First Glances at the Transversity Parton Distribution through Dihadron Fragmentation Functions

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We present first observations of the transversity parton distribution based on an analysis of pion-pair production in deep-inelastic scattering off transversely polarized targets. The extraction of transversity relies on the knowledge of dihadron fragmentation functions, which we take from electron-positron annihilation measurements. This is the first attempt to determine the transversity distribution in the framework of collinear factorization.

The distribution of quarks and gluons inside hadrons can be described by means of parton distribution functions (PDFs). In a parton-model picture, PDFs describe combinations of number densities of quarks and gluons in a fast-moving hadron. The knowledge of PDFs is crucial for our understanding of quantum chromodynamics (QCD) and for the interpretation of high-energy experiments involving hadrons.

If parton transverse momentum is integrated over, in the Bjorken limit the partonic structure of the nucleon is described in terms of only three PDFs: the well-known unpolarized, $f_1^q(x)$, and helicity, $g_1^q(x)$, distribution functions, and the transversity distribution function $h_1^q(x)$, which measures the transverse polarization of quarks with flavor q and fractional momentum x in a transversely polarized nucleon [1]. Intuitively, helicity and transversity give two orthogonal pictures of the partonic structure of polarized nucleons. They have very different properties, and transversity is much less known. In this work, we present an extraction of the h_1^q.

Transversity is related to the interference of amplitudes with different helicities of partons and of the parent nucleon. In jargon, it is called a chiral-odd function. There is no transversity for gluons in a nucleon, and h_1^q has a pure nonsinglet scale evolution [2]. From transversity one can build the nucleon tensor charge, which is odd under charge conjugation and can be computed in lattice QCD [3] (for a review on transversity, see Ref. [4] and references therein).

Transversity is particularly difficult to measure because it must appear in cross sections combined with another chiral-odd function. An example is the cross section for single-particle inclusive deep inelastic scattering (DIS), where h_1^q appears in a convolution with the chiral-odd Collins fragmentation function $H_1^{\perp q}$ [5], which describes the correlation between the transverse polarization of a fragmenting quark with flavor q and the transverse momentum distribution of the detected unpolarized hadron. The convolution $h_1^q \otimes H_1^{\perp q}$ gives rise to a specific azimuthal modulation of the cross section. The amplitude of the modulation has been measured by the HERMES and COMPASS Collaborations [6]. In order to extract the transversity distribution from this signal, the Collins function should be determined through the measurement of azimuthal asymmetries in the distribution of two almost back-to-back hadrons in e^+e^- annihilation [7]. The Belle Collaboration has measured this asymmetry [8], making the first-ever extraction of h_1^q possible from a simultaneous analysis of $ep^\uparrow \to e'\pi X$ and $e^+e^- \to \pi\pi X$ data [9].

In spite of this achievement, some questions still hinder the extraction of transversity from single-particle-inclusive measurements. The most crucial issue is the treatment of evolution effects, since the measurements were performed at very different energies. The convolution $h_1^q \otimes H_1^{\perp q}$ involves the transverse momentum of quarks. Hence, its evolution should be described in the framework of the transverse-momentum-dependent factorization [10]. Quantitative explorations in this direction suggest that neglecting evolution effects could lead to overestimating transversity [11].

In this context, it is of paramount importance to extract transversity in an independent way, requiring only standard collinear factorization where the above complications are absent (see, e.g., Refs. [12] and references therein). Here, we come for the first time to this result by considering the semi-inclusive deep-inelastic production of two hadrons with small invariant mass.

In this case, the transversity distribution function is combined with a chiral-odd dihadron fragmentation function (DiFF), denoted as $H_1^{\sphericalangle q}$ [13], which describes the correlation between the transverse polarization of the fragmenting quark with flavor q and the azimuthal orientation of the plane containing the momenta of the detected hadron pair. Contrary to the Collins mechanism, this effect survives after integration over quark transverse momenta and can be analyzed in the framework of collinear factorization. This process has been studied from different

perspectives in a number of papers [13–16]. The only published measurement of the relevant asymmetry has been presented by the HERMES Collaboration for the production of $\pi^+\pi^-$ pairs on transversely polarized protons [17]. Preliminary measurements have been presented by the COMPASS Collaboration [18]. Related preliminary measurements in proton-proton scattering have been presented by the PHENIX Collaboration [19].

