Exploring the Thermodynamic Limits of Computation in Integrated Systems: Magnetic Memory, Nanomagnetic Logic, and the Landauer Limit

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Nanomagnetic memory and logic circuits are attractive integrated platforms for studying the fundamental thermodynamic limits of computation. Using the stochastic Landau-Lifshitz-Gilbert equation, we show by direct calculation that the amount of energy dissipated during nanomagnet erasure approaches Landauer's thermodynamic limit of $kT\ln(2)$ with high precision when the external magnetic fields are applied slowly. In addition, we find that nanomagnet systems behave according to generalized formulations of Landauer's principle that hold for small systems and generic logic operations. In all cases, the results are independent of the anisotropy energy of the nanomagnet. Lastly, we apply our computational approach to a nanomagnet majority logic gate, where we find that dissipationless, reversible computation can be achieved when the magnetic fields are applied in the appropriate order.

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Introduction.—In spite of their fundamental importance to the fields of computer engineering and information science, Landauer's principle [1] and related work on the thermodynamic limits of computation [2–5] have not been the subject of direct experimental investigation. This is due in part to the lack of integrated circuit devices with energy efficiencies that approach these theoretical limits. In conventional semiconductor electronics, energy dissipation is dominated by resistive losses due to significant electron currents, meaning their efficiency is technology dependent and generally far lower than the fundamental limits. Moreover, the potential energies of their two logical states, V_{high} and V_{low}, are different, which presents additional challenges to their efficient operation such as charge leakage. What is needed instead is a device architecture in which both of the logical states are degenerate in energy and that operates in a thermodynamically reversible manner, by which we mean the device remains in or near its minimum potential energy configuration throughout a computation.

One way to meet both of these requirements is to use electron spin, rather than charge, to process and store information as is done in magnetic memory and emerging nanomagnetic logic applications. Of particular interest are technologies based on nanomagnets [6–10], defined here as lithographically patterned magnetic thin films with sub-200 nm critical dimensions. Because of strong exchange interactions between electronic spins, a nanomagnet can be treated as a single collective spin with moment $M_S V$, where M_S is the saturation magnetization of the material and V is the nanomagnet volume. In the absence of external magnetic fields, nanomagnets have two logical states, both at the same potential energy, corresponding to parallel and antiparallel magnetization along the magnetic easy axis. Changes in the magnetization direction occur by nearly uniform rotation of the collective moment rather than domain wall motion, making thermodynamically reversible logic operations possible [11]. Indeed, both Landauer [1] and Bennett [2] have cited nanomagnets as prototypical bistable logic elements in which energy efficiency near the fundamental limits might be observed. More recently, it has been experimentally demonstrated that binary information can propagate along a chain of dipole-coupled nanomagnets without any energetic input, instead using thermal energy to move the information via Brownian motion [12].

Here we show by direct calculation that energy efficiency of information erasure in nanomagnets can approach Landauer's thermodynamic limit of $kT\ln(2)$ per bit with high precision. Additionally, we show that it is possible to carry out dissipationless, reversible computations in logic gates build from interacting nanomagnets. Our work differs from previous studies [13] that have demonstrated energy dissipation on the order of $kT\ln(2)$ per nanomagnet when switching the state of the magnet using precessional dynamics. Unlike erasure, precessional switching, which is equivalent to a logical NOT operation, does not reduce the phase space of the bit and is therefore not required to dissipate energy by Landauer's principle.

Numerical calculations.—Landauer erasure, sometimes called the "restore to one" operation, involves driving a bit that is initially in either its "zero" or "one" state with equal probability to the one state with unity probability. To execute Landauer erasure in a nanomagnet, two magnetic fields are required, one along the magnetic hard axis to lower the energy barrier between the two states and the other along the easy axis to drive the nanomagnet into the one state. This means the total energy dissipation in the nanomagnet is equivalent to the sum of the area of two hysteresis loops, one along each in-plane axis.

To generate the hysteresis loops of interest, the external magnetic fields must first be specified as a function of time. Taking into consideration optimal energy efficiency and mathematical simplicity, we choose to manipulate the two fields independently of one another using the timing sequence shown in Fig. 1. Applying the fields in this manner causes the operation to split naturally into four stages in which, during any given stage, one of the fields is held fixed while the other ramps linearly from zero to its maximum value or vice versa. The application of H_x ensures that the energy dissipation is independent of the nanomagnet's anisotropy energy barrier, as this barrier is removed prior to switching.

