Photon-Photon Gates in Bose-Einstein Condensates

Arnaud Rispe, Bing He, and Christoph Simon
Institute for Quantum Information Science and Department of Physics and Astronomy, University of Calgary, Calgary T2N 1N4, Alberta, Canada
(Received 30 September 2010; published 18 July 2011)

It has recently been shown that light can be stored in Bose-Einstein condensates for over a second. Here we propose a method for realizing a controlled phase gate between two stored photons. The photons are both stored in the ground state of the effective trapping potential inside the condensate. The collision-induced interaction is enhanced by adiabatically increasing the trapping frequency and by using a Feshbach resonance. A controlled phase shift of π can be achieved in 1 s or less.

DOI: 10.1103/PhysRevLett.107.043601 PACS numbers: 42.50.Ex, 03.67.Lx, 03.75.Gg, 42.65.Hw

Photons are ideal carriers of quantum information over long distances. It is interesting to explore their potential for the implementation of quantum information processing as well. This is particularly relevant for quantum repeaters [1–3], which would allow one to distribute quantum states over distances that are inaccessible by direct transmission. Quantum repeaters require both the capability to store photons for relatively long times and to perform efficient quantum gates between them [3]. Potential architectures where storage and quantum gates can be achieved in the same system are particularly attractive. Recently it was shown that light can be stored for over a second in a Bose-Einstein condensate (BEC) [4], making condensates a very interesting candidate system for the implementation of quantum memories. Quantum repeaters can tolerate long gate times in the subsecond range, since repetition rates are in any case limited by other factors such as communication times and transmission probabilities. It is therefore of great interest to explore the potential for photon-photon gates in BECs, where interactions between stored excitations are weak, but nonzero. This also leads one to consider fascinating experiments at the interface of quantum optics and cold-atom physics [5].

The simplest controlled phase gate between two light modes corresponds to the operation $|0\rangle_1|0\rangle_2 \rightarrow |0\rangle_1|0\rangle_2$, $|0\rangle_1|1\rangle_2 \rightarrow |0\rangle_1|1\rangle_2$, $|1\rangle_1|0\rangle_2 \rightarrow |1\rangle_1|0\rangle_2$, $|1\rangle_1|1\rangle_2 \rightarrow e^{i\phi}|1\rangle_1|1\rangle_2$, where the states $|0\rangle$ and $|1\rangle$ correspond to zero- and one-photon states of the two modes (labeled 1 and 2) [6]. The controlled phase ϕ only occurs for the state where there is a photon in each mode. For quantum information processing a controlled phase $\phi = \pi$ is desirable. Here we will show that this can be achieved by having the two modes interact in a BEC. Since the phase is due to a photon-photon interaction, no phase is accumulated if no photons or only one photon is present. In the following we therefore focus on the case where there is one photon in each mode.

Our theoretical treatment is inspired by Ref. [7], which discussed processing optical information in BECs, and by Ref. [8], which studied two-component interactions in BECs. The present proposal led us to extend these theoretical approaches into the quantum (i.e., few-excitation) regime. Based on this theory, we show that phase shifts of π due to the interaction between two stored photons can be achieved on subsecond time scales, by combining a Feshbach enhancement of the relevant scattering length and an adiabatic compression of the trap after the photons have been stored. The fidelity of photon-photon gates can be affected by unwanted multimode effects (see, e.g., Ref. [9]). In the present proposal these effects are greatly suppressed by the fact that the interaction is much weaker than the confinement, ensuring high-fidelity operations.

Let us assume that the two photons have orthogonal polarization. Their propagation inside the BEC can be controlled by two independent control beams, leading to storage in two different atomic levels 1 and 2, where the BEC was prepared in level 0 (see Fig. 1). Slow and stopped light in BECs has been thoroughly investigated [4,7,10–12]. Because of the linearity of the equations of motion, the physics of storage and retrieval is the same at the single-photon level as for weak classical probe pulses [13,14]. Inside the medium and in the presence of the appropriate control beam, the photon is converted into a slowly moving polariton, which can be stopped by adiabatically switching off the control beam, thus converting the photon into a stored atomic spin wave. Running the process in reverse allows the reconversion of spin waves into photons. Here we focus on the interaction between the

