

## Correlation between $\alpha$ -Decay Energies of Superheavy Nuclei Involving the Effects of Symmetry Energy

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A formula for the relationship between the  $\alpha$ -decay energies ( $Q$  values) of superheavy nuclei (SHN) is presented, which is composed of the effects of Coulomb energy and symmetry energy. It can be employed not only to validate the experimental observations and measurements to a large extent, but also to predict the  $Q$  values of heaviest SHN with a high accuracy generally which will be very useful for future experiments. Furthermore, the shell closures in superheavy region and the effect of the symmetry energy on the stability of SHN against  $\alpha$  decay are discussed with the help of this formula.

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The synthesis and identification of superheavy nuclei (SHN) have been receiving worldwide attention since the prediction of the existence of a superheavy island in the 1960s. But where the closed shells are located in the superheavy region is less certain, depending on the model employed. The experimental investigations are thus crucial and a series of experimental efforts so far have been focused on the direct production of SHN in heavy ion fusion reactions. The superheavy elements with  $Z = 107$ – $112$  have been successfully produced at GSI, Darmstadt, in cold-fusion reactions [1]. Several new elements with  $Z = 113$ – $118$  have been discovered at JINR-FLNR, Dubna, using hot-fusion evaporation reactions with the neutron-rich  $^{48}\text{Ca}$  beam and actinide targets [2]. The element 114 was independently confirmed recently by the LBNL in the USA [3] and GSI [4]. A superheavy element isotope  $^{285}114$  was observed in LBNL last year [5], and an isotope of  $Z = 113$  has been identified at RIKEN, Japan [6]. Thus up to now superheavy elements with  $Z = 104$ – $118$  have been synthesized in experiment and hence it offers the possibility to study the heaviest known nuclear island of stability with greater detail.

Superheavy nuclei with atomic numbers beyond 110 predominantly undergo sequential  $\alpha$  decay terminated by spontaneous fission [1], leading to  $\alpha$  decay that is one efficient approach to identify new nucleus via the observation of  $\alpha$ -decay chain and to extract some information about their stability. In experiment one usually measures the  $\alpha$ -decay  $Q$  values and half-lives, while one of the major goals of theory is to predict the half-lives to serve the experimental design. As one of the crucial quantity for a quantitative prediction of decay half-life,  $Q$  value strongly affects the calculation of the half-life due to the exponential law. Therefore, it is extremely important and necessary to obtain an accurate theoretical  $Q$  value in a reliable half-life prediction during the experiment design. However, the

existing microscopic nuclear many-body approaches do not achieve a very good accuracy. In this study, we propose a new approach to calculate the  $\alpha$ -decay energy with a high accuracy for the superheavy elements above 110. In our previous work [7], a formula was proposed for  $\alpha$ -decay  $Q$  value of SHN based on a liquid drop model. Taking no account of the shell energy it gives as

$$Q(\text{MeV}) = aZA^{-4/3}(3A - Z) + b\left(\frac{N - Z}{A}\right)^2 + e, \quad (1)$$

with  $a = 4a_c/3 = 0.9373$ ,  $b = -4a_{\text{sym}} = -99.3027$  and  $e = -27.4530$  [7]. Here,  $Z$ ,  $N$ , and  $A$  are the proton, neutron, and mass numbers of the parent nuclei, respectively. The first two terms on the right-hand side are the contributions of Coulomb energy and symmetry energy, respectively. The nuclear symmetry energy plays an important role in astrophysics [8,9], the structure of exotic nuclei and the dynamics of heavy ion reactions [10–12]. In this Letter, the effect of symmetry energy on the stability of SHN against  $\alpha$  decay is going to be shown.