Similarly to the single-hadron case, we need to independently determine $H_1^{\sphericalangle q}$ by looking at correlations between the azimuthal orientations of two pion pairs in back-to-back jets in e^+e^- annihilation [20,21]. The measurement of this so-called Artru-Collins azimuthal asymmetry has recently become possible thanks to the Belle Collaboration [22].

In the HERMES publication [17], the asymmetry was denoted as $A_{UT}^{\sin(\phi_R+\phi_S)\sin\theta}$; for brevity and without ambiguity, here we will use the notation A_{DIS}. The data set was collected in bins of the variables x (the momentum fraction of the initial quark), z (the fractional energy carried by the $\pi^+\pi^-$ pair), and M_h (the invariant mass of the pair). Since our interest here lies mainly on the transversity distribution, and to avoid problems when dealing with three different projections of the same data set, we consider only the x binning. In Table I, we reproduce the data for convenience indicating for each bin also the average hard scale $\langle Q^2\rangle$ and fractional beam energy loss $\langle y\rangle$.

The A_{DIS} measured by HERMES [17] can be interpreted as [24]

$$A_{DIS}(x, Q^2) = -C_y \frac{\sum_q e_q^2 h_1^q(x, Q^2) n_q^\uparrow(Q^2)}{\sum_q e_q^2 f_1^q(x, Q^2) n_q(Q^2)}, \quad (1)$$

where (neglecting target-mass corrections)

$$C_y = \frac{\langle 1-y\rangle}{\langle 1-y+y^2/2\rangle} \approx \frac{1-\langle y\rangle}{1-\langle y\rangle+\langle y\rangle^2/2}. \quad (2)$$

In Eq. (1), we also introduced the following quantities

$$n_q(Q^2) = \int dz\, dM_h^2\, D_1^{q\to\pi^+\pi^-}(z, M_h^2, Q^2),$$
$$n_q^\uparrow(Q^2) = \int dz\, dM_h^2\, \frac{|R|}{M_h} H_{1,sp}^{\sphericalangle q\to\pi^+\pi^-}(z, M_h^2, Q^2), \quad (3)$$

TABLE I. Semi-inclusive DIS data of the asymmetry A_{DIS} from HERMES [17]. The errors are mainly statistical (we added the systematic errors in quadrature). The average values of the variables are taken from Table 5.1 of Ref. [23]. The other variables have been integrated in the range $0.5 \le M_h \le 1$ GeV and $0.2 \le z \le 1$.

Bin boundaries	$\langle x\rangle$	$\langle y\rangle$	$\langle Q^2\rangle$ (GeV^2)	A_{DIS}
$0.023 < x < 0.040$	0.033	0.734	1.232	0.015 ± 0.010
$0.040 < x < 0.055$	0.047	0.659	1.604	0.002 ± 0.011
$0.055 < x < 0.085$	0.068	0.630	2.214	0.035 ± 0.011
$0.085 < x < 0.400$	0.133	0.592	4.031	0.020 ± 0.010

with $|R|/M_h = \sqrt{1/4 - m_\pi^2/M_h^2}$. $D_1^{q\to\pi^+\pi^-}$ is the unpolarized DiFF describing the hadronization of a quark q into a $\pi^+\pi^-$ pair plus any number of undetected hadrons, averaged over quark polarization and pair orientation. Finally, $H_{1,sp}^{\sphericalangle q\to\pi^+\pi^-}$ is a chiral-odd DiFF, and denotes the component of $H_1^{\sphericalangle q\to\pi^+\pi^-}$ that is sensitive to the interference between the fragmentation amplitudes into pion pairs in relative s wave and in relative p wave, from which comes the common name of interference fragmentation functions [15]. Intuitively, if the fragmenting quark is moving along the $\hat{z}$ direction and is polarized along $\hat{y}$, a positive $H_{1,sp}^{\sphericalangle q\to\pi^+\pi^-}$ means that π^+ is preferentially emitted along $-\hat{x}$ and π^- along $\hat{x}$. Since in this case no ambiguities arise, in the following we shall conveniently simplify the notation by using D_1^q and $H_1^{\sphericalangle q}$ to denote the relevant DiFFs.

