Efficiency of Heat Transfer in Turbulent Rayleigh-Bénard Convection

P. Urban,[1] V. Musilová,[1] and L. Skrbek[2]

[1]*Institute of Scientific Instruments ASCR, v.v.i., Královopolská 147, Brno, Czech Republic*

[2]*Faculty of Mathematics and Physics, Charles University, Ke Karlovu 3, 12116 Prague, Czech Republic*

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We present an experimental study of turbulent Rayleigh-Bénard convection (RBC) in a cylindrical cell of height 0.3 m, diameter 0.3 m. It is designed to minimize the influence of its structure on the convective flow of cryogenic ^{4}He gas of Prandtl number $\mathrm{Pr} \approx 1$, with the aim of resolving existing contradictions in Nusselt (Nu) versus Rayleigh number (Ra) scaling. For $7.2 \times 10^6 \leq \mathrm{Ra} \lesssim 10^{11}$ our data agree with suitably corrected data from similar cryogenic experiments and are consistent with $\mathrm{Nu} \propto \mathrm{Ra}^{2/7}$. On approaching $\mathrm{Ra} \approx 10^{11}$ our data display a crossover to $\mathrm{Nu} \propto \mathrm{Ra}^{1/3}$ that approximately holds up to $\mathrm{Ra} \cong 4.6 \times 10^{13}$; there is no sign of a transition to the ultimate Kraichnan regime. Differences in Nu(Ra) scaling observed in similar RBC experiments for $\mathrm{Ra} \gtrsim 10^{11}$ cannot be explained due to the difference in Pr, but seem to depend also on experimental details.

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The ideal laterally infinite Rayleigh-Bénard convection (RBC) is a model for fundamental studies of buoyancy-driven flows. It occurs in a (Boussinesq) fluid layer confined between infinite perfectly conducting plates heated from below in a gravitational field, and it is characterized by the Rayleigh, Ra, and the Prandtl numbers, Pr. The convective heat transfer efficiency is expressed by the Nusselt number, $\mathrm{Nu} = \mathrm{Nu}(\mathrm{Ra}, \mathrm{Pr})$. These dimensionless numbers are defined as

$$\mathrm{Nu} = \frac{LH}{\lambda \Delta T}, \qquad \mathrm{Ra} = g\frac{\alpha}{\nu\kappa}\Delta T L^3, \qquad \mathrm{Pr} = \frac{\nu}{\kappa}, \quad (1)$$

where H is the total convective heat flux density, g stands for the acceleration due to gravity, and ΔT is the temperature difference between the parallel bottom and top plates separated by vertical distance L. The properties of the working fluid are characterized by its heat conductivity λ and by the combination $\alpha/(\nu\kappa)$, where α is the isobaric thermal expansion, ν is the kinematic viscosity, and κ denotes the thermal diffusivity. Functional dependence $\mathrm{Nu} = \mathrm{Nu}(\mathrm{Ra}, \mathrm{Pr})$ at high Ra, usually expressed as a scaling law $\mathrm{Nu} \propto \mathrm{Ra}^\gamma \mathrm{Pr}^\beta$, is intensively studied theoretically, numerically, and experimentally [1].

The scaling law with $\gamma \approx 1/3$ corresponds to a model where all ΔT occurs across the boundary layers (thin in comparison with L at high Nu) adjacent to the plates, while in the central turbulent region the working fluid is effectively mixed. Heat transfer is controlled by thermal conduction of the boundary layers and the convective heat flux does not depend on L. At very high Ra the boundary layer should undergo a laminar-turbulent transition when convection enters the "ultimate," "asymptotic" regime, with $\mathrm{Nu} \propto \mathrm{Ra}^{1/2}\mathrm{Pr}^{-1/4}(\log\mathrm{Ra})^{-3/2}$ and $0.15 < \mathrm{Pr} < 1$, as predicted by Kraichnan [2].

