New Parity-Violating Muonic Forces and the Proton Charge Radius

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(Received 16 March 2011; published 30 June 2011)

The recent discrepancy between proton charge radius measurements extracted from electron-proton versus muon-proton systems is suggestive of a new force that differentiates between lepton species. We identify a class of models with gauged right-handed muon number, which contains new vector and scalar force carriers at the ~100 MeV scale or lighter, that is consistent with observations. Such forces would lead to an enhancement by several orders-of-magnitude of the parity-violating asymmetries in the scattering of low-energy muons on nuclei. The relatively large size of such asymmetries, $O(10^{-4})$, opens up the possibility for new tests of parity violation in neutral currents with existing low-energy muon beams.

DOI: 10.1103/PhysRevLett.107.011803 PACS numbers: 12.60.Cn, 13.88.+e, 14.20.Dh, 14.60.Ef

Introduction.—There has been much interest as of late in the possibility of new gauge forces existing in the MeV-GeV scale, stimulated in part by the prospect of a light mediator between dark matter and the standard model (SM) (see, e.g., [1]). While many models of this type can be explored, a great deal of attention has been given to a new U(1) gauge boson V kinetically mixed with hypercharge [2]. At low energies V appears as a massive copy of the ordinary photon,

$$\mathcal{L} = -\frac{1}{4}V_{\mu\nu}^2 + \frac{1}{2}m_V^2 V_\mu^2 + \kappa V_\mu J_\mu^{EM}, \tag{1}$$

where κ is the mixing angle parameter. The conservation of the electromagnetic current and the absence of any intrinsic parity, flavor, or CP violation in the interaction of V with the SM fermions can hide this force from very powerful symmetry tests. The model (1), while perhaps the simplest, is not the unique possibility for new gauge interactions below the weak scale [3].

While the astroparticle physics incentives are rather speculative, an additional motivation for a new light gauge boson is provided by terrestrial experiments. Among several discrepant low-energy measurements, the recent determination of the proton charge radius using muonic hydrogen [4] and the long-standing measurement of the anomalous magnetic moment of the muon [5] may be manifestations of a new sub-GeV scale force carrier that couples preferentially to muons. In this Letter we argue that if such discrepancies are caused by a new muon-specific gauge force, one should expect parity nonconservation (PNC) in the scattering of muons on nuclei far above the SM level. We point out the feasibility of a dedicated search for the PNC asymmetry, enhanced to $O(10^{-4})$ level, with existing low-energy muon beams.

With our present understanding of the strong interactions, the charge radius of the proton r_p cannot be computed from first principles but instead must be extracted from experiment. The comparison of r_p values obtained using different experimental methods provides a consistency check of QED theory and constrains a variety of new physics scenarios. Currently, there are three competitive ways of determining r_p: (1) high-precision measurements of the atomic levels in hydrogen and deuterium, (2) direct electron-proton scattering experiments, and (3) the measurement of the Lamb shift in muonic hydrogen. The most precise determinations currently read [4,6,7]

$$r_{p,1} = 0.8768(69)\ \text{fm} \quad \text{atomic H, D}, \tag{2}$$

$$r_{p,2} = 0.879(8)\ \text{fm} \quad e\text{-}p\ \text{scattering}, \tag{3}$$

$$r_{p,3} = 0.84184(67)\ \text{fm} \quad \text{muonic H}. \tag{4}$$

The r_p values obtained from e-p systems are consistent with each other and significantly differ from the r_p value extracted from the muonic hydrogen Lamb shift,

$$\begin{gathered} r_{p,1} \simeq r_{p,2} > r_{p,3}, \\ \Delta r^2 \equiv (r_p)^2_{e\text{-}p\ \text{results}} - (r_p)^2_{\mu\text{-}p\ \text{results}} \simeq 0.06\ \text{fm}^2. \end{gathered} \tag{5}$$

The difference between $r_{p,1}$ [6] and $r_{p,3}$ [4] is 5σ while the difference between $r_{p,2}$ [7] and $r_{p,3}$ is 4.6σ (for an up-to-date theoretical analysis see Ref. [8]). Part of this discrepancy may be related to the model dependence of the proton form factor used in various extractions of r_p [9], and it is conceivable that further scrutiny of SM predictions can close this gap [10]. At the moment, however, this discrepancy stands and has stimulated investigations of new interactions that could potentially be responsible for the difference [11–13]. The difficulties associated with such an enterprise stem from the fact that the difference (5) requires the strength of the new interactions to be on the order of $O(10^4 G_F)$, which is impossible to attain without new light states below 1 GeV.