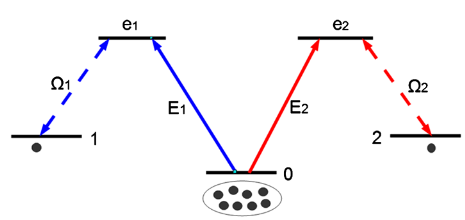

FIG. 1 (color online). Level scheme for photon-photon gate. The BEC is prepared in level 0. The single photons in modes E_1 and E_2 can be independently stored as delocalized excitations in levels 1 and 2, using the control beams Ω_1 and Ω_2.

two spin waves, once the control beams have been turned off. Because of the weakness of the collision-induced interactions the time scale for the storage and retrieval processes is much shorter than the time scale on which significant interaction occurs in the photon-photon regime.

The dynamics of the atomic spin waves is governed by the collisional interactions between atoms in combination with the external trapping potential. Spin waves in levels 1 and 2 experience an effective trapping potential and effective collisional interactions that depend on the differences between the atomic scattering lengths in the various atomic levels [8]. These differences, which are usually small, can be enhanced by Feshbach resonances [15–17]. We consider a situation where both spin waves experience the same effective trapping potential, and where they are both in its ground state. The latter condition can be achieved by carefully matching the pulse duration and width of the incoming photons and the intensity of the control beams (which determines the propagation speed and thus the longitudinal extent of the polaritons inside the condensate during the storage process) to the parameters of the effective trapping potential. We focus on the regime where the stored spin waves are localized well inside the condensate (cf. Fig. 2).

The interaction strength, and thus the accumulated controlled phase shift for a given time, then strongly depends not only on the scattering lengths, but also on the size of the

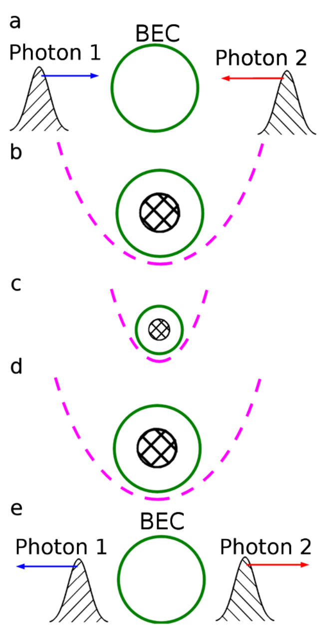

FIG. 2 (color online). Principle of photon-photon gate. (a) Photons 1 and 2 are independently absorbed by the BEC. (b) Both are converted into atomic excitations that are in the ground state of the effective trapping potential (see text). (c) The collision-induced interaction between the atomic spin waves is enhanced by adiabatically increasing the trapping potential, thus reducing the size of the ground state wave functions (and of the BEC). (d) The trapping potential is adiabatically brought back to its original value. (e) The photons can be read out independently.

ground state wave packets. During storage and retrieval, this size has to be significantly larger than a wavelength, due to focusing restrictions for the transverse dimensions and in order to justify the slowly varying envelope description which underlies the polariton picture for the longitudinal dimension. However, in between storage and retrieval it is possible to adiabatically increase the trapping potential, thus reducing the size of the ground state wave packets while keeping the spin waves in the ground state (see Fig. 2). This enhances the interaction strength, making controlled phase shifts of π achievable on 1 s time scales. Note that the basic ingredients of the present proposal are similar to those of single-atom quantum gates schemes based on cold collisions such as Refs. [18,19].

We now describe our proposal in more quantitative terms. Our treatment of the spin waves inside the BEC (once the control beams have been turned off) is based on Refs. [8,20]. The Gross-Pitaevskii equation for the macroscopic wave function ψ_0 of the condensate is

$$i\hbar\frac{\partial\psi_0}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + U_{00}|\psi_0|^2 + U_{01}|\psi_1|^2 + U_{02}|\psi_2|^2\right)\psi_0, \quad (1)$$

where m is the atomic mass, V is the trapping potential, U_{00}, U_{01}, U_{02} are the collisional interaction potentials, which are related to the corresponding scattering lengths a_{00}, a_{01}, a_{02} by $U_{0j} = \frac{4\pi\hbar^2 a_{0j}}{m}$, and ψ_1, ψ_2 are the macroscopic wave functions for levels 1 and 2. We will make the transition to a single-quantum description for the latter in a moment.

