Electroweak S and T Parameters from a Fixed Point Condition

M. Fabbrichesi,[1] R. Percacci,[2,1] A. Tonero,[2,1] and L. Vecchi[3]

[1]*INFN, Sezione di Trieste, Trieste, Italy*
[2]*SISSA, via Bonomea 265, 34136 Trieste, Italy*
[3]*LANL, Los Alamos, New Mexico 87545, USA*
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We consider the standard model without the Higgs boson, where the Goldstone modes are described by a nonlinear sigma model. We study the renormalization group flow of the sigma model coupling $\tilde{f}$ and of the electroweak parameters S and T. The condition that the couplings reach a fixed point at high energy leaves the low energy values of $\tilde{f}$ and T arbitrary (to be determined experimentally) and fixes S to a value compatible with electroweak precision data.

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The nonlinear sigma model with values in a coset space G/H arises whenever a symmetry G is spontaneously broken to H. The best known application is the chiral model with $G = SU(2) \times SU(2)$ and $H = SU(2)$ (the diagonal, or vector, subgroup), which describes the low energy dynamics of pions, regarded as the (pseudo)-Goldstone bosons arising from spontaneous breaking of chiral symmetry in the theory of strong interactions with two massless fermion flavors. An equally important realization of the same geometry describes the Goldstone bosons that break the electroweak (EW) $SU(2) \times U(1)$ to $U(1)$. In this case the Goldstone bosons do not correspond to physical states; rather, they are transformed into the longitudinal components of the W and Z bosons by the Higgs phenomenon. Essentially, all we currently know about EW interactions can be encoded in an effective field theory of Goldstone bosons coupled to gauge fields and fermions [1]. This would be the minimal option: In fact, it would have no Higgs boson in the Lagrangian and hence one *less* degree of freedom than the standard model (SM).

Because of its perturbative nonrenormalizability, the nonlinear sigma model is usually regarded as a mere low energy effective field theory. In fact, in the case of strong interactions, the UV completion of the chiral model is QCD, so there is no reason to look further. In the EW case, however, things are not yet settled, and it is important to consider all options. The simplest possibility is to embed the nonlinear sigma model into a complex doublet transforming linearly under $SU(2)_L$; this renders the theory perturbatively renormalizable (though not UV complete, due to the positive beta function of the scalar coupling). Technicolor provides a dynamical way of breaking the EW group. Another possibility that we shall consider here is that the theory is renormalizable in a nonperturbative sense, namely, at a nontrivial fixed point (FP) of the renormalization group flow [2]. This idea has been developed mostly in the context of gravity [3]. For other applications to the SM, see [4]. This approach has the disadvantage that perturbation theory is at best a rough guide. If in spite of this one considers the one-loop beta functions, or some resummation thereof, it is easy to see that a nontrivial FP is present [5]. It persists when one considers in addition terms with four derivatives of the Goldstone bosons [6] or the coupling to gauge fields [7]. It is not there in the presence of Yukawa coupling to fermions, but it reappears if one also adds four-fermion contact interactions [8]. In the same spirit, we will consider here the compatibility of this hypothesis with precision EW data. The effect of physics beyond the SM on the gauge bosons can be tested by calculating the oblique parameters S and T [9] and comparing with their experimental bounds. We will study this issue by calculating the renormalization group flow of the effective couplings representing these parameters in the EW effective theory.

We will restrict ourselves to the bosonic sector of the EW effective theory. We use a geometrical description of the Goldstone bosons as coordinates $\varphi^\alpha(x)$ of a field $U(x)$ taking values in $SU(2) \times U(1)/U(1) \sim SU(2)$. The lowest order terms in the Euclidean action are

$$S = \frac{1}{2f^2}\int d^4x h_{\alpha\beta} D_\mu \varphi^\alpha D^\mu \varphi^\beta + \frac{1}{4g^2}\int d^4x W^I_{\mu\nu} W_I^{\mu\nu} + \frac{1}{4g'^2}\int d^4x B_{\mu\nu} B^{\mu\nu}, \tag{1}$$

where W^I_μ are the $SU(2)_L$ gauge fields and B_μ is the $U(1)_Y$ gauge field; g and g' are the gauge couplings, and f is the (dimension -1) chiral coupling. The covariant derivative acting on φ is