Experimentally, RBC is often studied in cylindrical cells of height L and diameter D, characterized by the aspect ratio $\Gamma = D/L$ and $\mathrm{Nu} = \mathrm{Nu}(\mathrm{Ra}, \mathrm{Pr}, \Gamma)$ (for details of scaling laws, in particular, the significance of $\gamma = 2/7$, see [1]). The existence of the ultimate regime and its position in the Ra, Pr, Γ parameter space is a challenging open question, in view of its utmost importance for understanding many large scale convective flows in Nature. Although Kraichnan himself assumed an onset at extremely high Ra not yet achieved in any laboratory, for $\Gamma \approx \mathrm{Pr} \approx 1$ Grossmann and Lohse (G-L theory, see [1]) estimated the transition to the $\mathrm{Nu} \propto \mathrm{Ra}^{1/2}$ regime at $\mathrm{Ra} \approx 10^{13} - 10^{14}$.

Utilization of cryogenic ^{4}He gas as a working fluid offers an outstanding possibility to achieve very high Ra thanks to the extremely large value of its fluid-properties ratio $\alpha\nu^{-1}\kappa^{-1}$ near the critical point ($T_c = 5.1953$ K, $p_c = 227.46$ kPa, $\rho_c = 69.641$ kg/m^3) [3]. Moreover, this ratio can easily be tuned over a wide range *in situ* within a single experimental run. The highest values of Ra using cryogenic gaseous ^{4}He have been achieved in Chicago [4,5], Grenoble [6,7], and Oregon (Eugene) [8], albeit with controversial results on Nu(Ra) scaling at high Ra. The Chicago and Oregon groups used $\Gamma = 1/2$ cylindrical cells ($L = 40$ cm and 1 m) and studied the convective heat transport up to $\mathrm{Ra} \approx 10^{14}$ and $\approx 10^{17}$, respectively; the observed γ did not exceed 1/3. This is in striking contrast with the Grenoble results from the $\Gamma = 1/2$ cell of smaller height $L = 0.2$ m: $\gamma \approx 2/7$ was found for $\mathrm{Ra} < 10^{11}$ and a transition to a regime characterized by $\gamma \approx 0.4$ above $\mathrm{Ra} \approx 10^{11}$, interpreted as a transition into Kraichnan ultimate regime [6]. Moreover, in subsequent experiments in Grenoble, Roche, Gauthier, Kaiser, and Salort observed similar transitions above $\mathrm{Ra} \approx 10^{11}$ with seven different $\Gamma = 1/2$ convection cells (with smooth and rough Cu plates, smooth brass bottom plate, mean flow restrictions, tilted cell) and also with the $\Gamma = 1.14$ cell [7]. In Trieste, Niemela and Sreenivasan performed additional experiments (using essentially the Oregon apparatus) with a $\Gamma = 1$ cell up to Ra $\approx 10^{15}$ and a $\Gamma = 4$ cell up to Ra $\approx 2\times10^{13}$ [9], and claimed to find a transition from one Nu $\approx \chi$Ra$^{1/3}$ regime (with $\chi \approx 0.064$) to another (with $\chi \approx 0.078$) [10]. Although the results of all these cryogenic experiments are difficult to evaluate with precision (in view of corrections needed due to, e.g., the finite conductivity and heat capacity of the plates and sidewalls, proximity to the critical point, parasitic heat leaks, adiabatic gradient [1]), the Trieste cryogenic experiments and the experiments of Funfschilling, Bodenschatz, and Ahlers with gaseous He, N_2, and SF_6 at nearly ambient temperature (although indicating different, not yet fully understood transitions at even higher Ra [10,11]) did not display transition to the ultimate Kraichnan regime [12]. Hence there was a clear call to design a suitable cryogenic Rayleigh-Bénard cell to resolve these controversies.