In our analysis, we make the following assumptions (valid only for $\pi^+\pi^-$ pairs) based on isospin symmetry and charge conjugation [24]:

$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}}, \quad (4)$$

$$D_1^s = D_1^{\bar{s}}, \qquad D_1^c = D_1^{\bar{c}}, \quad (5)$$

$$H_1^{\sphericalangle u} = -H_1^{\sphericalangle d} = -H_1^{\sphericalangle \bar{u}} = H_1^{\sphericalangle \bar{d}}, \quad (6)$$

$$H_1^{\sphericalangle s} = -H_1^{\sphericalangle \bar{s}} = H_1^{\sphericalangle c} = -H_1^{\sphericalangle \bar{c}} = 0. \quad (7)$$

We also assume $D_1^s \equiv N_s D_1^u$ and we consider the two scenarios $N_s = 1$ and $N_s = 1/2$. The second choice is suggested by the output of the PYTHIA event generator [25]. Our final results will not depend strongly on this choice.

The above assumptions allow us to turn Eq. (1) into the following simple relation (neglecting charm quarks)

$$xh_1^{u_v}(x, Q^2) - \frac{1}{4}xh_1^{d_v}(x, Q^2)$$
$$= -\frac{A_{DIS}(x, Q^2)}{C_y} \frac{n_u(Q^2)}{n_u^\uparrow(Q^2)} \sum_{q=u,d,s} \frac{e_q^2 N_q}{e_u^2} xf_1^{q+\bar{q}}(x, Q^2), \quad (8)$$

where $N_u = N_d = 1$ and $f_1^{q+\bar{q}} = f_1^q + f_1^{\bar{q}}$, $h_1^{q_v} = h_1^q - h_1^{\bar{q}}$. Our goal is to derive from data the difference between the valence up and down transversity distributions by computing the right-hand side (rhs) of the above relation.

The PDFs in Eq. (8) can be estimated using any parametrization of the unpolarized distributions. We chose to employ the MSTW08LO PDF set [26]. We checked that using different sets makes no significant change. We also checked that the charm contribution is irrelevant.

The only other unknown term on the rhs of Eq. (8) is the ratio $n_u/n_u^\uparrow$. We extract this information from the recent measurement by the Belle Collaboration [22] of the Artru-Collins azimuthal asymmetry $A^{\cos(\phi_R+\bar{\phi}_R)}$ [20,21,27] (denoted as a_{12R} in the experimental paper). As in the previous case, without ambiguity we simplify the notation and refer to this asymmetry as $A_{e^+e^-}$. Using the assumptions (4)–(7), the asymmetry can be written as

$$A_{e+e-}(z, M_h^2, \bar{z}, \bar{M}_h^2, Q^2) = - \frac{\langle \sin^2\theta_2 \rangle}{\langle 1 + \cos^2\theta_2 \rangle} \frac{\langle \sin\theta \rangle \langle \sin\bar{\theta} \rangle 5 \frac{|R|}{M_h} H_1^{\sphericalangle u}(z, M_h^2, Q^2) \frac{|\bar{R}|}{M_h} H_1^{\sphericalangle u}(\bar{z}, \bar{M}_h^2, Q^2)}{(5 + N_s^2) D_1^u(z, M_h^2, Q^2) D_1^u(\bar{z}, \bar{M}_h^2, Q^2) + 4 D_1^c(z, M_h^2, Q^2) D_1^c(\bar{z}, \bar{M}_h^2, Q^2)}, \quad (9)$$

where θ_2 is the angle between the direction of the lepton annihilation and the thrust axis, and θ is the angle between the momentum of one hadron in the c.m. of the hadron pair and the total momentum of the pair in the laboratory [16].

