Relaxation Mechanisms in the Unfolding of Thin Sheets

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When a thin sheet is crumpled, creases form in which plastic deformations are localized. Here we study experimentally the relaxation process of a single fold in a thin sheet subjected to an external strain. The unfolding process is described by a quick opening at first and then a progressive slow relaxation of the crease. In the latter regime, the necessary force needed to open the folded sheet at a given displacement is found to decrease logarithmically in time, allowing its description through an Arrhenius activation process. We accurately determine the parameters of this law and show its general character by performing experiments on both Mylar and paper sheets.

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The physics of folding displays interesting phenomena in connection with mechanical and geometrical properties of low-dimensional objects such as rods or thin plates [1]. Folding problems arise, for instance, in the study of protein conformations [2], in the modeling of biological or artificial membranes [3], or in the optimization of self-deployable structures such as solar sails [4], insect wings [5], or plant leaves [6]. While folding procedures appear quite random and very complex, the statistical properties of crumpled thin sheets seem to have generic features limited by two main physical constraints. First, the topology and self-avoiding interactions are important physical factors during folding [1]. Second, a thin material crumples so that the largest part of the folding energy is concentrated in the network of narrow creases that meet in pointlike vertices [7,8]. Using the theory describing individual sharp vertices and ridges allows us to determine scaling properties of the folded state as a function of the confinement force, material dimensions, and its mechanical properties [9]. Hence, the mechanical response of the crumpled sheet to external loading might be used as a probe of the geometrical and topological properties of the crease network. Along this line of thinking, experiments on a crumpled thin sheet placed under fixed compressive force have shown that its size decreases logarithmically in time [10], suggesting an Arrhenius activation mechanism occurring in the crease network. Similarly, a crumpled surface [11] maintained at a fixed compressive strain involves a slow stress relaxation. However, these experiments share the same disadvantage: They involve friction due to the rigid constraint of self-avoidance during the crumpling process and between the material and the experimental setup. Apart from revealing thermally activatedlike behavior, the study of crumpling under compressive conditions does not allow us to discriminate between the effect of friction (which exhibits similar relaxation behavior [12]) and of the mechanical response of the fold network. An alternative approach to avoid self-contact problems would consist in studying the physics of unfolding. Indeed, it is commonly known that an initially crumpled sheet can freely unfold by quickly opening at first and then inflates slowly and progressively. This behavior is exclusively due to the relaxation of the fold network created during the crumpling process. This Letter is a step toward the understanding of this phenomenon by providing a thorough study of the relaxation properties of a single fold.

Figure 1 shows a typical evolution of a single crease made by folding a thin sheet of Mylar. The experiment consists in holding the fold vertically (to avoid gravity effects) from one side, keeping the other one free. The angle $\theta(t)$ of the crease was measured by following the

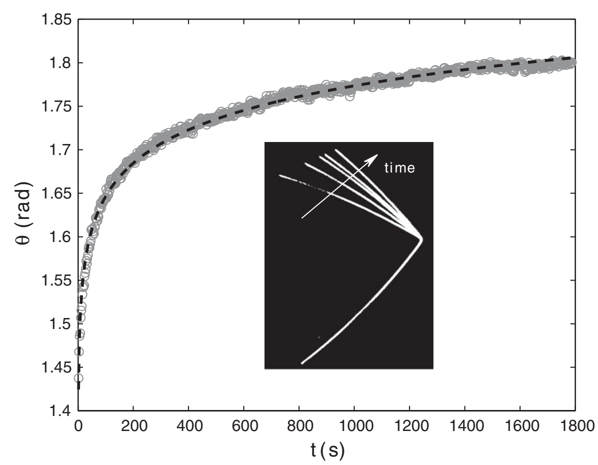

FIG. 1. Uncontrolled experiment: time evolution of the angle of a crease made in a rectangular sheet of Mylar of thickness 0.35 mm (see the inset). The dashed line is a logarithmic fit $\theta(t) = a\log(t) + b$ with $a = 0.055$ and $b = 1.392$. Inset: Superimposed images of the sheet during the relaxation process.

time evolution of the edge of the sheet by using a slow cadenced camera. It can be observed that the relaxation of the fold is described by a logarithmic law $\theta(t) = a\log(t) + b$. However, the constants a and b are very dependent on the folding protocol and fluctuate between runs with no monotonic evolution. Apart from showing logarithmic behavior of the relaxation, this experiment does not provide further insights on the unfolding process. We then proceeded to a more elaborate experiment.