$$D_\mu \varphi^\alpha = \partial_\mu \varphi^\alpha + W^I_\mu R^\alpha_I - B_\mu L^\alpha_3, \tag{2}$$

while the gauge field strength tensors are $W^I_{\mu\nu} = \partial_\mu W^I_\nu - \partial_\nu W^I_\mu + \epsilon^I_{JM} W^J_\mu W^M_\nu$ and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The indices $\alpha, \beta = 1, 2, 3$ run over the target space coordinates while $I, J, M = 1, 2, 3$ are $SU(2)$ Lie-algebra indices. We denote R^α_I and L^α_I the right- and left-invariant vector fields on $SU(2)$, respectively. In particular, R^α_I generate $SU(2)_L$ and L^α_3 generates $U(1)_Y$.

The gauge invariance of the SM demands that the metric $h_{\alpha\beta}$ be invariant under the action of these vector fields but not necessarily under the $SU(2)_R$ transformations generated by L_1^α and L_2^α. The most general metric of this type is of the form

$$h_{\alpha\beta} = L^1_\alpha L^1_\beta + L^2_\alpha L^2_\beta + (1-2a_0)L^3_\alpha L^3_\beta, \tag{3}$$

where L^I_α is the basis of left-invariant one-forms dual to L^α_I. The parameter a_0 measures the violation of the "custodial" symmetry $SU(2)_R$ and vanishes in the bare SM Lagrangian. Radiative corrections then induce a small nonvanishing effective value for a_0. It is therefore customary to assume that the metric $h_{\alpha\beta}$ is bi-invariant and to consider the $SU(2)_R$ breaking as due to a separate term in the effective Lagrangian:

$$\frac{a_0}{f^2}(\mathrm{tr}\sigma_3 U^\dagger DU)^2 = \frac{a_0}{f^2} D_\mu\varphi^\alpha D^\mu\varphi^\beta L^3_\alpha L^3_\beta. \tag{4}$$

The action contains further terms. Among these we shall be interested, in particular, in the term

$$a_1 \tfrac{1}{2} B_{\mu\nu}\mathrm{tr}\sigma_3 U^\dagger W^{\mu\nu}U = a_1\tfrac{1}{2}B^{\mu\nu}W^I_{\mu\nu}R_{I\alpha}L^\alpha_3. \tag{5}$$

These definitions agree with those of Refs. [10,11] except for the rescaling of the gauge fields with the gauge couplings. The running couplings a_0 and a_1 are related to the oblique parameters S and T by

$$S = -16\pi a_1(m_Z) + \frac{1}{6\pi}\left[\frac{5}{12} - \log\left(\frac{m_H}{m_Z}\right)\right], \tag{6}$$

$$T = \frac{2}{\alpha}a_0(m_Z) - \frac{3}{8\pi\cos^2\theta_W}\left[\frac{5}{12} - \log\left(\frac{m_H}{m_Z}\right)\right]. \tag{7}$$

The second term on the right-hand side corresponds to subtracting the contribution of the Higgs field with mass m_H [12].

In this Letter, we will be concerned with the renormalization group running of the gauge couplings g and g', the sigma model coupling f, and the parameters a_0 and a_1.

It will be instructive to consider first the ungauged $SU(2)\times U(1)/U(1)$ sigma model, with couplings f and a_0. Quite generally, the beta function of the sigma model is given by a kind of Ricci flow [5]:

$$\frac{d}{dt}\left(\frac{1}{f^2}h_{\alpha\beta}\right) = \frac{1}{(4\pi)^2}k^2 R_{\alpha\beta}, \tag{8}$$

where $t = \log k$. In the basis of the right-invariant vector fields, the Ricci tensor of the metric $h_{\alpha\beta}$ is $R_{11} = R_{22} = \frac{1}{2} + a_0$, $R_{33} = \frac{1}{2} - a_0$, so the beta functions of $\tilde{f}^2 = f^2k^2$ and a_0 are

$$\frac{d\tilde{f}^2}{dt} = 2\tilde{f}^2 - \frac{1}{(4\pi)^2}\tilde{f}^4\left(\frac{1}{2} + a_0\right), \tag{9}$$

$$\frac{da_0}{dt} = \frac{1}{2}\frac{1}{(4\pi)^2}\tilde{f}^2 a_0(1-2a_0). \tag{10}$$