In Eq. (8) we need $n_u^\uparrow / n_u$ at the experimental values of $\langle Q^2 \rangle$ of Table I and integrated over the HERMES invariant-mass range $0.5 \leq M_h \leq 1$ GeV. We will get to this number in two steps: first, we estimate the ratio $n_u^\uparrow / n_u$ integrated over $0.5 \leq M_h \leq 1$ GeV and at the Belle scale (100 GeV^2), then we address the problem of changing Q^2.

We consider the Belle asymmetry integrated over $(z, \bar{z})$ and binned in $(M_h, \bar{M}_h)$. We restrict our attention only on the bins between $0.5 \leq (M_h, \bar{M}_h) \leq 1.1$ GeV (neglecting the small difference with the HERMES upper limit). We weight the contribution of each bin by the inverse of the statistical error squared, which should be to a good approximation proportional to the denominator of the asymmetry in each bin. By summing over all bins in the considered range, we get the total asymmetry

$$\begin{aligned} A_{e+e-} &= \frac{-\langle \sin^2\theta_2 \rangle}{\langle 1 + \cos^2\theta_2 \rangle} \frac{\langle \sin\theta \rangle \langle \sin\bar{\theta} \rangle 5 (n_u^\uparrow)^2}{(5 + N_s^2) n_u^2 + 4 n_c^2} \\ &= -0.0307 \pm 0.0011. \quad (10) \end{aligned}$$

From the Belle analysis, we know that

$$\frac{\langle \sin^2\theta_2 \rangle}{\langle 1 + \cos^2\theta_2 \rangle} = 0.753, \qquad \langle \sin\theta \rangle \langle \sin\bar{\theta} \rangle = 0.871,$$

$$\frac{4 n_c^2}{(5 + N_s^2) n_u^2} = 0.415 \pm 0.047.$$

Therefore we obtain

$$n_u^\uparrow / n_u (100 \text{ GeV}^2) = -0.273 \pm 0.007_{\text{ex}} \pm 0.009_{\text{th}}, \quad (11)$$

where the second error comes from using the two different values of the s-quark normalization N_s. We assumed the sign of the ratio to be negative in order to obtain a positive u-quark transversity distribution. To verify the reliability of this procedure, we repeated the calculation estimating the denominator of the asymmetry using the PYTHIA event generator [25] without acceptance cuts (courtesy of the Belle Collaboration). The result falls within the errors quoted in Eq. (11).

The last step in the procedure is to address the Q^2 evolution of $n_u^\uparrow / n_u$, since the Belle scale is very different from the HERMES one (see Table I). DiFFs must be connected from one scale to the other via their QCD evolution equations [28]. In order to do this, we need to know the z dependence of $H_1^{\sphericalangle u}$ and D_1^q for each M_h value. For $H_1^{\sphericalangle u}$, we fit the Belle data for A_{e+e-} binned in (z, M_h) and integrated over $(\bar{z}, \bar{M}_h)$, multiplied by the inverse of the statistical error squared.

The D_1^q should be obtained from global fits of unpolarized cross sections, similarly to what is done for single-hadron fragmentation functions [29]. In the absence of published data, we extract D_1^q by fitting the unpolarized cross section as produced by the PYTHIA event generator [25], which is known to give a good description of the total cross section. Following the assumptions introduced in Eqs. (4) and (5), we describe the unpolarized cross section for the production of a hadron pair with [21]

$$\frac{d\sigma}{dz dM_h^2} = \frac{4\pi\alpha^2}{Q^2} \left[\frac{10}{9} D_1^u + \frac{2}{9} D_1^s + \frac{8}{9} D_1^c \right], \quad (12)$$

where α is the fine structure constant. We assume the integration over $\cos\theta_2$ to be complete in the Monte Carlo sample.