In the following, we are interested in the behavior of a fold maintained at a fixed ''strain,'' allowing simultaneous measurements of the force and opening angle. The experiment sketched in Fig. 2 consists in an initially folded sheet that is clamped from both sides to metallic plates. One of the plates is fixed, and the other one is held by a highly resolved 0–5N dynamometer free to move horizontally (normal to the direction of the crease) via a microprecision displacement system. As previously, the evolution of the angle $\theta(t)$ is measured by visualizing the sheet from the side. For all the experiments, the crease preparation was performed by applying a constant load on a folded sheet into two equal parts during a given time (typically between 10 and 20 min). After clamping each edge of the fold to a plate, a given displacement $2y$ is quickly imposed. The force $F(t)$ and the fold angle $\theta(t)$ were measured simultaneously during a time lapse ranging from 10 min to a few hours. Various clamped configurations have been tested in order to estimate the role of the curvature change at the clamped edges. It turned out that this does not affect the results. We used rectangular thin sheets of both Mylar and paper of length $2L = 18$ cm, width $W = 14$ cm, and various thicknesses h and bending rigidities B. It is worth noticing that, in contrast with previous studies, the present system is completely frictionless, allowing us to focus only on the proper mechanics of the fold.

Figure 3 displays a typical evolution in time of the force and the crease angle for both Mylar and paper experiments. The results show evidence of a two-step relaxation regime. At short time scales, F (respectively, θ) follows a fast decrease (respectively, increase) with time that is well described by an exponential function. At long time scales, both F and θ show a logarithmic evolution which is similar to the case of the free relaxation experiment. One can wonder if the relaxation behaviors of the force and the angle are related and representative of the same phenomenon. By taking advantage of the experimental configuration, it seems then possible to determine a relation between the force F, the angle θ, and the control parameters of the experiment. Moreover, solving such a problem could shed light on the possible mechanisms governing the two different relaxation regimes. The relaxation process is very slow compared to characteristic times associated with elastic wave propagation in the material. Therefore, the experimental configuration of Fig. 2 is always at equilibrium, and the shape of the elastic sheet is governed by the elastica equation $BW\psi_{ss} - F\cos\psi = 0$, where s is the curvilinear coordinate along the plate and $\psi(s)$ its local slope. When considering one side of the fold (see Fig. 2), the boundary conditions read $\psi(0) = 0$ and $\psi(L) = \theta/2$. The magnitude of the force F is fixed through the condition $y = \int_0^L \sin\psi\, ds$, and the spacing is imposed experimentally. The numerical resolution of this problem is straightforward, and a relation $F(\theta)$ is computed for a fixed value of the dimensionless spacing y/L.

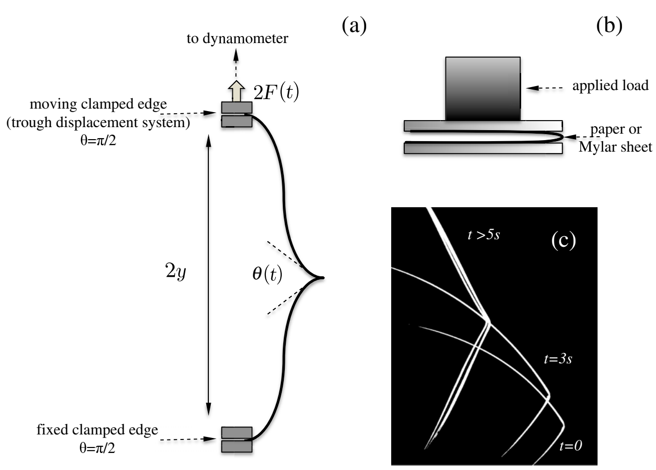

FIG. 2 (color online). (a) Schematics of the controlled experiment. A folded sheet is clamped on both sides and maintained at a fixed strain $2y$. The force and angle are measured simultaneously. (b) Illustration of the protocol for the fold preparation. The crease is obtained by applying a constant load on a folded sheet into two equal parts during a given time (typically between 10 and 20 min). (c) Superimposed images of the initial stage of the sheet during the unfolding and relaxation processes.