For a sufficiently large condensate, and keeping in mind that the perturbation due to the spin waves in levels 1 and 2 is extremely weak in our case, the solution for ψ_0 will be essentially stationary, and the stationary equation for a chemical potential μ can be solved in the Thomas-Fermi approximation (i.e., neglecting the kinetic term) [20], giving

$$|\psi_0|^2 = \frac{1}{U_{00}}(\mu - V - U_{01}|\psi_1|^2 + U_{02}|\psi_2|^2), \quad (2)$$

where μ is the chemical potential. This solution of Eq. (2) can now be inserted into the Gross-Pitaevskii equations for ψ_1 and ψ_2. Corrections to the Thomas-Fermi approximation mainly affect the boundary layer of the condensate [21]. We therefore expect the present treatment to be correct under the above-mentioned condition that the spin waves in levels 1 and 2 are localized well inside the BEC.

In order to describe the few-excitation regime, we replace the macroscopic wave functions ψ_1, ψ_2 by quantum field operators $\hat{\psi}_1$, $\hat{\psi}_2$ satisfying commutation relations $[\hat{\psi}_i(\mathbf{x}), \hat{\psi}_j^\dagger(\mathbf{x}')] = \delta_{ij}\delta^{(3)}(\mathbf{x}-\mathbf{x}')$, in analogy to the transition from classical to quantum nonlinear optics [22]. They fulfill the equations (neglecting a constant energy shift that depends on μ)

$$i\hbar\frac{\partial\hat{\psi}_1}{\partial t}=\Big[-\frac{\hbar^2}{2m}\nabla^2+\tilde{V}_1(\mathbf{x})+\tilde{U}_{11}\hat{\psi}_1^\dagger\hat{\psi}_1+\tilde{U}_{12}\hat{\psi}_2^\dagger\hat{\psi}_2\Big]\hat{\psi}_1,$$
$$i\hbar\frac{\partial\hat{\psi}_2}{\partial t}=\Big[-\frac{\hbar^2}{2m}\nabla^2+\tilde{V}_2(\mathbf{x})+\tilde{U}_{22}\hat{\psi}_2^\dagger\hat{\psi}_2+\tilde{U}_{12}\hat{\psi}_1^\dagger\hat{\psi}_1\Big]\hat{\psi}_2, \quad (3)$$

where $\tilde{V}_i=(1-\frac{a_{0i}}{a_{00}})V$ are the effective trapping potentials and $\tilde{U}_{ij}=\frac{4\pi\hbar^2}{m}(a_{ij}-\frac{a_{0i}a_{0j}}{a_{00}})$ are the effective interaction potentials, which are all modified due to the interaction with the background condensate in level 0. These equations are analogous to those obtained in Ref. [8] for the two-level case. Here we have assumed that the bare trapping potential V is the same for all atomic levels. Moreover for simplicity we will assume that $a_{01}=a_{02}$ implying $\tilde{V}_1=\tilde{V}_2=\tilde{V}$. We require $a_{01}<a_{00}$ in order for $\tilde{V}$ to be attractive, provided that V is attractive [23].

The quantum fields $\hat{\psi}_1$ and $\hat{\psi}_2$ in Eq. (3) describe quantum-level excitations in levels 1 and 2. Equation (3) can be used to describe the dynamics of any number of excitations. However, we are interested in the case where there is exactly one excitation in each level. It is then convenient to introduce the two-particle wave function $\psi_{12}(\mathbf{x}_1,\mathbf{x}_2)=\langle 0|\hat{\psi}_1(\mathbf{x}_1,t)\hat{\psi}_2(\mathbf{x}_2,t)|\Phi\rangle$, where $|0\rangle$ is the state without any excitations (i.e., the state where there is just the condensate in level 0), and