Here we study the relationship between the  $Q$  values of the neighboring SHN taking Eq. (1) as the starting point but we do not use the parameters in Ref. [7] any longer. With  $\beta = (N - Z)/A$  denoting the isospin asymmetry and  $Z = A(1 - \beta)/2$ , we obtain

$$\frac{Q_2 - Q_1}{\beta_2 - \beta_1} \approx \frac{\partial Q}{\partial \beta} = -\frac{2}{3}a_cA^{2/3}(\beta + 2) - 8a_{\text{sym}}\beta. \quad (2)$$

Once the decay energy  $Q_1$  of a reference nucleus  $^AZ_1$  is known, the  $Q_2$  values of the other nucleus  $^AZ_2$  (target nucleus) with the same mass number  $A$  can be estimated by

$$Q_2 = Q_1 - (\beta_2 - \beta_1) \left[ \frac{2}{3}a_cA^{2/3}(\beta + 2) + 8a_{\text{sym}}\beta \right], \quad (3)$$

with  $\beta = (\beta_1 + \beta_2)/2$  and  $a_c = 0.71$ . The mass dependence of the symmetry energy coefficient is given by

Danielewicz and Lee [13] as  $a_{\text{sym}} = c_{\text{sym}}(1 + \kappa A^{-1/3})^{-1}$ , where  $c_{\text{sym}}$  is the volume symmetry energy coefficient of the nuclei and  $\kappa$  is the ratio of the surface symmetry coefficient to the volume symmetry coefficient. Here  $c_{\text{sym}} = 31.1$  and  $\kappa = 2.31$  are taken from the results of Ref. [14] without including the uncertainty.

Apart from  $(A, \beta)$  discussed above,  $(Z, \beta)$  or  $(N, \beta)$  can be also adopted as variables. By an analogous derivation, the correlation between the  $Q$  values of the nuclei belonging to an isotope chain with a proton number  $Z$  is given by

$$Q_2 = Q_1 - (\beta_2 - \beta_1) \times \left[ \frac{2^{5/3}}{9} a_c Z^{2/3} (1 - \beta)^{-2/3} (1 + 2\beta) + 8a_{\text{sym}}\beta \right] \quad (4)$$

and that of the nuclei belonging to an isotone chain with a neutron number  $N$  is given by

$$Q_2 = Q_1 - (\beta_2 - \beta_1) \times \left[ \frac{2^{5/3}}{9} a_c N^{2/3} (1 + \beta)^{-5/3} (11 + 5\beta + 2\beta^2) + 8a_{\text{sym}}\beta \right] \quad (5)$$

In general, if one selects  $\xi = xZ + yN$  and  $\beta$  as variables, the relationship between the  $Q$  values of  $\alpha$  decay can be written as

$$Q_2 = Q_1 - (\beta_2 - \beta_1) \left\{ \frac{2^{5/3}}{9} a_c \xi^{2/3} [(1 - \beta)x + (1 + \beta)y]^{-5/3} \times [(1 + \beta - 2\beta^2)x + (11 + 5\beta + 2\beta^2)y] + 8a_{\text{sym}}\beta \right\} \quad (6)$$

where  $x$  and  $y$  are integers and  $|x|^2 + |y|^2 \neq 0$  with  $Z = (1 - \beta)\xi / [(1 - \beta)x + (1 + \beta)y]$  and  $N = (1 + \beta)\xi / [(1 - \beta)x + (1 + \beta)y]$ . Here only the differences of the symmetry energy effect ( $a_{\text{sym}}$  term) together with the differences of Coulomb energy effect ( $a_c$  term) between a reference nucleus and a target one contribute to this correlation. The isospin dependence of the symmetry energy coefficient  $a_{\text{sym}}$  is neglected here because the  $a_{\text{sym}}$  changes quite slightly between the neighboring nuclei. With this formula, the  $Q$  values of target nuclei can be obtained by any neighboring nuclei. Equations (3)–(5) are the special cases of Eq. (6)—that is,  $x = y = 1$  for Eq. (3),  $x = 1, y = 0$  for Eq. (4) and  $x = 0, y = 1$  for Eq. (5), respectively.