These beta functions admit a Gaussian FP, with $\tilde{f} = 0$ and arbitrary a_0, and two nontrivial fixed points: an $SU(2)_R$-symmetric one at $a_0 = 0$, $\tilde{f} = 8\pi \approx 25.13$ and another one with strongly broken $SU(2)_R$ at $a_0 = 1/2$, $\tilde{f} = 4\sqrt{2}\pi \approx 17.8$. The FP at $a_0 = 0$ is UV-repulsive, and the one at $a_0 = 1/2$ is UV-attractive. If $a_0 < 0$, corresponding to an elongated three-sphere, a_0 decreases with increasing energy; if $0 < a_0 < 1/2$, corresponding to a mildly squashed three-sphere, a_0 increases with energy towards the FP at $a_0 = 1/2$; if $a_0 > 1/2$, corresponding to a strongly squashed three-sphere, a_0 decreases with energy towards the FP at $a_0 = 1/2$.

Coming to the gauged case, we begin by considering the subsystem of the couplings g, g', and f, keeping $a_0 = a_1 = 0$. This is a slight generalization of a calculation described in detail in Ref. [7]. The beta functions of the gauge couplings are

$$\frac{dg^2}{dt} = \frac{g^4}{(4\pi)^2}\frac{1}{1+\tilde{m}_W^2}\left[-\frac{16}{(1+\tilde{m}_W^2)^2} + \frac{3}{2}\right], \tag{11}$$

$$\frac{dg'^2}{dt} = \frac{1}{6}\frac{g'^4}{(4\pi)^2}\frac{1}{1+\tilde{m}_W^2}, \tag{12}$$

where $\tilde{m}_W^2 = m_W^2/k^2 = g^2/\tilde{f}^2$. The fractions represent the effect of thresholds and automatically switch off the beta functions when k becomes smaller than m_W. Aside from these thresholds, the difference with the SM is due only to the absence of the Higgs particle and is quite small, so g is asymptotically free, while g' has a Landau pole at a trans-Planckian energy.

The beta function of $\tilde{f}^2$ is

$$\begin{aligned}\frac{d\tilde{f}^2}{dt} = 2\tilde{f}^2 - \frac{1}{(4\pi)^2}\Bigg\{&\frac{1}{4}\frac{\tilde{f}^4}{(1+\tilde{m}_W^2)^2} + \frac{1}{4}\frac{\tilde{f}^4}{(1+\tilde{m}_Z^2)^2} \\ &+ \frac{2g^2\tilde{f}^2}{(1+\tilde{m}_W^2)^3} + \frac{g'^2\tilde{f}^2}{(1+\tilde{m}_W^2)(1+\tilde{m}_B^2)} \\ &\times\left[\frac{1}{(1+\tilde{m}_W^2)} + \frac{1}{(1+\tilde{m}_B^2)}\right] + \frac{g^2\tilde{f}^2}{(1+\tilde{m}_W^2)(1+\tilde{m}_Z^2)} \\ &\times\left[\frac{1}{(1+\tilde{m}_W^2)} + \frac{1}{(1+\tilde{m}_Z^2)}\right]\Bigg\},\end{aligned} \tag{13}$$

where $\tilde{m}_Z^2 = m_Z^2/k^2 = (g^2 + g'^2)/\tilde{f}^2$ and we also define the shorthand $\tilde{m}_B^2 = g'^2/\tilde{f}^2$. The whole expression simplifies drastically when the mass terms can be neglected. In practice, one can use this approximation when $k > m_Z$, the heaviest mass in the theory, while for $k < m_B = g'/f$, the lightest mass in the theory, the beta function reduces to the first (classical) term. In the following, we will use this approximation.

Because of the positive beta function for g', strictly speaking this system does not have a FP. However, the running of the gauge couplings is very slow, and for our purposes it is a good approximation to treat them as constants. Setting $g = 0.65$ and $g' = 0.35$, we find an approximate UV-attractive FP at $\tilde{f} = 25.08$. As expected, it is very close to the FP of the ungauged model.