To calculate the time-dependent magnetization state of a nanomagnetic bit during erasure, we numerically solved the stochastic Landau-Lifshitz-Gilbert (LLG) equation using the finite difference midpoint technique discussed in Ref. [14]. For our simulations we selected a ramp time of 50 ns and temperature of 300 K. The nanomagnets were modeled as circular disks with a diameter of 10 nm and thickness of 2 nm, uniaxial anisotropy energy density of 0.26 eV ($10kT$ at 300 K), saturation magnetization of 800 kA/m, and Gilbert damping constant of 1. All of the selected parameters are consistent with real magnetic materials except for the Gilbert damping constant, which is typically less than 0.1. A damping constant of 1 minimizes the time it takes for the nanomagnet to reach thermal equilibrium; had we instead used a damping constant less than 0.1, the ramp time needed to ensure the validity of the quasistatic approximation would have increased by perhaps an order of magnitude or more. A discussion of the dependence of energy dissipation in nanomagnets on ramp time when the quasistatic approximation does not hold is available in Ref. [15].

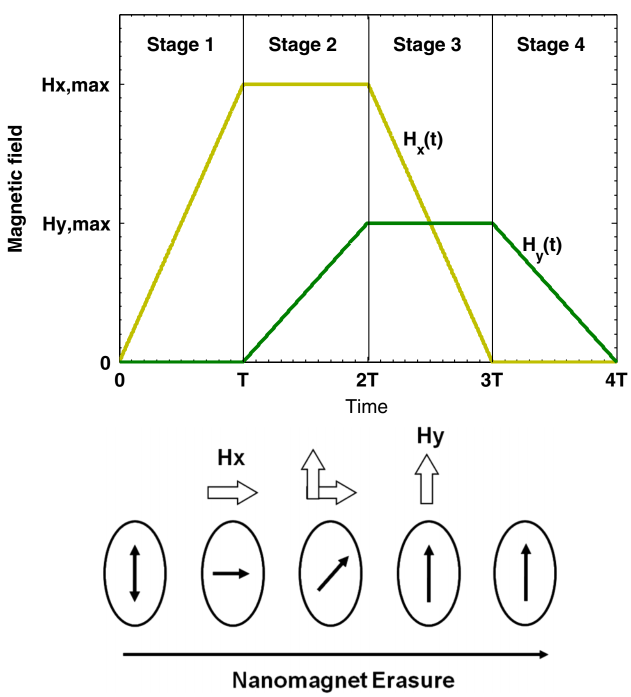


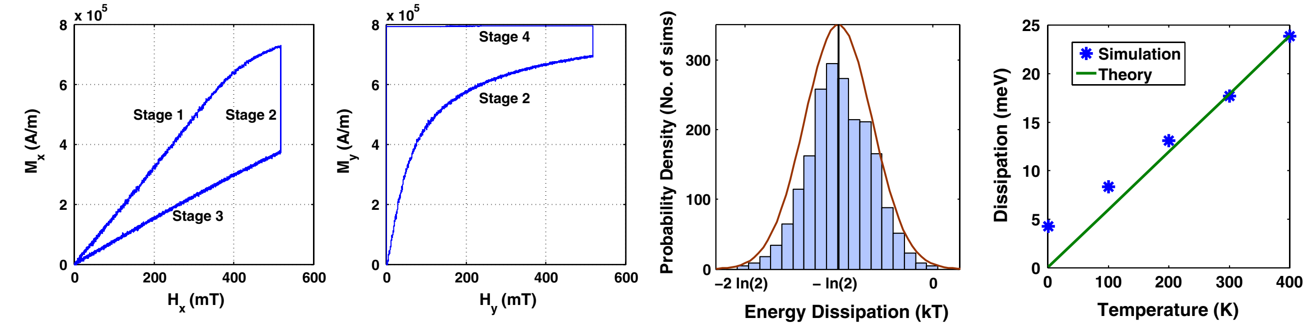
FIG. 1 (color online). Top: timing diagram for external magnetic fields applied during "restore to one." H_x is applied along the magnetic hard axis to remove the uniaxial anisotropy barrier, while H_y is applied along the easy axis to force the magnetization to one. Bottom: illustration of the magnetization of the nanomagnet at the beginning and end of each stage, and the direction of the applied field in the x-y plane.

Using the parameters noted above, we ran 2000 simulations, initializing the nanomagnet to one in half of the simulations and zero in the other half. In all simulations, the final state of the magnet was one. Although we give the nanomagnets a well-defined initial magnetization state, the external fields are applied in exactly the same manner (i.e., the same timing, direction, and magnitude) in both cases. This ensures that the system (including the magnetic field generators) does not contain any trace of the original information stored in the nanomagnet after the erasure operation is complete, as required by Landauer's principle.

The outputs of the simulation were the vectors $\mathbf{M}(t)$ and $\mathbf{H}(t)$. The hysteresis loops, averaged over the 2000 simulations, are plotted in Figs. 2(a) and 2(b), and a histogram of the energy dissipation, calculated from the area of the hysteresis loops from each simulation, is plotted in Fig. 2(c). The mean energy dissipation was found to be $0.6842kT$, which corresponds to a 95% confidence interval of $0.6740kT$ to $0.6943kT$. These values are in very close agreement with the Landauer limit, $kT\ln(2) = 0.6932kT$. The energy dissipation had no statistically significant dependence on whether the nanomagnets initialized to one or zero; the mean energy dissipation for the separate cases were $0.6809kT$ and $0.6875kT$, respectively. The distribution around the mean is consistent with the generalized formulation of Landauer's principle for small systems given by Dillenschneider *et al.* [5], which states that heat fluctuations at the nanoscale make it possible for individual erasure events to dissipate less than $kT\ln(2)$ even though the average dissipation cannot be more efficient than Landauer's limit. Similar simulations were carried out for a range of temperatures from 0 to 400 K to verify the linear dependence of energy dissipation on temperature, plotted in Fig. 2(d).