In this Letter, we present Nu(Ra, Pr, $\Gamma = 1$) (see Fig. 1) for $7.2\times10^6 \le$ Ra $\le 4.6\times10^{13}$. Although our apparatus (upper inset in Fig. 1; for a detailed technical description see [13]) is capable of reaching much higher Ra, we limit ourselves to this range because (i) it covers adequately the range of the main controversy, (ii) some of the above-mentioned corrections are minimized for our cell that was designed and built especially for this purpose, and (iii) the working fluid—cryogenic helium gas—can be kept sufficiently far from its critical point. The $\Gamma = 1$ cell with thin ($\delta = 0.5$ mm) stainless steel sidewalls of relatively low thermal conductivity λ_w is $2R = 0.3$ m in diameter. The top and bottom plates are made of 28 mm thick annealed OFHC copper of thermal conductivity λ_p at least 2 kW m^{-1} K^{-1}. The upper plate is thermally connected to the liquid helium (LHe) vessel via a heat exchange chamber (HEC) filled with gaseous ^{4}He. The total external parasitic heat leak to the cell (both radiative and conductive) is suppressed to <1% of the lowest convective heat flux used in the experiment, measured to ±0.5%. Design of the heaters ensures better than 1 mK temperature homogeneity of the internal side of plates, under the assumption that the heat is uniformly supplied or removed. Four calibrated Lake Shore GR-200A-1500-1.4B Ge temperature sensors (5 mK absolute accuracy guaranteed by the manufacturer for two of them, additional calibration [13] allows determination of ΔT within 2 mK) are imbedded in the center and near the edge of Cu plates. The pressure in the cell is measured with an MKS Baratron 690 A (calibration traceable to NIST) with 0.08% reading accuracy. Helium properties are gained from the NIST database [3,14], based on the actual pressure in the cell and the mean temperature T_m assessed as arithmetic average of the plate temperatures. Two stainless steel tubes thermally anchored to the LHe vessel [15] are used for venting.

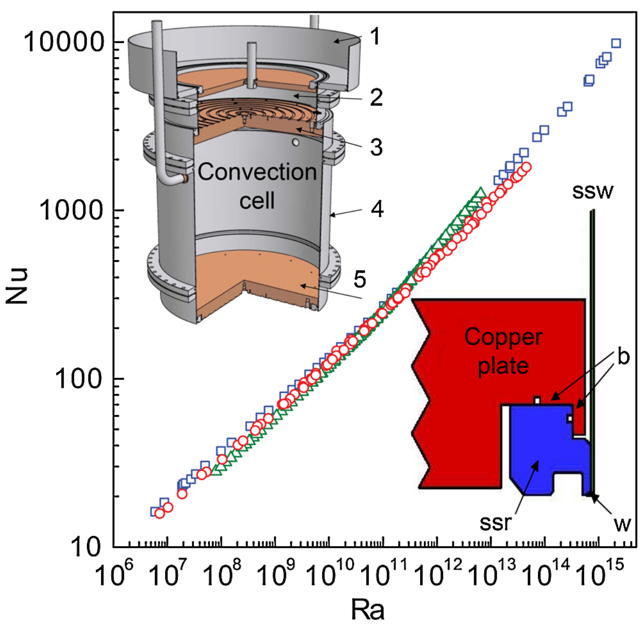

FIG. 1 (color online). Observed Nu(Ra) values [open (red) circles] in comparison with the data sets obtained with the $\Gamma = 1$ cell in Trieste [9] [open (blue) squares] and with the $\Gamma = 1.14$ cell in Grenoble [7] [open (olive) triangles]. The upper inset shows our cell: 1—LHe vessel; 2—heat exchange chamber, 3,5—Cu plates, 4—exchangeable part of the sidewall. The lower inset shows how the thick Cu plate joins the thin stainless steel wall (ssw), welded (w) to a stainless steel ring (ssr) that is brazed (b) to the Cu plate.