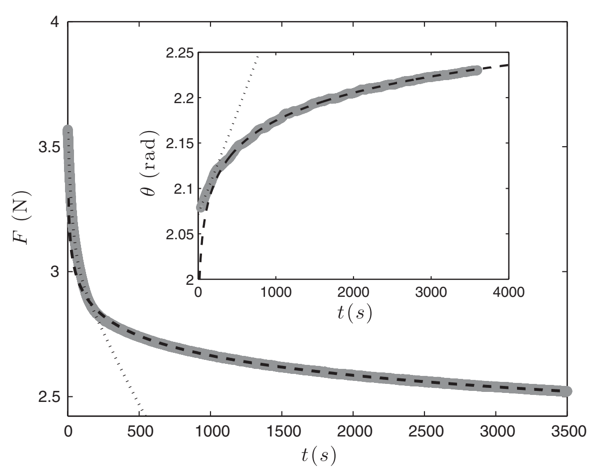

FIG. 3. Evolution of the force F as the fold relaxes in a Mylar sheet of thickness $h = 0.35$ mm and for a spacing $2y = 6.5$ cm between the plates. At short times ($t \lesssim 150$ s), the force decreases following an exponential law $\gamma\exp(-t/t_0)$ (thin dashed curve), and then a slow relaxation regime described by a logarithmic law $a\log(t) + b$ (thick dashed curve) takes place. Inset: The corresponding evolution of the angle $\theta(t)$ of the crease showing the same features as $F(t)$.

Figure 4 shows the evolution of the dimensionless force $\tilde{F}(\theta) = y^2 F(\theta)/\mathrm{BW}$, for two different experiments using Mylar of the same properties and imposing the same spacing y. The difference between runs lies in the preparation of the fold (10 and 15 min waiting time, respectively). Again, two different regimes can be distinguished and compared to the elastica predictions. The first regime, corresponding to the short time behavior, does not follow the solution of the elastica for a fixed dimensionless displacement y/L, while in the second regime, the evolution of $\tilde{F}(\theta)$ follows closely the predictions of the elastica for a fixed y/L which is slightly smaller than the imposed one. The point where $\tilde{F}$ and θ change from exponential to logarithmic relaxation behavior as shown in Fig. 3 corresponds precisely to the change of regimes observed in Fig. 4. The adequation between the elastica solution computed at a constant y/L and the experimental data in the second regime indicates that the relaxation occurs in the crease only.

These results show the presence of two distinct regimes of relaxation and suggest the following interpretation: At short times, the dynamics of the relaxation is dominated by viscous flow inside the creased region where strong strains are experienced (we performed experiments under the same conditions on the same material but without making a crease, and plastic flow was found to be negligible). Once the stretching of the fold ceases, an intrinsic relaxation due to an Arrhenius-like relaxation mechanism takes place. Moreover, Fig. 4 shows that the angle of the fold relaxes within a narrow range leading to a useful quasilinear relation between $\tilde{F}$ and θ. Therefore, force measurements, which are easier to perform experimentally, are sufficient to characterize the relaxation dynamics. In the following, we focus on the logarithmic relaxation regime of the force $F(t) = a \log t + b$, by studying the fluctuations of the constants a and b and their dependence on the experimental conditions. For this purpose, two different experimental protocols (I and II) have been followed.

First, we increased the spacing between plates by steps of Δy and measured the relaxation of the force for given time intervals Δt between successive events [Fig. 5(a)]. Protocol II consisted of measuring the response of the fold to periodic solicitations of amplitude $+\Delta y$ and $-\Delta y$ alternatively [Fig. 5(b)]. Both protocols raise the question of the time origin for the long time logarithmic force relaxation measurements. In a first study, we chose to fit it by $F(t) = a \log(t - t_0) + b$, where t_0 corresponds to the time of application of each new spacing. In a second study, the time just after the preparation of the crease was imposed as a unique reference and the force was fitted by $F(t) = a_\star \log(t) + b_\star$. As observed earlier, fitting the experimental curve is not straightforward because of the short time behavior. In order to capture the intrinsic relaxation of the fold, a logarithmic interpolation was performed on the force signal by excluding an increasing short time data interval until the convergence of the fitting parameters is reached.

