Confirmation of the Copernican Principle at Gpc Radial Scale and above from the Kinetic Sunyaev-Zel'dovich Effect Power Spectrum

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(Received 19 September 2010; revised manuscript received 14 April 2011; published 21 July 2011)

The Copernican principle, a cornerstone of modern cosmology, remains largely unproven at the Gpc radial scale and above. Here will show that violations of this type will inevitably cause a first order anisotropic kinetic Sunyaev-Zel'dovich effect. If large scale radial inhomogeneities have an amplitude large enough to explain the "dark energy" phenomena, the induced kinetic Sunyaev-Zel'dovich power spectrum will be much larger than the Atacama Cosmology Telescope and/or South Pole Telescope upper limit. This single test confirms the Copernican principle and rules out the adiabatic void model as a viable alternative to dark energy.

DOI: 10.1103/PhysRevLett.107.041301 PACS numbers: 98.80.Es, 95.36.+x, 98.70.Vc

Introduction.—The Copernican principle has been a fundamental tenet of modern science since the sixteenth century and is also a cornerstone of modern cosmology. It states that we should not live in a special region of the Universe. Cosmic microwave background (CMB) observations verify the statistical homogeneity of the last scattering surface [1]. Galaxy surveys verify the radial homogeneity up to the Gpc scale [2]. However, radial homogeneity at larger scales remains unproven.

Testing the Copernican principle is of crucial importance for fundamental cosmology. If the Copernican principle is violated such that we live in or near the center of a large ($\sim$ Gpc) void as described by a Lemaître-Tolman-Bondi (LTB) space-time [3] in which the matter distribution is spherically symmetric, the apparent cosmic acceleration [4,5] can be explained without a cosmological constant, dark energy, or modifications of general relativity [6]. We will restrict our Letter to this type of violation of the Copernican principle. Various tests of the Copernican principle have been proposed and a large class of void models has been ruled out (e.g., [7–9]). Here we propose a powerful single test which confirms the Copernican principle at Gpc radial scale.

The KSZ test.—A generic consequence of violating the Copernican principle is that some regions will expand faster or slower than others and as photons transit between these regions there will be a relative motion between the average matter frame and the CMB. When relative motions between free electrons and photons exist, the inverse Compton scattering will induce a shift of the brightness temperature of CMB photons via the kinetic Sunyaev-Zel'dovich (KSZ) effect [10]. This temperature shift will be anisotropic on our sky, tracing the anisotropy of the projected free electron surface density. This test of the Copernican principle has been applied to cluster KSZ observations [7,9], where the electron surface density is high. However, this effect applies to all free electrons which exist in great abundance everywhere in the Universe up to the reionization epoch at redshift $z \gtrsim 6$ (and comoving distance $\gtrsim 6h^{-1}$ Gpc), whereas clusters are rare above $z \sim 1$. So one can expect a more sensitive test from blank field CMB anisotropy power spectrum measurements than from cluster measurements as has been demonstrated for the "dark flow" [11] induced small scale KSZ effect [12].

Free electrons have local motion $\vec{v}_L$ with respect to the average matter frame and the subscript "L" refers to "local." It vanishes when averaging over a sufficiently large scale. However, when the Copernican principle is violated at a large scale, electrons will have relative motion $\vec{v}_H$ between the average matter frame and the CMB, which does not vanish even when averaging over the Hubble scale. Correspondingly the induced KSZ temperature fluctuation [10,12] has two contributions,

$$\Delta T(\hat{n}) = \Delta T_L(\hat{n}) + \Delta T_H(\hat{n}). \tag{1}$$

The first term on the right-hand side is the conventional KSZ effect,

$$\Delta T_L(\hat{n}) = T_{\rm CMB} \int [1 + \delta_e(\hat{n}, z)] \frac{\vec{v}_L(\hat{n}, z) \cdot \hat{n}}{c} d\tau_e. \tag{2}$$