It is easy to see that the kinetically mixed vector (1) cannot explain the observed pattern. In the presence of V

exchange, the inferred r_p would actually depend on the effective momentum transfer $|q|$ involved [14]. For $r_{p,1}$ ($r_{p,3}$), this corresponds to the inverse Bohr radius $\sim\alpha m_e$ (αm_μ), while for $r_{p,2}$ the momentum transfer is much larger. The effect of the extra attraction generated by V will be interpreted as the *largest negative* correction to the charge radius for the experiment that involves the *smallest* $|q|$. Therefore, a kinetically mixed vector predicts $r_{p,1} < r_{p,3} < r_{p,2}$, which is not consistent with the observed pattern (5). One can easily show that the inclusion of several kinetically mixed vectors does not change this pattern. Another logical possibility is a repulsive Yukawa force between protons and muons or electrons, e.g., as may occur if there is a new force with gauged baryon number and kinetic mixing with photons. However, in this case the natural pattern will be $r_{p,2} < r_{p,3} < r_{p,1}$, which again disagrees with (5).

In Refs. [12,13] a purely phenomenological approach to explain (5) was taken, in which dimension 6 operators $(\bar{\mu}\gamma^\alpha\mu)(\bar{p}\gamma_\alpha p)$ or $(\bar{\mu}\mu)(\bar{p}p)$ are mediated by the exchange of a new light vector or scalar particle. Scalar mediators of this type are reminiscent of a very light Higgs boson and will face stringent constraints from rare meson decays and neutron-nucleus scattering [15]. Vector mediators are more promising, but in order to be integrated with the rest of the SM, the following conditions must be met: (i) The interactions must be formulated in terms of SM fermion representations. (ii) No new interactions stronger than G_F can exist between neutrinos and nucleons or electrons. (iii) No new electrically charged elementary particles with masses below 100 GeV can exist. (iv) The model must have the possibility of a UV completion at or above the weak scale. (v) The model must be consistent with a variety of tests from QED and particle physics in the MeV energy range.

The second condition comes from the wealth of data on neutrino scattering in the $E \sim 10$ MeV energy range and neutrino oscillations and is emphasized here because it serves as a powerful model discriminator. Indeed, the interaction of a new particle V with the lepton vector current may be viewed as a subset of the interaction with left- and right-handed SM fermion currents,

$$V_\alpha \bar{l}\gamma_\alpha l \subset V_\alpha(c_1\bar{L}\gamma_\alpha L + c_2\bar{R}\gamma_\alpha R), \qquad c_1 \neq -c_2. \quad (6)$$

The left-handed fermion doublet L includes a neutrino field, so the requirement (ii) is equivalent to $c_1 = 0$. This forces V to couple to the pure right-handed fermion current. The absence of large neutral right-handed currents for electrons follows from PNC tests in the electron sector, and we therefore conclude that the most promising coupling of a vector particle that can explain (5) is to the right-handed muon.

Models with gauged μ_R.—We now focus on the class of models based on a new $U(1)_R$ gauge symmetry with quantized μ_R number. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4}V_{\alpha\beta}^2 + |D_\alpha\phi|^2 + \bar{\mu}_R i\not{D}\mu_R - \frac{\kappa}{2}V_{\alpha\beta}F^{\alpha\beta} - \mathcal{L}_m. \quad (7)$$