Figure 6(a) displays the constants a and b for several experimental conditions. Results of protocol I show that the larger the strain is (which is related to the value of b), the faster the relaxation is (which is captured by the constant a). Results of protocol II show that the system is weakened and tends to a faster relaxation when the crease is submitted to cyclic folding and unfolding. However, both protocols suggest that a and b fluctuate and depend

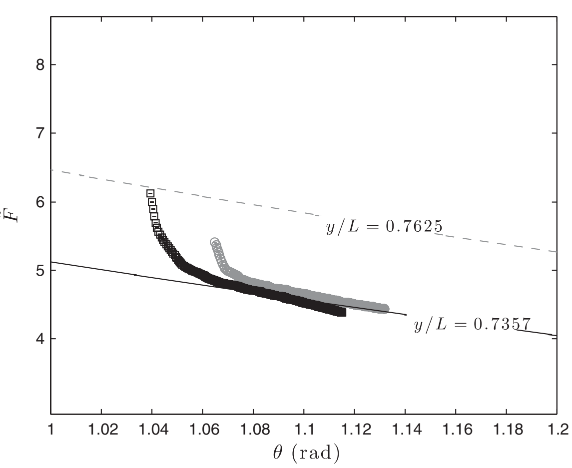

FIG. 4. Evolution of the dimensionless force $\tilde{F}(\theta)$ measured for two different runs (○). Experiments were performed in Mylar sheets of thickness $h = 0.35$ mm at the same imposed initial spacing $y/L = 0.7625$. The predictions of the elastica for the experimentally imposed spacing (dashed line) and for $y/L = 0.7357$ (continuous line) are also shown.

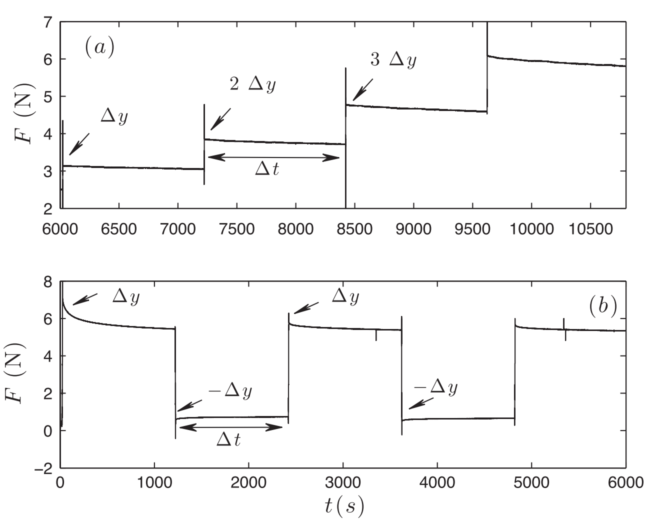

FIG. 5. Experimental runs showing the two protocols followed for the characterization of the force relaxation. (a) The fold spacing is increased periodically by an increment Δy starting from an initial spacing y_0. (b) The fold spacing is alternately increased and decreased by an increment Δy. The time interval Δt between two successive events and Δy are the control parameters of the experiment.