$$|\Phi\rangle=\int d^3x_1d^3x_2\phi_0(\mathbf{x}_1)\phi_0(\mathbf{x}_2)\hat{\psi}_1^\dagger(\mathbf{x}_1)\hat{\psi}_2^\dagger(\mathbf{x}_2)|0\rangle \quad (4)$$

is the initial state (after storage), which consists of one atomic excitation in each level (1 and 2), both of which are in the ground state ϕ_0 of the effective trapping potential $\tilde{V}$. In the Heisenberg picture for the quantum field theory the state remains constant, but the field operators evolve according to Eq. (3). As a consequence, the two-particle wave function ψ_{12} defined above evolves according to

$$i\hbar\frac{\partial}{\partial t}\psi_{12}(\mathbf{x}_1,\mathbf{x}_2,t)=\Big[-\frac{\hbar^2}{2m}(\nabla_1^2+\nabla_2^2)+\tilde{V}(\mathbf{x}_1)+\tilde{V}(\mathbf{x}_2)+\tilde{U}_{12}\delta^{(3)}(\mathbf{x}_1-\mathbf{x}_2)\Big]\psi_{12}(\mathbf{x}_1,\mathbf{x}_2,t), \quad (5)$$

with the initial condition $\psi_{12}(\mathbf{x}_1,\mathbf{x}_2,0)=\phi_0(\mathbf{x}_1)\phi_0(\mathbf{x}_2)$. We assume a spherically symmetric harmonic potential $\tilde{V}(\mathbf{x})=\frac{1}{2}m\tilde{\omega}^2\mathbf{x}^2$, implying $\phi_0(\mathbf{x})=(\frac{m\tilde{\omega}}{\pi\hbar})^{3/2}e^{-(m\tilde{\omega}\mathbf{x}^2)/2\hbar}$.

It is convenient to transform to center-of-mass and relative coordinates defined by $\mathbf{X}=\frac{\mathbf{x}_1+\mathbf{x}_2}{\sqrt{2}}$ and $\mathbf{r}=\frac{\mathbf{x}_1-\mathbf{x}_2}{\sqrt{2}}$. In these coordinates the wave function is separable at all times, $\psi_{12}(\mathbf{X},\mathbf{r},t)=e^{-i(\tilde{\omega}/2)t}\phi_0(\mathbf{X})\psi(\mathbf{r},t)$. The center-of-mass wave function exactly remains in the ground state of $\tilde{V}$. The relative coordinate wave function $\psi(\mathbf{r},t)$ fulfills the equation

$$i\frac{\partial}{\partial t}\psi(\mathbf{r},t)=\Big[-\frac{\hbar^2}{2m}\nabla^2+\tilde{V}(\mathbf{r})+\bar{U}_{12}\delta^{(3)}(\mathbf{r})\Big]\psi(\mathbf{r},t), \quad (6)$$

where $\bar{U}_{12}=\tilde{U}_{12}2^{-(3/2)}$. The interaction between the two spin wave excitations inside the BEC is thus reduced to a fairly simple problem in one-particle quantum mechanics.

In practice the interaction energy associated with the $\bar{U}_{12}$ term is 2 to 3 orders of magnitude smaller than the harmonic oscillator energy scale $\hbar\tilde{\omega}$. As a consequence, the use of perturbation theory is well justified. Because of the large separation between the two energy scales, $\psi(\mathbf{r},t)$ remains essentially proportional to the ground state (see below). However, the interaction term in Eq. (6) causes a shift of the ground state energy, which is given by

$$\Delta E=\langle\phi_0|\bar{U}_{12}\delta^{(3)}(\mathbf{r})|\phi_0\rangle=\bar{U}_{12}|\phi_0(\mathbf{0})|^2=\bar{U}_{12}s^{-3}, \quad (7)$$

where $s=\sqrt{\frac{\pi\hbar}{m\tilde{\omega}}}$ is the characteristic length scale of the ground state wave function, which is related to its full width at half maximum l by $s=\sqrt{\frac{\pi}{8\ln 2}}l$. This energy shift is the basis of our quantum gate proposal. Since it is due to the interaction, it only occurs if there are two excitations in the condensate, allowing one to realize a controlled phase gate as described in the introduction. The gate naturally has high fidelity [9] because the correction terms to the ground state wave function have amplitudes of order $\frac{\Delta E}{\hbar\tilde{\omega}}\sim\frac{a_{12}}{s}$, which is smaller than 10^{-2} even for the largest scattering length and smallest ground state size that we will consider. This means that, apart from the phase, the overlap with the initial state remains extremely high, which is exactly what is required for high-fidelity operation [9].

