)_f$. From the results of [25] and (4) we then derive N_f independent CMWs of flavored chiral charge densities $j^{0,f}_{L,R}$ with velocities given by

$$v^f_\chi = q_f \frac{N_c eB}{4\pi^2} \left(\frac{\partial \mu^f_L}{\partial j^{0,f}_L}\right) \equiv q_f \frac{N_c eB\alpha^f}{4\pi^2}. \tag{5}$$

We obtain v^f_χ and D^f_L from the computation in Ref. [25] performed in the framework of the Sakai-Sugimoto model in the large N_c quenched approximation. Each quark of flavor f interacts with the magnetic field of effective magnitude $q_f eB$; we replace eB with $q_f eB$ in the arguments of v_χ and D_L as functions of eB:

$$v^f_\chi = v_\chi(eB \to q_f eB), \qquad D^f_L = D_L(eB \to q_f eB). \tag{6}$$

We evaluate the densities of u and d flavors at the time of the plasma creation in the Au-Au collisions, and introduce the corresponding initial chemical potentials $\mu^u_V + \mu^d_V = 2\mu_B/3$. The shape of the initial "almond" of QCD matter produced in a heavy ion collision is taken by using the phenomenologically successful Kharzeev-Levin-Nardi (KLN) model [28] based on parton saturation and k_T factorization. Au-Au collisions have been simulated, with realistic Woods-Saxon nuclear densities. The axial chemical potentials at the initial time are set to zero.

We then solve the CMW equation numerically and find that it generates the separation of chiral charge, as shown in Fig. 1—the quark-gluon plasma acquires a "chiral dipole moment".

We evaluate the electric charge distribution by superimposing the waves of different flavors weighted by their charges,

$$j^0_e = \sum_f q_f (j^{0,f}_L + j^{0,f}_R). \tag{7}$$

The resulting distribution is shown in Fig. 2; for clarity, we have subtracted the charge density distribution without the CMW. As argued above qualitatively, the quark-gluon plasma indeed acquires an electric quadrupole moment. The poles of the produced fireball (pointing outside of the reaction plane) acquire additional positive electric charge, and the "equator" acquires additional negative charge. It is very important to note that this pattern of charge separation does not depend on the orientation of the magnetic field. This means that the effect should survive even after the event averaging.

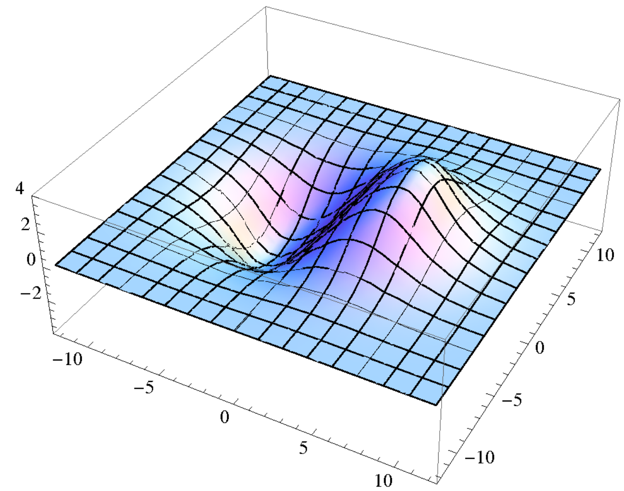

FIG. 1 (color online). Chiral charge density in the plane transverse to the beam axis. Magnetic field strength $eB = m^2_\pi$, lifetime of magnetic field $\tau = 10$ fm, temperature $T = 165$ MeV, impact parameter $b = 3$ fm.

From the electric quadrupole moment to charge-dependent elliptic flow.—The expansion of the quark-gluon plasma produced in heavy ion collisions is characterized by a strong collective flow driven by the gradients of pressure that transforms the spatial anisotropy of produced matter into the momentum anisotropy of the produced hadrons. Since the fireball of quark-gluon plasma produced in an off-central heavy ion collision has an elliptical almondlike shape, the gradients of pressure make it expand predominantly along the minor axis, i.e., in the reaction plane—this is the "elliptic flow" (for a review, see [29]). As a result, the electric quadrupole deformation of the plasma described above will increase the elliptic flow of