In order to test the applicability of Eq. (6), we compute the  $Q$  values of recently synthesized heaviest SHN with the help of their neighbors, and the results are listed in Fig. 1 compared with experimental ones. The results obtained with Eqs. (3)–(5) are marked by distinguishable symbols. For the nuclei except  $^{294}\text{118}$ ,  $^{290}\text{115}$ ,  $^{282}\text{113}$  and  $^{280}\text{111}$ , our approach reproduces the measured values quite accurately

with a root-mean-square deviation  $\sqrt{\langle \sigma^2 \rangle} = 0.077$  MeV and an average deviation  $\langle \sigma \rangle = 0.064$  MeV for central values from 380 reference-target combinations. It is thus very practical that Eq. (6) can be reliably applied to the  $Q$  values of the as-yet-unobserved SHN with the help of known nuclei which is the most effective method to the  $Q$  values at present. As three simple cases of Eq. (6), Eqs. (3)–(5) work even better with  $\sqrt{\langle \sigma^2 \rangle} = 0.052$  MeV and  $\langle \sigma \rangle = 0.043$  MeV from 96 reference-target combinations, which are very convenient to be used and are sufficient for predictions of  $Q$  values generally, though they are simple in formalism. In addition, the agreement between the experimental and theoretical values has additional significance. Since the  $Q$  values of the reference nuclei are taken from the experimental measurements in calculations, the agreement suggests that the experimental data themselves are consistent with each other, which indicates that the experimental observations and measurements of the SHN are reliable to a great extent. These SHN still await independent verification by other laboratories, which is not easy because the new SHN form an isolated island that tends to be not linked through  $\alpha$ -decay chains with any known nuclei, making the theoretical supports become important and necessary. For the nuclide  $^{290}\text{115}$ , the experimental value of  $10.14 \pm 0.41$  MeV carries a large uncertainty while the  $Q$  value is about 10.4 MeV according to Eq. (6), which requires a more precise experimental measurement.

The shell closures should play a particular important role in the superheavy system. However, modern theoretical approaches disagree on the position of the closed shells. For instance, the macroscopic-microscopic models with various parameterizations predict the shell gaps at  $Z = 114$  and  $N = 184$  [15,16]. Skyrme-Hartree-Fock calculations favor  $Z = 124, 126$  and  $N = 184$  [17,18] while the relativistic mean field models favor  $Z = 120, N = 172$  [18–20] and  $Z = 120, N = 184$  [21]. The magic numbers  $Z = 132$  and  $N = 194$  were predicted from the discontinuity of the volume integral at shell closures [22]. The reason for this uncertainty lies in incomplete knowledge of the nuclear force and the difficulty of many-body techniques. It is well known that the shell effect on the  $\alpha$  radioactivity is related to the  $Q$  value. For the  $\alpha$ -decay of the nuclei being not close to the shell closures, due to a parent nucleus and its daughter nucleus sharing the same oddity of both the proton and neutron numbers, the shell correction (also pairing correction) energies to their masses could be canceled to a large extent leading to a small correction to a  $Q$  value compared with the contributions of the Coulomb and symmetry energies within semiempirical formulas [7], and even these small shell energies to the  $Q$  values turn out to be nearly a constant in a local region of particle numbers which confirms that the shell energies hardly take effect in Eq. (6). Most importantly, the agreements between the estimated and experimental results in