Here V is the $U(1)_R$ gauge boson, ϕ is a new Higgs field, neutral under the SM gauge group and charged under $U(1)_R$, that condenses $\langle\phi\rangle \equiv v_R/\sqrt{2}$, $D = \partial + ig_R Q_R V + ieQ_{EM}A$, and κ is the mixing angle parameter. The mass term for the muon is necessarily a higher-dimensional operator involving both ϕ and the SM Higgs field H_{SM} generated at a high scale Λ,

$$\mathcal{L}_m = \bar{L}\mu_R H_{SM}\frac{\phi}{\Lambda} + \text{H.c.} \to \bar{\mu}\mu\frac{vv_R}{2\Lambda}, \quad (8)$$

with $v_R/(\sqrt{2}\Lambda)$ entering as an effective SM-like Yukawa coupling for the muon. As we shall see below, the range for v_R suggested by the charge radius phenomenology is fully consistent with Λ being at the weak scale. Therefore, we are not concerned with building an explicit model that provides a UV completion to $\mathcal{L}_m$. The physical excitation of ϕ is a new muon-specific Higgs scalar S in the mass range $m_S \lesssim v_R$.

The model (7) suffers from gauge anomalies involving the photon and V. It is possible to restore gauge invariance by introducing dynamical scalar "gauge" degrees of freedom. The price for maintaining gauge invariance is that the theory becomes nonrenormalizable, with a UV cutoff Λ_{UV} above which calculability of the theory is lost. The estimate for Λ_{UV} may be obtained, e.g., from the radiative three loop vector self-energy diagram [16]:

$$\Lambda_{UV} \leq \frac{(4\pi)^3}{eg_R^2}m_V \sim 700\ \text{GeV}\left(\frac{m_V}{10\ \text{MeV}}\right)\left(\frac{g_R}{e}\right)^{-2}. \quad (9)$$

We observe that vectors in the range $m_V \sim 10 - 100$ MeV with couplings $g_R \sim 0.01 - 0.1$ are consistent with a UV cutoff well above the TeV scale. There are of course examples of perturbative cancellations of the anomalies, such as quantized $\mu_R + s_R - c_R$. Such a scenario faces severe constraints from quark flavor physics, $c\bar{c}$ resonance decays, and parity-violating tests involving nucleons and appears to be thoroughly excluded. Therefore, we choose the model (7) as the best candidate to describe new muon-specific forces, which is a consistent effective field theory valid below Λ_{UV} (9).

Phenomenological constraints.—From the Lagrangian (7) we obtain the couplings of the new vector and scalar particles to fermions,

$$g_V^\mu = -e\kappa - \frac{g_R}{2}; \quad g_A^\mu = -\frac{g_R}{2}; \quad g_V^p = -g_V^e = e\kappa$$
$$g_A^{e,p,n} = g_V^n = 0; \quad g_S^\mu = |g_R|m_\mu/m_V, \quad (10)$$

where $e = \sqrt{4\pi\alpha}$ is the positron charge. With the couplings (10), we calculate the corrections to the energy levels of the ordinary and muonic hydrogen that will be interpreted as corrections to the proton charge radius,

$$\Delta r_p^2|_{e\text{-H}} = -\frac{6\kappa^2}{m_V^2}; \quad \Delta r_p^2|_{\mu\text{-H}} = -\frac{6(\kappa^2+\eta)}{m_V^2} f(am_V) \quad (11)$$

where $a = (\alpha m_\mu m_p)^{-1}(m_\mu + m_p)$ is the μ-H Bohr radius, $f(\hat{x}) \equiv \hat{x}^4(1+\hat{x})^{-4}$, and $\eta \equiv \kappa g_R/(2e)$. The difference $\Delta r_p^2|_{e\text{-H}} - \Delta r_p^2|_{\mu\text{-H}}$ must be consistent with the observed pattern (5) and requires η to be positive. In the scaling regime of $am_V \gg 1$ one has

$$\frac{\eta}{m_V^2} \simeq \frac{\Delta r^2}{6} \simeq 0.01\ \text{fm}^2 \simeq \frac{2.5\times 10^{-5}}{(10\ \text{MeV})^2}. \quad (12)$$

In the same regime, the model predicts that future experiments with μ-He would detect the effective charge radius of the helium nucleus shifted down by $\Delta r_{\text{He}}^2 = -0.06\ \text{fm}^2$.