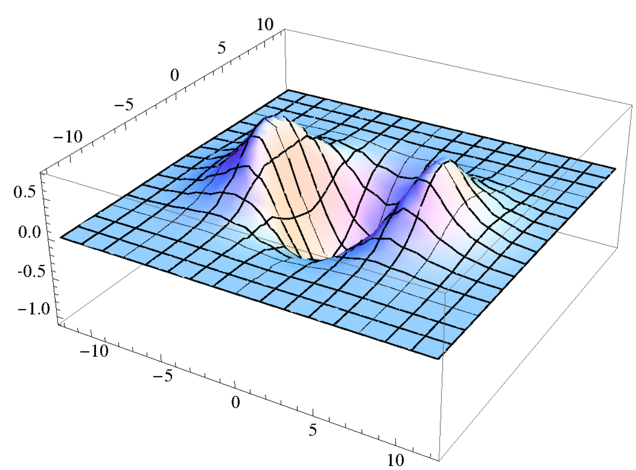

FIG. 2 (color online). Electric charge density in the transverse plane (background subtracted, see text). Same parameters as in Fig. 1.

negative hadrons, and decrease the elliptic flow of positive hadrons, leading to $v_2^+ < v_2^-$ as demonstrated in Fig. 3. However, the large differences in the absorption cross sections of antiprotons and protons, and of negative and positive kaons in hadronic matter at finite baryon density, are likely to mask or reverse this difference in the hadron resonance "afterburner" phase of a heavy ion collision. On the other hand, the smaller difference in the absorption cross sections of negative and positive pions potentially may make it possible to detect the electric quadrupole moment of the plasma through the difference of elliptic flows of pions, $v_2(\pi^+) < v_2(\pi^-)$.

The effect can be estimated by noting that a strong radial flow aligns the momenta of the emitted hadrons along the direction of the radial flow (see Fig. 3). The asymmetry of the electric charge distribution in the expanding plasma is then translated into the asymmetry in the azimuthal distribution of the positive and negative hadrons:

$$N_+(\phi) - N_-(\phi) \propto \int j_e^0(R, \phi) R dR. \tag{8}$$

This asymmetry has a 0th Fourier harmonic (monopole) originating from a nonzero net charge density:

$$\bar{\rho}_e = \int R dR d\phi \, j^0_{e,B=0}(R, \phi). \tag{9}$$

In addition there is a 2nd harmonic (quadrupole) of the form $2q_e \cos(2\phi)$ due to the CMW contribution:

$$q_e = \int R dR d\phi \cos(2\phi)[j_e^0(R, \phi) - j^0_{e,B=0}(R, \phi)]. \tag{10}$$

The ratio of the two $r \equiv \frac{2q_e}{\bar{\rho}_e}$ can be used to parametrize the asymmetry in the azimuthal distributions of positive and negative hadrons:

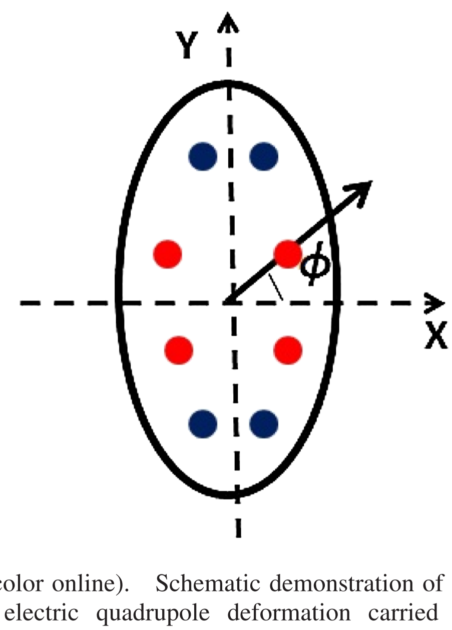

FIG. 3 (color online). Schematic demonstration of the CMW-induced electric quadrupole deformation carried by strong radial flow.

$$N_+(\phi) - N_-(\phi) = (\bar{N}_+ - \bar{N}_-)[1 - r\cos(2\phi)], \tag{11}$$

where $\bar{N}_\pm$ are the multiplicities of positive and negative hadrons. Therefore the hadron azimuthal distributions including the "usual" elliptic flow are

$$\begin{aligned} \frac{dN_\pm}{d\phi} &= N_\pm[1 + 2v_2\cos(2\phi)] \\ &\approx \bar{N}_\pm[1 + 2v_2\cos(2\phi) \mp A_\pm r\cos(2\phi)]. \end{aligned} \tag{12}$$

In the second line we assume that both v_2 and the charge asymmetry $A_\pm \equiv (\bar{N}_+ - \bar{N}_-)/(\bar{N}_+ + \bar{N}_-)$ are small.