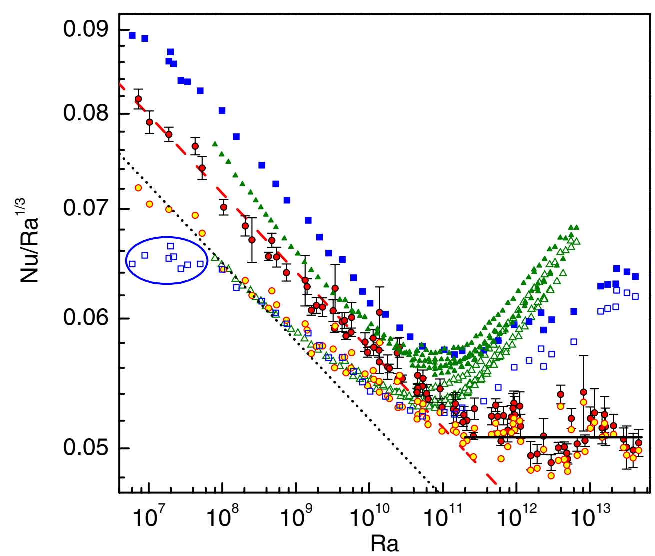

FIG. 2 (color online). The compensated NuRa$^{-1/3}$ plot versus Ra: our measured data (without wall correction) are shown as (red filled) circles with error bars representing the total uncertainty in NuRa$^{-1/3}$ caused by uncertainties in the determination of T_m (4 mK), p (0.1%), ΔT (2 mK) and heat power to the bottom plate (0.5%); (red, yellow filled) circles are our data with the wall corrections applied as described in the text; (olive) triangles and open (olive) triangles represent the uncorrected and corrected ($\Gamma = 1.14$) Grenoble data set [7]; solid (blue) squares and open (blue) squares are the uncorrected and corrected ($\Gamma = 1$) data sets from Trieste ($T_m = 5.34 \pm 0.02$ K) [9]. The dashed (red) line is functional dependence Nu $= 0.172$Ra$^{2/7}$, the dotted line Nu $= 0.156$Ra$^{2/7}$, and the solid line Nu $= 0.0508$Ra$^{1/3}$.

The presented Nu data are obtained with densities from 0.04 kg/m^3 to 37 kg/m^3 and 50 mK $\leq \Delta T \leq$ 1.7 K [16]. The compensated plot in Fig. 2 shows that, without applying any wall corrections, for Ra $\lesssim 10^{11}$ we obtain Nu $\propto$ Ra^γ with the power exponent $\gamma \approx 2/7$. For higher Ra the observed Nu(Ra) dependence is consistent with a power law with $\gamma \approx 1/3$. This crossover in γ takes place together with slight increase in Pr (see Fig. 3). Note that for Pr = 1 the viscous boundary layer and the thermal layer are equally thick.

The reason for showing the uncorrected data is twofold. (i) As already mentioned, our cell is designed to minimize sidewall and parasitic heat leak corrections; we do not expect any significant corrections due to finite conductivity of plates [17,18], and the measuring protocol is chosen to stay sufficiently away from the critical point. (ii) The quantitatively correct form for all these corrections, due to the complexity of the problem, is generally not known.

In order to compare our data with cryogenic experiments in cylindrical cells in Trieste ($\Gamma = 1$) [9,10] and Grenoble ($\Gamma = 1.14$) [7], Fig. 2 includes our $\mathrm{NuRa}^{-1/3}$ corrected with respect to sidewall effects. As suggested by Roche *et al.* [19] and numerically tested by Verzicco [20], one way to estimate the influence of the sidewall is via the wall parameter $W = 2\lambda_w \delta \lambda^{-1} R^{-1}$ [19]. For our cell

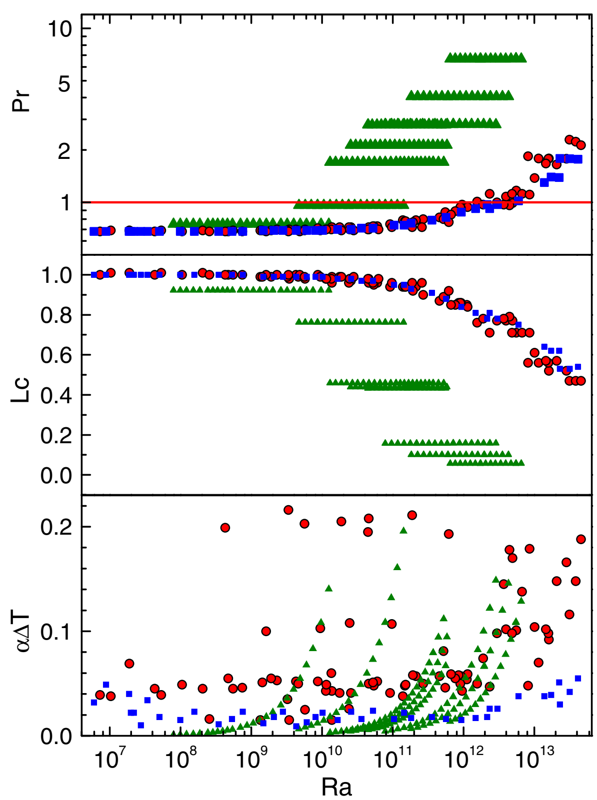

FIG. 3 (color online). Comparison of characteristic working fluid parameters for all experimental data points of three discussed cryogenic experiments (symbols correspond to those in Fig. 2): Prandtl number, the distance L_c from the critical point (T_c, p_c) and parameter $\alpha \Delta T$ plotted versus Ra.