Here, $\hat{n}$ is the radial direction on the sky. τ_e is the mean Thomson optical depth to the corresponding redshift and δ_e is the fractional fluctuation in the free electron number density. The last term in Eq. (1) is new and does not vanish in a non-Copernican universe,

$$\Delta T_H(\hat{n}) = T_{\rm CMB} \int [1 + \delta_e(\hat{n}, z)] \frac{\vec{v}_H(\hat{n}, z) \cdot \hat{n}}{c} d\tau_e$$
$$= 9.1\ \mu{\rm K}\left[\int \frac{\vec{v}_H \cdot \hat{n}}{10^4\,{\rm km/s}} \frac{\delta_e(\hat{n}, z)}{0.1} \frac{d\tau_e}{0.001}\right]. \tag{3}$$

The last expression neglects the $\int \vec{v}_H \cdot \hat{n} d\tau_e$ term, which has no direction dependence in LTB models in which we live at the center, and is therefore not observable. $\vec{v}_H$ varies slowly along radial direction and does not suffer the cancellation of $\vec{v}_L$ in the conventional KSZ effect [13,14]. The small scale anisotropy power spectrum will be quadratic in the amplitude of δ_e (which does fluctuate about zero) so we can say that $\Delta T_H/T$ is first order in the density fluctuations. Throughout this Letter, unless otherwise specified, we will focus on this *linear* KSZ effect. We restrict ourselves to adiabatic voids in which the initial matter, radiation, and baryon densities track each other. This is what one would expect if baryogenesis and dark matter decoupling occurs after the process which generates the void inhomogeneity. We also restrict ourselves to voids outside of which both matter and radiation are homogeneous. Adding additional inhomogeneities will generically lead to larger values of v_H.

To explain the dimming of type Ia supernovae (SNe Ia) and hence the apparent cosmic acceleration without dark energy and modifications of general relativity, we shall live in an underdense region (void) of size $\gtrsim 1h^{-1}$ Gpc, with a typical outward velocity $v_H \gtrsim 10^4$ km/s (e.g., [9]). Given the baryon density $\Omega_b h^2 = 0.02 \pm 0.002$ from the big bang nucleosynthesis [15], $\tau_e > 10^{-3}$. Scaling the observed weak lensing rms convergence $\kappa \sim 10^{-2}$ at $\sim 7'$ [16], the rms fluctuation in δ_e projected over the Gpc length is $\gtrsim 0.1$ at such scale. Hence such a void generates a KSZ power spectrum $\Delta T_H^2 \gtrsim 80\ \mu\mathrm{k}^2$ at multipole $\ell = 3000$. This is in conflict with recent KSZ observations. The South Pole Telescope (SPT) collaboration [17] found $\Delta T^2 < 6.5\ \mu\mathrm{K}^2$ (95% upper limit) and the Atacama Cosmology Telescope (ACT) collaboration [18] found $\Delta T^2 < 8\ \mu\mathrm{K}^2$. This simple order of magnitude estimation demonstrates the potential discriminating power of the KSZ power spectrum measurement. It suggests that a wide range of void models capable of replacing dark energy are ruled out. This also demonstrates how purely empirical measurements of CMB anisotropies and the large scale structure (e.g., weak lensing) can in principle be combined to limit non-Copernican models without any assumptions of how the inhomogeneities vary with distance.

We perform quantitative calculation for a popular void model, namely, the Hubble bubble model ([8] and references therein). In this model, we live at the center of a Hubble bubble of constant matter density $\Omega_0 < 1$ embedded in a flat Einstein—de Sitter universe ($\Omega_m = 1$). The void extends to redshift $z_{\rm edge}$, surrounded by a compensating shell ($z_{\rm edge} < z < z_{\rm out}$) and then the flat Einstein–-de Sitter universe ($z > z_{\rm out}$). The KSZ effect in this universe has two components: (1) the linear KSZ arising from the large angular scale anisotropies generated by matter (a) inside the void, (b) in the compensating shell, and (c) outside the void and (2) the conventional KSZ effect quadratic in density fluctuation [14] and the KSZ effect from patchy reionization [19]. The contributions of each of these to the anisotropy power spectrum are uncorrelated. Hence the ACT and/or SPT measurements put an upper limit on the total. The latter contributes $\sim 3.5\ \mu\mathrm{K}^2$ [20], so what is left for the first component is $\lesssim 3\ \mu\mathrm{K}^2$. However, we will test the Copernican principle in a conservative way, by requiring the power spectrum of the first component generated by matter *inside the void* to be below the SPT upper limit $6.5\ \mu\mathrm{K}^2$ at $\ell = 3000$.