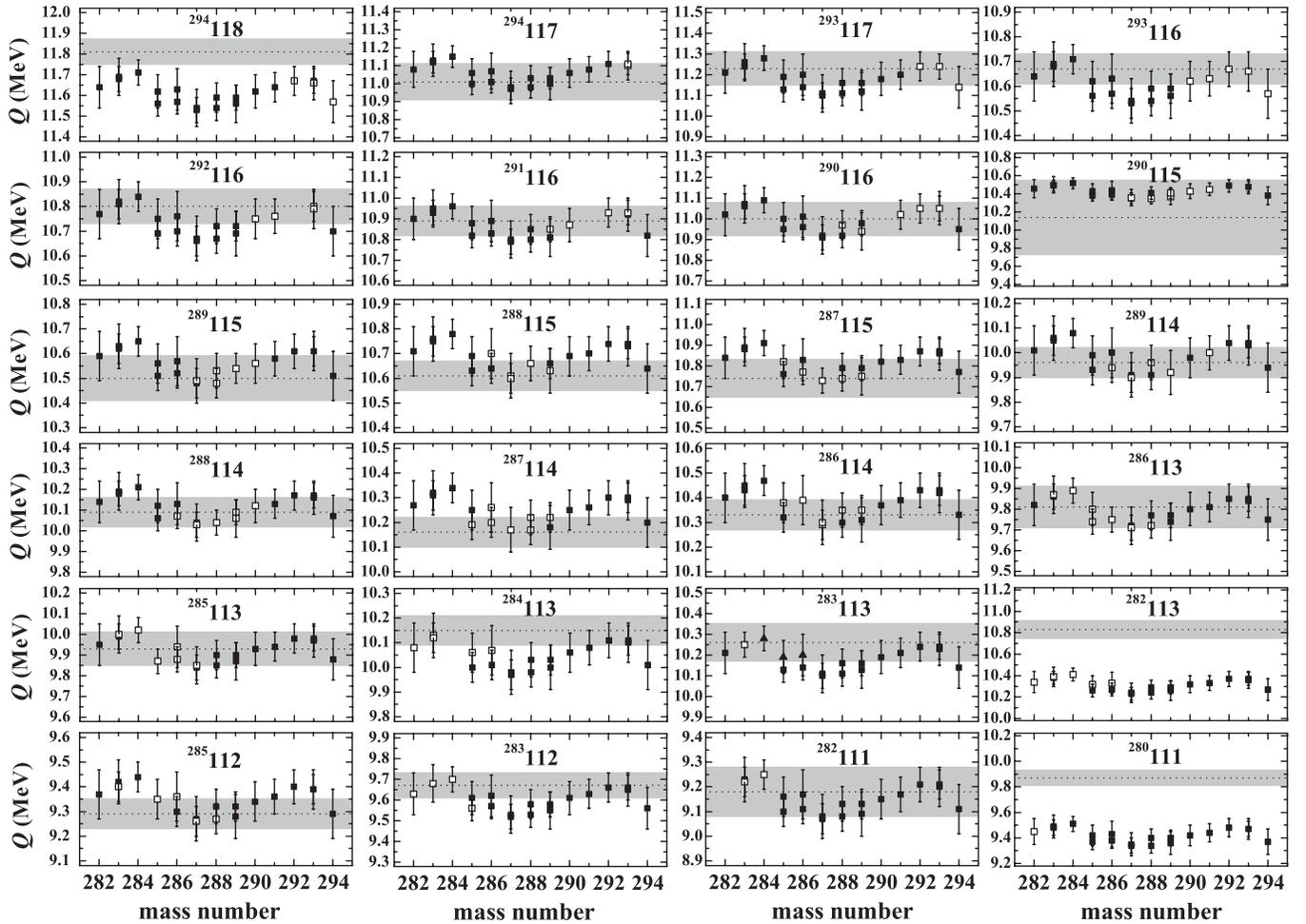


FIG. 1. Comparison of  $\alpha$ -decay  $Q$  values with Eq. (6) (the rectangles with error bars) and experimental ones [2] (shaded area) of recently synthesized heaviest SHN. The horizontal ordinate denotes the mass numbers of the reference nuclei. Since the experimental  $Q$  values of the reference nuclei include uncertainties, the calculated ones also display error bars. The results from Eqs. (3)–(5) which are special cases of Eq. (6), are presented separately marked by hollow rectangles.

turn show this point. Once the parent nucleus or the daughter one has neutron and/or proton magic numbers or the shell gaps are crossed, the  $Q$  value shows an irregular behavior. Since the shell energy is excluded in Eq. (6), it should show some discrepancies for nuclei around shell closures. Yet, it would help us to investigate the shell structure by comparing the experimental and calculated  $Q$  values. All the theoretical calculated  $Q$  values of  $^{294}\text{118}$ ,  $^{282}\text{113}$  and  $^{280}\text{111}$  based on Eq. (6) are lower than the experimental ones, which is possibly attributed to the likely shell gaps at  $Z = 120$  for  $^{294}\text{118}$ , and at  $N = 166$  for  $^{282}\text{113}$  and  $^{280}\text{111}$ . In Ref. [23], it is suggested that  $N = 166$  is a neutron shell gap in a certain region within relativistic mean field models. Yet, to confirm the existence of shell gaps positively is not easy due to the insufficient experimental observations. Apart from the shell effects, dramatic shape changes could also affect the energy of  $\alpha$  decay, as pointed out in Ref. [24]. Nevertheless, the deviations are not more than 0.5 MeV in general. However, it should be much easier to confirm the nonexistence of