Another important constraint comes from the measurement of g-2 of the muon, which currently displays a $\sim(2\text{–}4)\sigma$ discrepancy with the SM prediction depending on the estimate of the hadronic contribution [17]. The one-loop corrections to g-2 coming from the exchange of the vector and scalar particles is given by

$$a_\mu^{V,S} = (g_V^\mu)^2 I_V\left(\frac{m_V^2}{m_\mu^2}\right) + (g_A^\mu)^2 I_A\left(\frac{m_V^2}{m_\mu^2}\right) + (g_S)^2 I_S\left(\frac{m_S^2}{m_\mu^2}\right), \quad (13)$$

where $I_{V,A,S}$ refer to standard vector, axial-vector, and scalar one-loop integrals [18]. Because of the presence of the scalar and axial-vector couplings, the one-loop contributions from V and S are enhanced compared to the pure vector case [14] by m_μ^2/m_V^2 and of opposite sign, so that for the choice of parameters (12) they must cancel. This mutual cancellation must happen at a per-mille level, and should be considered as the main phenomenological drawback of the model (7). However, while $a_\mu^{V,S}$ in Eq. (13) depends on the parameters (g_R, κ, m_V, m_S), the charge radius corrections depend only on (g_R, κ, m_V). Thus, fixing (g_R, κ, m_V) to account for the charge radius discrepancy, we have the remaining freedom to adjust m_S in order to bring $a_\mu^{V,S}$ in agreement with the measured value. As such, the model can always accommodate the $(2\text{–}4)\sigma$ g-2 discrepancy while simultaneously explaining the r_p puzzle. While we have computed one-loop corrections to g-2, the precise cancellation required indicates that two-loop effects may be relevant in determining the exact allowed parameter values.

Other constraints that must be taken into account are the electron g-2 determination vs independent measurements of α [19] and tests of d-p transitions in muonic Si and Mg [20], for which no deviations from standard QED predictions were found. Table I displays three benchmark points for $m_V = 10, 50, 100$ MeV for which all constraints are satisfied. Vector masses $m_V \lesssim 10$ MeV are excluded by muonic Si,Mg data and tests of α.

Additional constraints on gauged μ_R theories depend on the decay channels of S and V. If no new states charged under $U(1)_R$ exist below $m_V/2$, the gauge boson V will decay to e^+e^- pairs and thus be subject to tests at lepton colliders and fixed target experiments [21]. In particular, a preliminary search for the rare decay mode $\phi \to \eta V$ would disfavor models with $\kappa \sim O(10^{-2})$ and m_V above 30 MeV [22]. If new decay channels for V are allowed these bounds can be relaxed. Among model independent probes, the different couplings of V to muons vs electrons (10) suggest nonuniversal leptonic branchings of J/ψ and other narrow vector resonances. Current data [23] is only sensitive to $\eta \gtrsim O(10^{-2})$, which does not probe the most interesting $m_V \lesssim 100$ MeV regime. An alternative way to search for V exchange is to study the $O(\eta)$ forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$ annihilation at medium energy high-luminosity facilities with longitudinally polarized beams.

TABLE I. Benchmark points for the model that pass all phenomenological constraints.

Parameter	Point A	Point B	Point C
m_V	10 MeV	50 MeV	100 MeV
m_S	102.84 MeV	90.44 MeV	84.97 MeV
g_R	0.01	0.05	0.07
κ	0.0015	0.0075	0.02
η	2.5×10^{-5}	6.2×10^{-4}	2.3×10^{-3}
v_R	1 GeV	1 GeV	1.4 GeV

New parity-violating effects.—Despite the existence of polarized muon sources, no tests of PNC in neutral currents involving low-energy muon beams have been performed. This is because the maximum muon intensity corresponds to $p = 29$ MeV/c, where the parity-violating asymmetry due to the weak interactions will not exceed $O(10^{-7})$. With the introduction of a new vector force coupled to μ_R, the PNC effects are greatly enhanced. For the scattering of semirelativstic muons on a heavy nuclear target, the asymmetry is given by

$$A_{\text{LR}} = \frac{d\sigma_{\text{L}} - d\sigma_{\text{R}}}{d\sigma_{\text{L}} + d\sigma_{\text{R}}} \simeq -\eta\beta \frac{Q^2}{Q^2+m_V^2} \frac{1+\cos(\theta)}{1-\beta^2\sin^2(\theta/2)}, \quad (14)$$

where Q is the momentum transfer of the elastic scattering, $Q^2/p^2 = 4\sin^2(\theta/2)$, $\beta = |p|/E$, and L(R) label the incoming muon's helicity. Notice that the same combination of couplings η governing the correction to r_p also determines the asymmetry. The asymmetry can vary in a broad range from 10^{-5} to 10^{-3}, becoming larger for $Q^2 > m_V^2$ due to the scaling relation (12).