For a general Hubble bubble $\vec{v}_H$ is determined by both Doppler and Sachs-Wolfe anisotropies generated by the void and depends qualitatively on the size of the void [8,21]. As we shall see below it is only Hubble bubbles with $z_{\rm edge} < 1$ which are consistent with both the SNe data and the proposed KSZ test, and for these a simple Doppler formula can be used [8,22],

$$v_H(z) \approx [H_i(z) - H_e] \frac{D_{A,\rm co}(z)}{1+z}, \tag{4}$$

where $H_i(z)$ is the Hubble expansion rate inside the void as a function of redshift, H_e gives the Hubble expansion rate exterior to the void at the same cosmological time, and $D_{A,\rm co}(z)$ is the comoving angular diameter distance to redshift z.

The temperature fluctuation at multipole ℓ generated by the linear KSZ effect inside of the Hubble bubble is, under the Limber approximation,

$$\Delta T_H^2(\ell) = (9.1\ \mu\mathrm{K})^2 \int_0^{z_{\rm edge}} \left[\frac{v_H(z)}{10^4\ \mathrm{km/s}}\right]^2 \left[\frac{d\tau_e/dz}{0.001}\right]^2 \times \left[\frac{\frac{\pi}{\ell}\Delta_e^2(\frac{\ell}{D_{A,\rm co}(z)}, z)}{0.1^2}\right] \frac{D_{A,\rm co}(z)}{c/H_i(z)} dz. \tag{5}$$

Here $\Delta T_H^2 \equiv T_{\rm CMB}^2 \ell(\ell+1) C_\ell/(2\pi)$, C_ℓ is the corresponding angular power spectrum, and $\Delta_e^2(k,z) = \frac{k^3}{2\pi^2} P_e(k,z)$ is the dimensionless electron number overdensity at wave number k and redshift z.

In our calculations we approximate P_e by the matter power spectrum P_m and approximate P_m by its form in a standard ΛCDM cosmology. It is nontrivial to calculate P_m in LTB models, since even at linear scales the expansion rate is locally anisotropic so the inhomogeneities will have an anisotropic power spectrum (see [23]), and since we are no longer assuming the cosmological principle one could also expect large scale variations in the initial inhomogeneities. The measured matter clustering and its evolution agree with the standard ΛCDM cosmology to a factor of ~ 2 uncertainty up to $z \sim 1$ [16,24], as do the galaxy clustering and evolution [25]. A minimalist approach is to simply use the ΛCDM predictions since any viable LTB models must be consistent with these data. If this assumption is not satisfied, then one should be able to obtain an even tighter constraint by considering these extra tests. Here we use P_m calculated by the CMBFAST package [26], and nonlinear clustering from the halofit formula [27], all using ΛCDM with $\Omega_m = 0.27$, $\Omega_\Lambda = 1 - \Omega_m$, $\Omega_b = 0.044$, $\sigma_8 = 0.84$, and $h = 0.71$. All other quantities such as τ_e and v_H are calculated based on the void model with the same Ω_b and $H_i(z=0) = 100h$ km/s/Mpc. The KSZ power spectrum is then computed using Eq. (5).

Constraints on the void model.—The ACT and/or SPT upper limit rules out large voids with low density (Fig. 1). Only those voids either with $\Omega_0 \to 1$ ($\Omega_0 \gtrsim 0.8$) or $z_{\rm edge} \to 0$ ($z_{\rm edge} \lesssim 0.2$, corresponding to void radius $\lesssim 0.6h^{-1}$ Gpc), survive this test (Fig. 1). These results agree fairly well with those in a more recent paper (Fig. 6, [28]), which used a more sophisticated treatment. (However, since [28] uses a different smooth void model, our results are not directly comparable.)