$0.22 > W > 0.15$ (depending on the actual value of λ for each data point), while estimated values for the Grenoble ($0.66 > W > 0.35$) and Oregon-Trieste ($0.78 > W > 0.40$ based on $\delta = 3$ mm) cells are higher, leading to larger sidewall corrections. Figure 2 shows that at low Ra $< 5 \times 10^{10}$ our corrected data [21] agree well with the Grenoble data corrected the same way. Having established that, we have to ask why these two corrected data sets lay substantially lower than the $\Gamma = 1$ Trieste data [9], despite all these sets for Ra $< 5 \times 10^{10}$ displaying very closely the same scaling exponent?

Figure 2 shows that for $10^8 \lesssim$ Ra $\lesssim 10^{11}$ all three data sets collapse when applying the same sidewall correction to the Trieste data, but based on an effective sidewall thickness $\delta_{\mathrm{eff}} = 6$ mm rather than on the nominal thickness $\delta = 3$ mm. This is very plausible result, as the Oregon-Trieste cell sidewall includes thick stainless steel rings making both upper and lower ends of the sidewall much thicker. The 3 mm stainless steel wall is welded to these rings that in turn are mounted via an In "O" ring to the inner surfaces of top and bottom copper plates. The effective wall thickness close the copper plates, i.e., where it matters most, is therefore substantially larger. The boundary layer [of thickness $\simeq L/(2\mathrm{Nu})$] is for most of the covered Ra range much thinner than the height of the stainless steel ring. However, it is about equally thick for the low end of Ra range, where quantitative estimates of the wall correction via a single value of δ_{eff} ought to fail as such a correction becomes too large. This most probably accounts for the difference between our and Trieste corrected low Ra data encircled in Fig. 2 [22].

Assuming that all the controversies among three different sets of the $\Gamma \simeq 1$ cryogenic data at relatively low Ra are resolved this way, it is striking that all of them display a change in γ at about Ra $\approx 10^{11}$ and differ distinctly above this value. Figure 2 clearly shows that while the Trieste and Grenoble data plotted as $\mathrm{NuRa}^{-1/3}$ increase with Ra above Ra $\approx 10^{11}$, our data (both corrected and uncorrected) do not appreciably deviate from the 1/3 power law up to Ra $= 4.6 \times 10^{13}$ (see also [9]).

As discussed intensely in the literature (for a review, see [1]) for fixed Ra the observed Nu may depend on Pr. Indeed, this is why an effort has been made at ambient temperature to reach high Ra using a series of working fluids of approximately constant Pr $\simeq 0.7-0.8$: He, N_2, and SF_6 [12]; these experiments of Funfschilling, Bodenschatz, and Ahlers indicate neither a transition to an ultimate regime with $\gamma \approx 1/2$ nor any other distinct change in scaling around Ra $= 10^{11}$. Rather surprisingly, they observed low values of $\gamma = 0.25$ above Ra $= 4 \times 10^{13}$ [12] and $\gamma = 0.17$ above Ra $= 3 \times 10^{14}$ [11]. In later experiments, they showed a transition to another regime ($\gamma = 0.355$) above Ra $\approx 10^{13}$ and revealed correlation of the diverse regimes with a change of the temperature external to the cell.