We next investigate the feasibility of achieving statistical sensitivity to A_{LR} in Eq. (14) in a Rutherford-type scattering setup. Since low-energy muons are easily stopped, counting rates are maximized with the use of high Z thin foil targets, while the optimal Z should be determined from the combined analysis of statistical and systematic errors. If, for example, a tungsten ($Z = 74$) foil of $d = 0.01$ mm thickness is used, the muons will lose only $\sim 5\%$ of their kinetic energy. Assuming a muon-counting detector with full azimuthal coverage and polar angle coverage in the range from 60° to 80° where the asymmetry is maximized, one obtains the following probability for the scattering of a muon at a large angle:

$$P = dN_{\rm atoms} V^{-1} \overline{\sigma_{\rm Rth}} \sim 6 \times 10^{-4}, \tag{15}$$

where V is the volume. With this probability, the time required to collect $N \sim (A_{\rm LR})^{-2}$ events is given by

$$t|_{N\sim 10^8} = \frac{N}{P\Phi_\mu} \sim 1600\ {\rm s} \times \frac{10^8\ {\rm muons/s}}{\Phi_\mu}, \tag{16}$$

where we have normalized the muon flux to the highest modern beam intensities [24]. It is thus apparent that the statistical uncertainty will not be a limiting factor in detecting parity-violating asymmetries of order 10^{-4}.

Another promising avenue in the search for anomalous PNC effects is the study of parity-violating decays of $2s$ states in muonic atoms, in which PNC will manifest in the enhanced one-photon rate of $2s$ decays. To illustrate, we assume η/m_V^2 is fixed by the scaling limit (12) and compute the $2s_{1/2}$-$2p_{1/2}$ mixing in μ^4He. The results for the mixing angle δ, ratio of $E1$ to $M1$ amplitudes for the one-photon decay of the $2s_{1/2}$ state, and rate for the one-photon decay are given by

$$\delta \simeq 3 \times 10^{-5}; \qquad \frac{A(E1)}{A(M1)} \simeq 60;$$
$$\Gamma^{\gamma}_{2s\to 1s} \simeq 1.9 \times 10^3 {\rm Hz}; \qquad \Gamma^{\gamma}_{2s\to 1s}/\Gamma^{\gamma\gamma}_{2s\to 1s} \simeq 0.018. \tag{17}$$

The rate of the one-photon decay is only marginally smaller than the one-photon quenching rate [25] at a gas pressure of 4hPa, at which the μ^4He Lamb shift experiment is planned, and the presence of such decays can be searched for at different gas pressures (with a modest improvement of the 2γ background rejection).

Finally, despite the absence of parity-violating couplings to the electron at tree level, an e-nucleus parity-violating amplitude will still occur at the two-loop level. Given the accuracy achieved in tests of PNC with electrons, it is therefore highly desirable to calculate this effect, which may lead to an independent constraint on gauged μ_R models.

To conclude, we have argued that the class of models with gauged μ_R represents one of the few possibilities in which the discrepancy between e-p and μ-p determinations of the proton charge radius can be reconciled with new physics. Although anomalous, these models constitute valid effective field theories which can in principle be UV completed at the weak scale. The simultaneous explanation of the r_p data and muon g-2 discrepancy requires a tight correlation between the scalar and vector masses. A striking consequence of this class of models is the existence of enhanced PNC effects in the muon sector that can be searched for at existing muon beam facilities.

We would like to thank Dr. A. Antognini, Dr. C. Burgess, Dr. A. Czarnecki, Dr. A. de Gouvêa, Dr. K. Kirch, Dr. W. Marciano, Dr. G. Marshall, Dr. A. Olin, Dr. M. Roney, Dr. J. Wacker, and Dr. I. Yavin for helpful discussions.

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