shell closures. Once the  $Q$  values behave quite regularly in a local range, a magic number should not appear here. For the eight nuclides of elements 116 and 114 ( $^{290-293}\text{116}$  and  $^{286-289}\text{114}$ ) together with the six nuclei with a neutron number  $N = 174$  ( $^{290}\text{116}$ ,  $^{289}\text{115}$  and  $^{288}\text{114}$ ) and  $N = 172$  ( $^{287}\text{115}$ ,  $^{286}\text{114}$  and  $^{285}\text{113}$ ), the experimental  $Q$  values can be reproduced very accurately that confirms  $Z = 114$  and  $N = 172$  are not shell closures in the considered region. Of course, one cannot rule out the possibility that they appear as magic numbers in other mass regions.

We now turn to the effect of the symmetry energy on  $\alpha$  decay. The  $a_{\text{sym}}$  term (difference of the symmetry energy effect between the reference and target nuclei) contributes by about 45% in Eq. (3), 80% in Eq. (4) and 35% in Eq. (5) to the  $\Delta Q = Q_2 - Q_1$  for the SHN in Fig. 1, suggesting its important role in the correlations between the  $Q$  values of SHN. The large contribution of  $a_{\text{sym}}$  term in Eq. (4) is of particular importance as discussed below. The most significant experimental conclusion is these observed

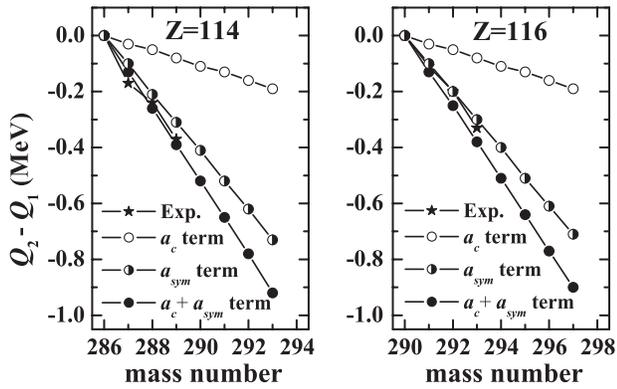


FIG. 2. Contributions of the  $a_{\text{sym}}$  and  $a_c$  terms to  $Q_2 - Q_1$  in Eq. (4) taking the elements 114 and 116 as examples ( $^{286}114$  and  $^{290}116$  as reference nuclei with decay energies  $Q_1$ , respectively). The experimental data, if available, are also shown for comparison.

superheavy elements generally display a trend of increased stability with larger neutron number, which is almost attributed to the larger symmetry energy that lowers the  $Q$  values. In order to illuminate this conclusion more obviously, we plot in Fig. 2 the contributions of the  $a_{\text{sym}}$  and  $a_c$  terms to  $Q_2 - Q_1$  in Eq. (4). One can find that the  $a_{\text{sym}}$  term contributes much more greatly than the  $a_c$  term. Therefore, due to the inclusion of the effect of symmetry energy, apart from the theoretical estimations being able to agree with the experimental values, the  $Q$  values reduce much more rapidly as  $N$  increases, and hence a superheavy element becomes longer-lived against  $\alpha$  decay with increasing  $N$ . In other words, it is the symmetry energy that primarily enhances the stability against  $\alpha$  decay with larger neutron number for these synthesized SHN not around shell closures.

We have investigated some aspects of the  $\alpha$  decay of SHN. The main conclusions are summarized as follows: (i) A simple formula for the correlation between the  $\alpha$  decay  $Q$  values of the SHN has been proposed, which works very well for an estimation of the  $\alpha$ -decay energies of the recently synthesized SHN. They thus allow us to reliably predict the  $Q$  values of the still unknown SHN with a good accuracy, and is going to be very useful for the future experiment design. Also, the agreements between the calculated and experimental values indicate the reliability of the experimental observations and measurements on these synthesized SHN to a great extent. (ii)  $Z = 114$  and  $N = 172$  turn out to be not shell closures for the presently observed superheavy region experimentally. (iii) The observed increase of  $\alpha$ -decay half-lives with increasing neutron number, i.e., the increased stability of these SHN not around shell closures with larger neutron number, is primarily attributed to the effect of the symmetry energy.