We are now ready to consider the effect of the couplings a_0 and a_1. As in the ungauged case, the beta functions of f and a_0 can be extracted from the geometric beta functional of the metric. For k much larger than all the masses ($g, g' \ll \tilde{f}$), the threshold fractions become equal to one, and the beta functions simplify to

$$\frac{d\tilde{f}^2}{dt} = 2\tilde{f}^2 - \frac{1}{2}\frac{\tilde{f}^2}{(4\pi)^2}[\tilde{f}^2(1 + 2a_0) + 6g^2 + 3g'^2], \quad (14)$$

$$\frac{da_0}{dt} = \frac{1}{2}\frac{1}{(4\pi)^2}\left(\tilde{f}^2 a_0(1 - 2a_0) + \frac{3}{2}g'^2\right). \quad (15)$$

We have neglected terms of order $g^2 a_0$ or $g'^2 a_0$, which are subleading relative to those of order $\tilde{f}^2 a_0$. They are not necessarily subleading relative to the terms of order g^2 and g'^2 that have been written, but they would be unimportant in what follows. Note that these beta functions reduce correctly to (9) and (10) in the ungauged case. The first term in (15) corresponds to a self-renormalization of the operator (4). Diagrammatically, it corresponds to a quadratically divergent Goldstone boson tadpole and cannot be seen in dimensional regularization. The second term agrees with the results of Ref. [11]; it is proportional to g'^2, consistent with the fact that the hypercharge coupling breaks the custodial symmetry. Its effect is to generate a nonzero a_0 even if initially $a_0 = 0$.

The fixed points of the ungauged case are slightly shifted by the gauge couplings. They occur at (FP I) $\tilde{f} = 25.1$, $a_0 = -0.000\,292$ and (FP II) $\tilde{f} = 17.7$, $a_0 = 0.501$. There is no longer a fixed point with $\tilde{f} = 0$. This flow is illustrated in Fig. 1.

The beta function of a_1 is

$$\frac{da_1}{dt} = \frac{1}{(4\pi)^2}\left(\tilde{f}^2 a_1 + \frac{1}{6}\right). \quad (16)$$

Also in this case the second term agrees with the one computed in Ref. [11], while the first comes from the self-renormalization of the operator (5). Introducing the FP values for $\tilde{f}^2$ discussed above, we find the FP values $a_1 = -0.000\,265$ for FP I and $a_1 = -0.000530$ for FP II. The eigenvalues and eigenvectors of the matrix describing the linearized flow around these FPs are given in Table I. Recall that negative eigenvalues correspond to UV-attractive (relevant) directions. The point FP I has one such direction, that to a good approximation can be identified with the parameter $\tilde{f}$. The point FP II has two relevant directions that lie almost exactly in the a_0-$\tilde{f}$ plane.

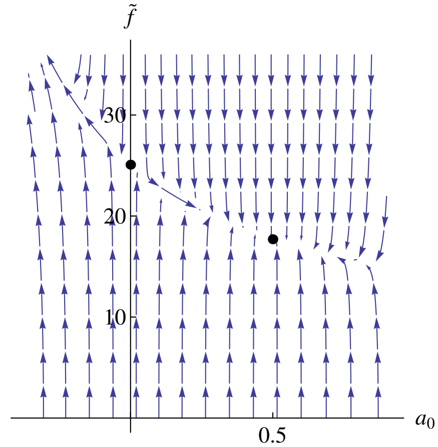

FIG. 1 (color online). Flow in the a_0-$\tilde{f}$ plane. The two dots mark the positions of FP I and FP II. Arrows point to increasing energy.

Within numerical errors we found a critical trajectory that starts from FP II in the UV approximately in the direction of (minus) its second eigenvector and reaches FP I in the IR from the direction of its second eigenvector. The origin is not a FP, but the beta functions become very small there. This almost FP is IR-attractive for $\tilde{f}$.

We now discuss the physics of these FPs. At $k = m_Z$ we have $\tilde{f} = 2m_Z/v = 0.7415$, and the experimentally allowed values for $a_0(m_Z)$ and $a_1(m_Z)$ are of the order of 10^{-3}. When one evolves the flow towards higher energies, $\tilde{f}$, a_0, or a_1 will generally diverge. This is a sign that "new physics" has to be taken into account. However, there may be trajectories that hit a FP in the UV: For them, the effective field theory description actually never breaks down. Such trajectories are said to be "renormalizable" or "asymptotically safe" [2], and they form the so-called "UV critical surface," which in the vicinity of a FP is spanned by the relevant couplings.