The remaining challenge is therefore to achieve a controlled phase shift of π. Let us begin by choosing parameter values that should be straightforwardly achievable. For example, one can choose level 0 in the $F=1$ submanifold of ^{87}Rb, and levels 1 and 2 in the $F=2$ submanifold, giving $a_{00}=5.39$ nm, $a_{01}=a_{02}=5.24$ nm, and $a_{12}=5.58$ nm [20], and a full width at half maximum for the ground state wave packet $l=8$ μm (corresponding to about ten wavelengths). With these values one finds that the time required for a phase of π is 6 min, which at first sight may seem rather discouraging. We now discuss how to overcome this difficulty by acting both on the $\bar{U}_{12}$ factor and the s^{-3} factor in Eq. (7).

The factor $\bar{U}_{12}=\frac{\sqrt{2}\pi\hbar^2}{m}(a_{12}-\frac{a_{01}a_{02}}{a_{00}})$ is very small for the values given above because there is a quasicancellation between the two terms in parentheses because all the scattering lengths are so similar. A moderate increase in a_{12}, which can be achieved using a Feshbach resonance [15–17], can therefore lead to a very large increase of $\bar{U}_{12}$. For example, increasing a_{12} by a factor of $F=3$, which was already demonstrated in Ref. [16] for ^{87}Rb, increases $\bar{U}_{12}$ by a factor of 24.

A comparable gain can be achieved by acting on the second factor in Eq. (7), i.e., on the size of the wave function, or equivalently on the trapping frequency. We

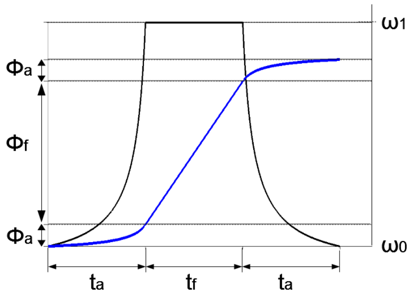

FIG. 3 (color online). Qualitative time dependence of the trap frequency ω and of the acquired phase ϕ. The total time is $2t_a + t_f$, where t_a is the time required for adiabatically changing the trap frequency from ω_0 to ω_1 or vice versa, and t_f is the time during which the trap frequency is held fixed at the high value ω_1. The corresponding phases are ϕ_a and ϕ_f, giving a total phase $2\phi_a + \phi_f$, where $\phi_f \gg \phi_a$ in the discussed regime.

already mentioned in the introduction that l (and thus s) cannot be too small during the storage process, because focusing becomes too difficult and the slowly varying envelope approximation breaks down. However, the trapping frequency can be increased once the photons have been stored (see Fig. 2) with the caveat that this increase has to be done adiabatically so that the spin waves remain in the ground state of the effective trapping potential. The mentioned $l = 8\ \mu$m corresponds to an effective frequency $\tilde{\omega} = 2\pi$ 10 Hz, which corresponds to a real trap frequency $\omega = 2\pi$ 50 Hz. This gives a condensate size of 17 μm for $N = 10^5$ atoms in the Thomas-Fermi approximation [24]. The effective frequency can be increased to $\tilde{\omega} = 2\pi$ 80 Hz in $t_a = 0.14$ seconds while exciting the system out of the ground state with a probability that is smaller than 0.002. At this frequency the ground state size l is 2.9 μm and the size of the condensate is 7.4 μm. For a Feshbach factor $F = 3$ a phase of order π can then be achieved with $t_f = 0.73$ seconds. Taking into account that one has to decrease the frequency before readout, the total gate time $2t_a + t_f$ is 1.01 seconds for this example. Note that there is also a small contribution to the total phase from the adiabatic compression and expansion periods (see Fig. 3). The peak density of the condensate in its compressed state is $6 \times 10^{14}/\mathrm{cm}^3$ in this case, which is compatible with typical three-body loss rates [25]. Shorter gate times could be achieved for smaller initial ground state sizes, higher compressed densities, or larger Feshbach enhancement factors. For example, with an initial effective frequency of $\tilde{\omega} = 2\pi 25$ Hz, which corresponds to $l = 5\ \mu$m, an effective frequency after compression $\tilde{\omega} = 2\pi 125$ Hz, and $F = 10$, one can achieve $2t_a = 0.10$ s (with a ground state excitation probability of 1.3×10^{-4}) and $t_f = 0.09$ s, giving a total gate time of 0.19 s. These values correspond to a final maximum density of $10^{15}/\ \mathrm{cm}^3$.