We start from a parametrization of D_1^q and $H_1^{\sphericalangle u}$ at $Q_0^2 = 1$ GeV^2. Taking inspiration from the model analysis of Ref. [24], for both DiFFs we consider two channels which are effective in the considered range $0.5 \leq M_h \leq 1.1$ GeV: the fragmentation into the ρ resonance decaying into $\pi^+ \pi^-$, and the continuum arising from the fragmentation into an incoherent $\pi^+ \pi^-$ pair. Then, we evolve the DiFFs at LO using the HOPPET code [30], which we suitably extended to include also chiral-odd splitting functions. Finally, we fit the cross section (12) and the numerator of the asymmetry (9) in the bins of interest. We checked that the final results are affected in a negligible way by the gluonic component $D_1^g(z, M_h; Q_0^2)$. A thorough analysis will be presented in a future publication [31].

By integrating the extracted DiFFs in the HERMES range $0.5 \leq M_h \leq 1$ GeV and $0.2 \leq z \leq 1$, we can calculate the evolution effects on $n_u^\uparrow / n_u$ at each $\langle Q^2 \rangle$ indicated in Table I. It turns out that the ratio is decreased by a factor 0.92 ± 0.08, where the error takes into account the difference of Q^2 in the HERMES experimental bins as well as the uncertainty related to different starting parametrizations at $Q_0^2 = 1$ GeV^2. In conclusion, for the extraction of transversity in Eq. (8) we use the number

$$n_u^\uparrow / n_u = -0.251 \pm 0.006_{\text{ex}} \pm 0.023_{\text{th}}. \quad (13)$$

In Fig. 1, the data points denote the combination $x h_1^{u_v} - x h_1^{d_v}/4$ of Eq. (8), plotted for each $\langle x \rangle$ and $\langle Q^2 \rangle$ listed in Table I. We studied the influence of the errors of each element in the rhs of Eq. (8). The only relevant contributions come from the experimental errors in the measurement of A_{DIS}, as reported in Table I, and from the 9% theoretical uncertainty on $n_u / n_u^\uparrow$.

In Fig. 1, the central line represents the best fit for the combination $x h_1^{u_v} - x h_1^{d_v}/4$, as deduced from the most recent parametrization of $h_1^{u_v}$ and $h_1^{d_v}$ extracted from the Collins effect [32]. The uncertainty band is obtained by
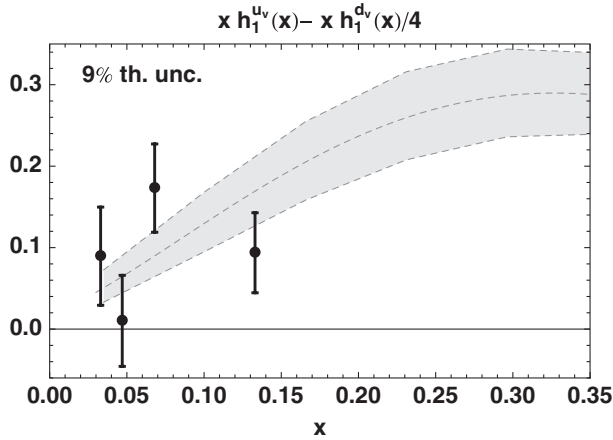
FIG. 1. The $xh_1^{u_v} - xh_1^{d_v}/4$ of Eq. (8) as a function of x. The error bars are obtained by propagating the statistical errors of each term in the equation. The uncertainty band represents the same observable as deduced from the parametrization of Ref. [32].

considering the errors on the parametrization and taking the upper and lower limits for the combination of interest. Our data points seem not in disagreement with the extraction. However, a word of caution is needed here: while the error bars of our data points correspond to 1σ deviation from the central value, the uncertainty on the parametrization [32] corresponds to a deviation $\Delta\chi^2 \approx 17$ from the best fit (see Ref. [33] for more details). In any case, to draw clearer conclusions more data are needed (e.g., from the COMPASS Collaboration [18]).

In summary, we have presented a determination of the transversity parton distribution in the framework of collinear factorization by using data for pion-pair production in deep-inelastic scattering off transversely polarized targets, combined with data of e^+e^- annihilations into pion pairs. The final trend of the extracted transversity seems not to be in disagreement with the transversity extracted from the Collins effect [32]. More data are needed to clarify the issue.

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