The KSZ test is highly complementary to other tests such as the supernova test. Our SNe Ia constraint follows Ref. [8] but uses the improved UNION2 data with 557 SNe Ia [29]. Not allowing for additional intrinsic dispersion of the SNe magnitudes we find a minimum χ^2 is 605.4. (Although this indicates a poor fit, including systematic errors and intrinsic magnitude dispersions would improve the fit.) Hubble bubble models within the 3σ contour typically have $\Delta T_H^2 > 10^3\ \mu\mathrm{K}^2$ at $\ell = 3000$, 2 orders of magnitude larger than the SPT upper limit 6.5 $\mu\mathrm{K}^2$ [17]. On the other hand, Hubble bubble models consistent with the SPT result have $\Delta\chi^2 > 209$ ($\chi^2 > 814$) for the SN Ia test and hence fail too. Thus the combination of SN Ia observations with small scale CMB anisotropy apparently rules out all Hubble bubble models.

Our KSZ calculation is based on these assumptions: (1) $\Omega_b h^2$ is the same as in the standard big bang nucleosynthesis analysis, (2) P_m is the same as in a ΛCDM model (based on the argument that any viable void model must reproduce the observed matter clustering), (3) $P_e = P_m$ (good to $\sim$10% accuracy [30]), (4) Eq. (4) is used for velocities (roughly accurate for subhorizon voids [22] which is required by CMB data [8,9]), (5) KSZ contributions are neglected from the compensating shell (which would only increase KSZ anisotropy), (6) there is a simple adiabatic Hubble bubble void, and (7) there is no CMB flow (intrinsic dipole) from nonadiabatic initial conditions outside the void. We expect that relaxing (1)–(6) in reasonable ways could not significantly reduce the tension imposed by the KSZ test, since for void models to explain the observed SN dimming, they must have the large scale gravitational potential of a large amplitude and hence must have large v_H and large KSZ effect. For example, [28] adopted a void model of a different profile and found much weaker SN constraint, but the generated KSZ power is nevertheless much larger than the ACT and/or SPT upper limit. This demonstrates the great discriminating power of the KSZ test. Completely relaxing (7) could change our conclusion for rather contrived initial conditions [31], but would generically lead to even larger and more unacceptable KSZ effect. Thus comparing KSZ with SNe is by far the most stringent test of the void models and the Copernican principle at Gpc scale and above. We conclude that any adiabatic void models capable of explaining the supernova Hubble diagram would likely generate too much KSZ power on the sky to be consistent with the ACT and/or SPT upper limit. This strengthens the evidence for cosmic acceleration and dark energy.

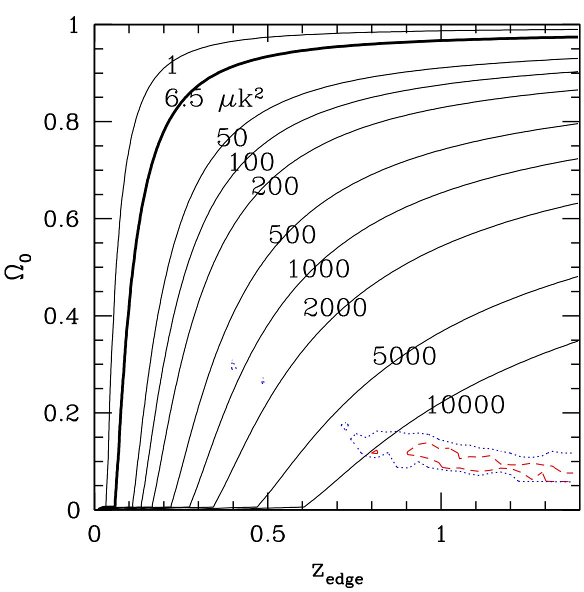

FIG. 1 (color online). The KSZ test. Black solid curves have constant $\Delta T_H^2(\ell = 3000)$. The thick one highlights the SPT 95% upper limit, $\Delta T^2 < 6.5\ \mu\mathrm{K}^2$ [17]. The KSZ test alone rules out large voids with low density and strongly supports the Copernican principle. The dashed and dotted contours are the 2-σ and 3-σ constraints from the UNION2 supernova data [29]. The KSZ test robustly excludes the Hubble bubble model as a viable alternative to dark energy.