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- [1] S. Hofmann and G. Münzenberg, *Rev. Mod. Phys.* **72**, 733 (2000), and references therein; S. Hofmann *et al.*, *Nucl. Phys.* **A734**, 93 (2004).
- [2] Yu. Ts. Oganessian *et al.*, *Phys. Rev. C* **69**, 021601(R) (2004); *Phys. Rev. C* **70**, 064609 (2004); *Phys. Rev. C* **72**, 034611 (2005); *Phys. Rev. C* **74**, 044602 (2006); *Phys. Rev. C* **76**, 011601(R) (2007); *Phys. Rev. Lett.* **104**, 142502 (2010).
- [3] L. Stavsetra *et al.*, *Phys. Rev. Lett.* **103**, 132502 (2009).
- [4] Ch. E. Düllmann *et al.*, *Phys. Rev. Lett.* **104**, 252701 (2010).
- [5] P. A. Ellison *et al.*, *Phys. Rev. Lett.* **105**, 182701 (2010).
- [6] K. Morita *et al.*, *J. Phys. Soc. Jpn.* **73**, 2593 (2004); K. Morita *et al.*, *J. Phys. Soc. Jpn.* **76**, 045001 (2007).
- [7] Jianmin Dong *et al.*, *Phys. Rev. C* **81**, 064309 (2010).
- [8] J. M. Lattimer and M. Prakash, *Phys. Rep.* **333–334**, 121 (2000); *Science* **304**, 536 (2004).
- [9] A. W. Steiner, M. Prakash, J. M. Lattimer, and P. J. Ellis, *Phys. Rep.* **411**, 325 (2005).
- [10] V. Baran, M. Colonna, V. Greco, and M. Di Toro, *Phys. Rep.* **410**, 335 (2005).
- [11] P. Danielewicz, R. Lacey, and W. G. Lynch, *Science* **298**, 1592 (2002).
- [12] B. A. Li, L. W. Chen, and C. M. Ko, *Phys. Rep.* **464**, 113 (2008).
- [13] P. Danielewicz and J. Lee, *Nucl. Phys.* **A818**, 36 (2009).
- [14] Min Liu *et al.*, *Phys. Rev. C* **82**, 064306 (2010).
- [15] P. Möller and J. R. Nix, *J. Phys. G* **20**, 1681 (1994).
- [16] A. Baran *et al.*, *Phys. Rev. C* **72**, 044310 (2005).
- [17] S. Ćwiok, J. Dobaczewski, P.-H. Heenen, P. Magierski, and W. Nazarewicz, *Nucl. Phys.* **A611**, 211 (1996).
- [18] A. T. Kruppa, M. Bender, W. Nazarewicz, P.-G. Reinhard, T. Vertse, and S. Ćwiok, *Phys. Rev. C* **61**, 034313 (2000).
- [19] M. Bender, K. Rutz, P.-G. Reinhard, J. A. Maruhn, and W. Greiner, *Phys. Rev. C* **60**, 034304 (1999).
- [20] K. Rutz, M. Bender, T. Bürvenich, T. Schilling, P.-G. Reinhard, J. A. Maruhn, and W. Greiner, *Phys. Rev. C* **56**, 238 (1997).
- [21] S. K. Patra, C.-L. Wu, C. R. Praharaaj, and R. K. Gupta, *Nucl. Phys.* **A651**, 117 (1999).
- [22] P. Mohr, *Phys. Rev. C* **73**, 031301(R) (2006).
- [23] G. A. Lalazissis, M. M. Sharma, P. Ring, and Y. K. Gambhir, *Nucl. Phys.* **A608**, 202 (1996).
- [24] S. Ćwiok, P.-H. Heenen, and W. Nazarewicz, *Nature (London)* **433**, 705 (2005).