Requiring that the world be described by a renormalizable trajectory leads to predictions for low energy physics. Since FP I has only one relevant direction, there is a single renormalizable trajectory that descends from it towards the origin. Since the beta functions go to zero for $k < m_Z$, we stop the flow at the scale m_Z (i.e., when $\tilde{f} = 0.7415$) and find, at that scale,

TABLE I. Properties of the fixed points.

FP	Eigenvalue	Eigenvector components $\tilde{f}$	a_0	a_1
I	-1.99	1.00	11.6×10^{-6}	14.1×10^{-6}
I	1.99	-0.997	0.0795	-42.2×10^{-6}
I	3.98	0	0	1
II	-1.99	1.00	66.0×10^{-6}	29.9×10^{-6}
II	-0.996	-0.998	0.0563	-40×10^{-6}
II	1.99	0	0	1

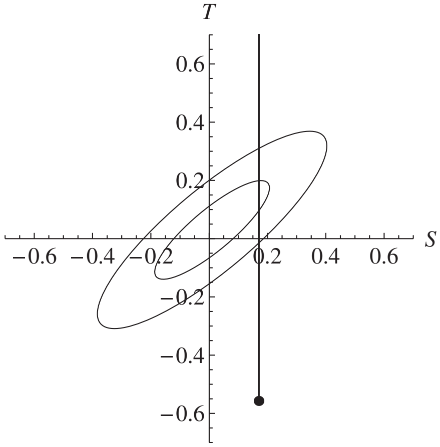

FIG. 2. The half-line and the dot show the values permitted by asymptotic safety. The ellipses show the 1 and 2σ experimental bounds with $m_H = 117$ GeV [13].

$$a_0(m_Z) = -0.0020, \qquad a_1(m_Z) = -0.0032, \tag{17}$$

which are 5σ away from the experimental values. The transition takes about four or five e-foldings (a change in scale by a factor e^4–e^5) which means that FP I would be reached at an energy scale of the order of 10 TeV.

The point FP II has two relevant directions, and therefore there is a one-parameter family of renormalizable trajectories that descend from it. From Fig. 1, we see that, for such a trajectory to come close to the origin, it has to be fine-tuned to first follow very closely the critical trajectory towards FP I and, hence, descend. Going upwards from $k = m_Z$, such a trajectory would take again four or five e-foldings to reach the vicinity of FP I and then another four e-foldings to cross over to FP II, placing the energy scale at which one arrives near FP II at 300–700 TeV. It is clear from Fig. 1 that these trajectories will have $a_0(m_Z) > -0.002$. Numerical analysis shows that the locus of end points of such trajectories satisfies

$$a_1(m_Z) = -0.003\,21 - 0.000\,52 a_0(m_Z). \tag{18}$$

For $a_0 \approx 0.5$, this relation is still true within a few percent.

Using Eqs. (6) and (7), this translates directly into a linear relation between S and T, which is shown in Fig. 2, and constitutes our main result. The dot corresponds to the UV critical surface of FP I (17), and the half-line to the UV critical surface of FP II. Note that the condition of asymptotic safety essentially fixes a_1, and hence S, leaving T arbitrary.

Standard model fermions would not change this conclusion, since their contribution is already included in the definition of S and T, but one has to make sure that they do not spoil the FP. We have shown in Ref. [8] that the FP of $\tilde{f}$ is preserved if four-fermion interactions are added. These interactions change the beta functions of S and T only at higher loops, so we expect our conclusions to remain valid.

Renormalizable trajectories represent UV complete theories. We see that within this model there are such trajectories that are in agreement with the experimental data: $S = 0.01 \pm 0.10$ and $T = 0.03 \pm 0.11$. They pass near FP I at scales ≈ 10 TeV and then veer towards FP II. There, the custodial symmetry is strongly broken, as witnessed by the large value $a_0 \approx 0.5$. This could be an important (and unexpected) clue about the UV behavior of the theory. In this model the conformal (FP) behavior sets in at energies that are probably too high to make a direct observation possible at the LHC, but there may be other signatures. We will return to this and related questions elsewhere.

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