Constraints on the Hubble flow.—Still, violation of the Copernican principle less dramatic than the above void models may exist [32]. For example, there could be large scale density modulation on the ΛCDM background. As long as the amplitude of the modulation is sufficiently small, it can pass the supernova test and the structure growth rate test. However, if unaccounted, it could bias the dark energy constraint. The KSZ test is able to put an interesting constraint on this type of violation. We take a model independent approach and parametrize the violation of the Copernican principle by $\Delta H(z)$, the deviation of the Hubble expansion of a mass shell of size Δz centered at redshift z from the overall expansion of the background universe. The ACT result constrains $|\Delta H(z)/H(z)| \lesssim 1\%$ for each mass shell of radial width $\sim 1h^{-1}$ Gpc (Fig. 2). This estimation neglects contributions from other mass shells so the actual constraint is tighter. This test can be carried out on each patch of the sky to test the isotropy of the Hubble flow.

The above test is not able to determine at which redshift a violation of the Copernican principle occurs, since the KSZ power spectrum is the sum over all contributions along the line of sight and hence has no redshift information. This problem can be solved with the aid of a survey of the large scale structure (LSS) with redshift information.

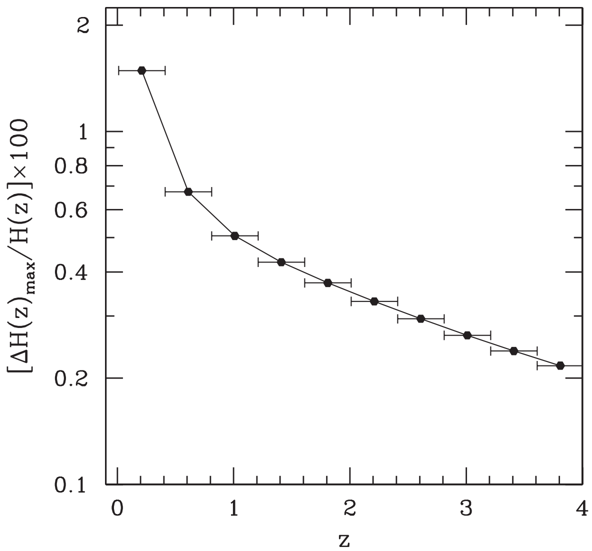

FIG. 2. The maximal deviation from the overall expansion allowed by the SPT observation, for each mass shell of $\Delta z = 0.4$, which corresponds to $1h^{-1}$ Gpc at $z \sim 0$, $0.7h^{-1}$ Gpc at $z \sim 1$, and $0.5h^{-1}$ Gpc at ~ 2.

The basic idea is the same as the one proposed by [12] to probe the dark flow through the KSZ-LSS density distribution two point cross correlation. This cross correlation is nonzero only in non-Copernican universes, since the velocity $\vec{v}_H$ varies slowly over the clustering length of the LSS and since the linear KSZ effect is linear in density. Since the cross correlation vanishes for the conventional KSZ effect, a nonvanishing cross correlation signal can serve as a smoking gun of violation of the Copernican principle. The thermal Sunyaev Zel'dovich effect contaminates the measurement. However, it can be largely removed by spectral fitting or observing at its null: 217 GHz. Since the redshift surveys can map the LSS with much higher S/N than KSZ measurements, this cross correlation can achieve much higher S/N than the KSZ autocorrelations. We thus expect that small scale CMB anisotropy surveys, such as ACT and SPT, in combination with deep LSS surveys will be able to put more stringent constraints on violations of the Copernican principle at each redshift and each direction of the sky.

We thank U. Pen, J. Fry, C. Clarkson, S. Das, P. Ferreira, and J. Zibin for useful discussions. P. Z. was supported by NSFC grants and the Beyond the Horizons program. A. S was supported by the DOE at Fermilab under Contract No. DE-AC02-07CH11359.

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