We have shown that a controlled phase of π between individual photons is achievable on the 1 s time scale under realistic conditions. We hope that our proposal will stimulate experimental work in this direction.

We thank D. Feder, A. I. Lvovsky, A. MacRae, and A. Sørensen for helpful comments. This work was supported by AI-TF and an NSERC Discovery Grant.

[1] H.-J. Briegel, W. Dür, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **81**, 5932 (1998).
[2] L.-M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature (London) **414**, 413 (2001).
[3] N. Sangouard, C. Simon, H. de Riedmatten, and N. Gisin, Rev. Mod. Phys. **83**, 33 (2011).
[4] R. Zhang, S. R. Garner, and L. V. Hau, Phys. Rev. Lett. **103**, 233602 (2009).
[5] A. V. Gorshkov *et al.*, Phys. Rev. Lett. **105**, 060502 (2010).
[6] It is straightforward to apply our proposed gate to dual-rail photonic qubits (I. L. Chuang and Y. Yamamoto, Phys. Rev. Lett. **76**, 4281 (1996)), which makes it easier to deal with photon loss.
[7] Z. Dutton and L. Vestergaard Hau, Phys. Rev. A **70**, 053831 (2004).
[8] Z. Dutton and C. W. Clark, Phys. Rev. A **71**, 063618 (2005).
[9] B. He, A. MacRae, Y. Han, A. I. Lvovsky, and C. Simon, Phys. Rev. A **83**, 022312 (2011).
[10] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature (London) **397**, 594 (1999).
[11] C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature (London) **409**, 490 (2001).
[12] N. S. Ginsberg, S. R. Garner, and L. V. Hau, Nature (London) **445**, 623 (2007).
[13] A. V. Gorshkov, A. André, M. Fleischhauer, A. S. Sørensen, and M. D. Lukin, Phys. Rev. Lett. **98**, 123601 (2007).
[14] K. Hammerer, A. S. Sørensen, and E. S. Polzik, Rev. Mod. Phys. **82**, 1041 (2010).
[15] S. Inouye, M. R. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper-Kurn, and W. Ketterle, Nature (London) **392**, 151 (1998).
[16] T. Volz, S. Dürr, S. Ernst, A. Marte, and G. Rempe, Phys. Rev. A **68**, 010702(R) (2003).
[17] A. M. Kaufman *et al.*, Phys. Rev. A **80**, 050701(R) (2009).
[18] D. Jaksch, H.-J. Briegel, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **82**, 1975 (1999).
[19] T. Calarco, E. A. Hinds, D. Jaksch, J. Schmiedmayer, J. I. Cirac, and P. Zoller, Phys. Rev. A **61**, 022304 (2000).
[20] Z. Dutton, Ph.D. thesis, Harvard University, 2002.
[21] A. L. Fetter and D. L. Feder, Phys. Rev. A **58**, 3185 (1998); F. Dalfovo, L. Pitaevskii, and S. Stringari, Phys. Rev. A **54**, 4213 (1996).
[22] M. Hillery, Acta Phys. Slovaca **59**, 1 (2009).
[23] These conditions are compatible with the phase-separation regime used in Ref. [4], provided that $a_{11}, a_{22} < \frac{a_{01}^2}{a_{00}}$.
[24] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999).
[25] D. M. Stamper-Kurn, M. R. Andrews, A. P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, and W. Ketterle, Phys. Rev. Lett. **80**, 2027 (1998).