The elliptic flow therefore becomes charge dependent:

$$v_2^\pm = v_2 \mp \frac{rA_\pm}{2}. \tag{13}$$

The magnitude of the effect: Numerical simulation.—As described above, we have computed the evolution of the right and left chiral components of the u and d quarks according to Eq. (3) (at zero rapidity) in a static plasma. For simplicity, we assume the temperature to be uniform within the almond. At the boundary of the plasma, the chiral symmetry is broken and therefore we set $v_\chi = 0$. In the transverse (with respect to the magnetic field) direction, we assume a diffusion with a diffusion constant D_T estimated [25] as $D_T = (2\pi T)^{-1}$ within the Sakai-Sugimoto model. The difference in the elliptic flows of positive and negative pions is given, within our approximation, by Eq. (13). In Fig. 4 we present the ratio $r = 2q_e/\bar{\rho}_e$ as a function of impact parameter b at different times. In this computation we took the impact parameter dependence of the magnetic field from [3], with the maximal value $eB|_{\max} = m_\pi^2$. To convert this ratio into the difference of the elliptic flows of positive and negative pions according to Eq. (13), we also have to estimate the electric charge asymmetry $A_\pm$ in the quark-gluon plasma that varies between 0 and 1. We do this using the baryon chemical potential and temperature at freeze-out extracted [30] from the data and evaluating the yields of baryons and charged mesons; for the energy of $\sqrt{s} = 11$ GeV we

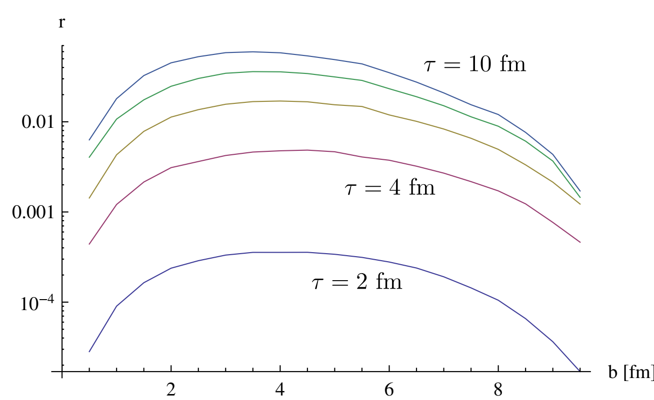

FIG. 4 (color online). The normalized electric quadrupole moment r, $eB|_{\max} = m_\pi^2$, $T = 165$ MeV.

estimate $A_\pm \simeq 0.3$. Note that at finite baryon density the asymmetry in the plasma and at freeze-out may differ. The lifetime of a magnetic field in the plasma is still uncertain: the initial pulse of a magnetic field rapidly falls off with time [3,31], but the induction in an electrically conducting quark-gluon plasma was estimated to drastically extend the lifetime of the magnetic field according to the Lenz's rule, perhaps making it last for the entire lifetime of the plasma [32]. On the other hand, it takes approximately $\tau \simeq 4$ fm to build a substantial quadrupole deformation (see Fig. 4). Choosing for the sake of estimate $\tau = 8$ fm and the corresponding $r \simeq 0.04$ for midcentral collisions in the plot of Fig. 4 and using $A_\pm \simeq 0.3$, we estimate the difference of π^- and π^+ elliptic flows in Au-Au collisions at $\sqrt{s} = 11$ GeV as

$$\Delta v_2^{\mathrm{CMW}} \equiv v_2(\pi^-) - v_2(\pi^+) \simeq rA_\pm \simeq 0.01. \tag{14}$$

The pion elliptic flow at $\sqrt{s} = 8.7$ GeV in midcentral Pb-Pb collisions is about $v_2 \simeq 0.03$ [33]. Therefore, the difference between the elliptic flows of positive and negative pions may be as big as $\sim 30\%$; this would clearly make it observable.

Uncertainties and outlook.—The main source of uncertainty in our computation is the lifetime of the magnetic field that will need to be evaluated numerically within relativistic magnetohydrodynamics. Our treatment of expansion and evolution of the plasma has been quite crude, and will also need to be refined by a magnetohydrodynamics computation. Possible backgrounds to the effect considered here have to be carefully analyzed, and include the Coulomb interaction of produced pions and the difference in absorption of negative and positive pions in dense resonance gas. Since the CMW can propagate only in the chirally symmetric phase, the electric quadrupole deformation of the plasma can provide a signature of chiral symmetry restoration in heavy ion collisions.

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