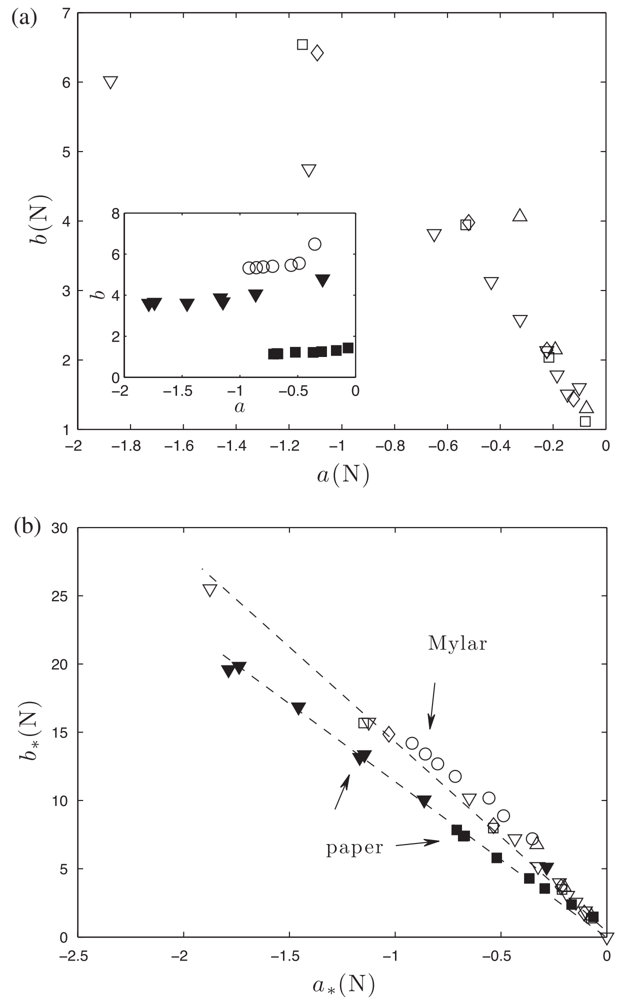

FIG. 6. Logarithmic relaxation of the force. (a) Results of protocol I experiments. We started from various initial spacings y_0 and applied an increment $\Delta y = 0.5$ cm during waiting times $\Delta t = 15$, 20, and 25 min. Inset: Protocol II experiments with $\Delta t = 20$ min and $\Delta y = 5.5$, 11, and 16.5 cm. The constants are determined from fittings by $a \log(t - t_0) + b$. (b) Constants determined from fittings by $a_\star \log(t) + b_\star$ of the same data as in (a). Experiments were performed by using 0.35 mm Mylar sheets (open symbols) and printing paper of thickness 0.1 (filled squares) and 0.2 mm (filled triangles).

on various constraints: fold preparation, experimental protocol, and memory effects. We then proceeded to a study of the time evolution of the force through the constants $a_\star$ and $b_\star$, which integrate time history, or memory, of the whole relaxation process. Figure 6(b) shows that modifying the reference time to $t_0 = 0$ independently of the experimental protocol makes the data collapse into a single linear curve $b_\star = -a_\star \log(\tau)$. The slope $\log(\tau)$ is a material property, in the sense that it depends neither on the protocol used nor on the experimental parameters. It is approximately equal to a month for a Mylar sheet and a day for paper. This main result allows us to write the time evolution of the force as $F = -|a_\star| \log(t/\tau)$, where τ is an intrinsic time scale of the material and $a_\star$ is a fluctuating force which depends on the experimental parameters and fold preparation. This also suggests an activated relaxation of the crease satisfying an Arrhenius behavior

$$\frac{dF}{dt} = -\frac{|a_\star|}{\tau} \exp\left[\frac{F}{|a_\star|}\right].$$

Here, $F/|a_\star|$ is the ratio between an activation energy and an effective thermal energy over a transformation volume. The characteristic time τ can be interpreted as the duration of rearrangements within the material due to plastic deformations that occur in the crease. The fact that this quantity is constant gives credit to the hypothesis of independent activation mechanisms without barriers.

In summary, we present a model experiment for the study of plastic deformation and related relaxation behavior. We characterized the slow relaxation process of a one-dimensional localized plastic zone during the unfolding of a thin sheet. We have shown a robust logarithmic relaxation law of the applied force (and consequently of the fold angle) and related it to an activated process. One notes that the identification of an intrinsic material constant such as τ has not been reported in previous crumpling experiments [10,11]. This can be attributed to the frictionless nature of the present experiment which allows us to characterize the behavior of the fold only. Finally, the characterization of the mechanical properties of the single fold is a step towards the understanding of the geometrical and mechanical properties of the crease network in crumpled sheets.

[1] T. A. Witten, Rev. Mod. Phys. **79**, 643 (2007).
[2] *Protein Folding Handbook*, edited by J. Buchner and T. Kiefhaber (Wiley, New York, 2005).
[3] *Statistical Mechanics of Membranes and Surfaces*, edited by D. Nelson, T. Piran, and S. Weinberg (World Scientific, Singapore, 2004).
[4] K. Miura, Int. J. Space Struct. **8**, 3 (1993).
[5] F. Haas and R. J. Wootton, Proc. R. Soc. B **263**, 1651 (1996).
[6] E. Couturier, S. C. du Pont, and S. Douady, PLoS ONE **4**, e7968 (2009).
[7] A. E. Loblovsky, S. Gentges, H. Li, D. Morse, and T. A. Witten, Science **270**, 1482 (1995).
[8] M. Ben Amar and Y. Pomeau, Proc. R. Soc. A **453**, 729 (1997).
[9] T. Tallinen, J. A. Aström, and J. Timonen, Nature Mater. **8**, 25 (2008).
[10] K. Matan, R. B. Williams, T. A. Witten, and S. R. Nagel, Phys. Rev. Lett. **88**, 076101 (2002).
[11] R. F. Albuquerque and M. A. F. Gomes, Physica (Amsterdam) **310A**, 377 (2002).
[12] T. Baumberger, P. Berthoud, and C. Caroli, Phys. Rev. B **60**, 3928 (1999).