Figure 3 compares Pr as a function of Ra from all three cryogenic experiments. We have obtained the Nu(Ra) data for $0.67 < \mathrm{Pr} \leq 1$ up to $\mathrm{Ra} \approx 3 \times 10^{12}$; above this, Pr spans from 1 to 2.4. For $\mathrm{Ra} > 10^{10}$ our Pr are lower than those of Grenoble, equal or slightly higher than those of the Trieste $\Gamma = 1$ experiment [9]. However, the observed Nu (Ra) scaling exponent is appreciably steeper here for the Trieste data and much more so for the Grenoble data. It seems improbable, therefore, to attribute the different $\mathrm{Nu}(\mathrm{Ra} > 10^{11})$ scaling in these cryogenic experiments to the difference in Pr.

In the close vicinity of the critical point the properties of helium vary significantly and might influence the results. The dimensionless distance from the critical point could conveniently be defined as [10]

$$L_c = \left[\left(\frac{\rho - \rho_c}{\rho_c}\right)^2 + \left(\frac{T - T_c}{T_c}\right)^2\right]^{1/2}. \tag{2}$$

Figure 3 shows this quantity for all these three cryogenic experiments. Note that for our data $L_c > 0.45$; thus the reasoning developed to explain the recently observed transition displayed by the recent Trieste data at somewhat higher Ra via parameter ξ [10] cannot be applied here. Figure 3 also compares the "Boussinesq parameter" $\alpha \Delta T$ for these cryogenic experiments. For all our data in Fig. 2, $\alpha \Delta T \lesssim 0.2$; for most of them, $\alpha \Delta T < 0.1$ [23].

In conclusion, being motivated by contradictory results of various RBC experiments, we have remeasured the Nu (Ra) dependence for $7.2 \times 10^6 \leq \mathrm{Ra} \leq 4.6 \times 10^{13}$ at $0.67 < \mathrm{Pr} < 2.4$ using a cell designed to minimize the influence of its structure on the studied convective flow [13]. High Ra are attained with cryogenic helium gas sufficiently far away from its critical point. The measured Nu (both corrected and uncorrected values) obey, at least approximately, Nu(Ra) power law scaling with exponent $\gamma \approx 2/7$ in the region $7.2 \times 10^6 \leq \mathrm{Ra} \lesssim 10^9$ where $\mathrm{Pr} < 1$; within the next 2–3 decades of Ra the power exponent slowly increases and approaches $\gamma \approx 1/3$ which, on slightly increasing Pr, holds approximately up to $\mathrm{Ra} \approx 4.6 \times 10^{13}$. By applying suitable sidewall corrections, we show full agreement among $\Gamma \simeq 1$ cryogenic experiments for Ra up to about 10^{11}, while at higher Ra all these sets of data differ considerably. We claim that distinctly different Nu(Ra) scaling as observed in various high Ra experiments [1,6–9,11,12] can hardly be explained by the difference in Pr. Other tiny experimental details such as influence of the parasitic heat leak, additional corrections to finite heat conductivity and heat capacity of plates (including perhaps adjacent parts of sidewall) [11] and/or rather more physical reasons such as various quantitative measures of Boussinesq conditions in virtue of [10] will have to be carefully considered. We aim to use our apparatus for detailed investigations of heat transport by turbulent convection up to $\mathrm{Ra} \approx 10^{15}$, having in mind peculiar transitions and bimodality as reported in [10,11]. We hope that the results presented here will stimulate further effort leading to better understanding of turbulent buoyancy-driven flows.

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[15] The test at $T_m \approx 10$ K reveal a spurious increase in Nu that could be explained by convection inside the filling tubes between the cell inlet (at $\approx T_m$) and the thermal anchor at the LHe vessel 140 mm above.
[16] Correction to the adiabatic temperature gradient is less than 1 mK and thus for $\Delta T > 50$ mK it hardly matters.
[17] Criterion due to finite λ_p [18] for the discussed experiments yields for our cell the lowest correction in Nu.
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[22] Mylar foil on the inner sidewall surface ([9], App. B) of heat conductivity $\lambda_M(5\ \mathrm{K}) \simeq 0.01\ \mathrm{W\,m^{-1}K^{-1}}$ (according to the Cryocomp database), comparable with that of helium, cannot alter the sidewall correction substantially.
[23] Several test measurements at $\alpha \Delta T \approx 0.3$ resulted in a slight ($\approx 2\%$) decrease in $\mathrm{NuRa}^{-1/3}$ for $\mathrm{Ra} < 3 \times 10^{11}$.