By inspection of Fig. 2, we note that it is possible to traverse the hysteresis loops in the reverse direction in the case that the initial state of the nanomagnet is known to be one. This is accomplished by applying the external magnetic fields as defined in Fig. 1 in reverse order. The resulting sign change of $d\mathbf{H}$ when computing the area of the hysteresis loops implies that the reverse erasure operation recovers, rather than dissipates, $kT\ln(2)$. Such reverse operations have been previously considered by Maroney [16] to motivate a more general formulation of Landauer's principle [3] that applies to generic logic operations in addition to the special case of bit erasure.

Nanomagnetic logic.—Nanomagnetic logic, which achieves universal logic functionality using dipole-coupled chains of nanomagnets, has been previously demonstrated experimentally [9]. Nanomagnetic logic circuits consist of interconnected majority logic gates (MLGs), which compute the majority vote of three input nanomagnets and

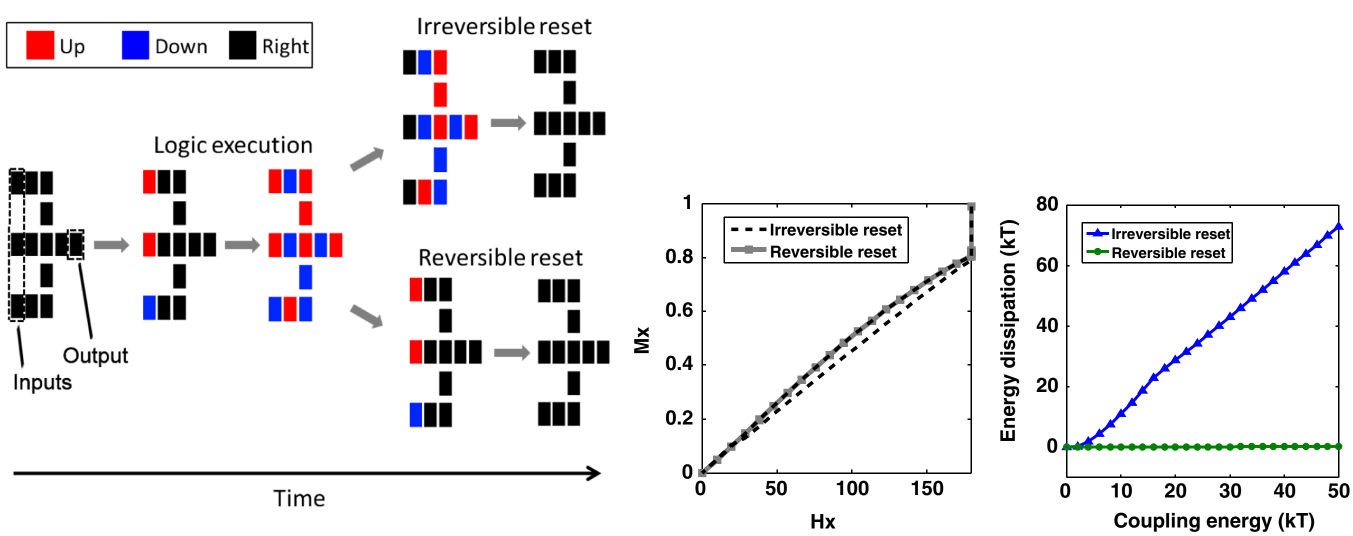


(a)X-axis hysteresis loop (b)Y-axis hysteresis loop (c)Probability density function (d)Energy dissipation vs. temperature

FIG. 2 (color online). (a) X-axis and (b) y-axis hysteresis loops of a nanomagnet during the "restore to one" operation at 300 K, obtained by solving the stochastic LLG equation. Each point on the curve is the average of 2000 simulations. The total area of the loops is very close to $kT\ln(2)$. Stage numbers correspond to the raising and lowering of external applied fields as described in Fig. 1. (c) Histogram of the energy dissipated at 300 K. The envelope curve was obtained using the Boltzmann thermal distribution calculation described in the supplementary material [17]. (d) Energy dissipation versus temperature. The solid line is the Landauer limit, $kT\ln(2)$, and the discrete points are the average dissipation over 2000 simulations carried out at each temperature. The simulations deviate from theory at low temperatures because the nanomagnets take longer to reach thermal equilibrium, weakening the quasistatic approximation.

write the result to an output nanomagnet. By calculating the relevant hysteresis loops associated with MLG operation, we show that although Landauer efficiency is not achieved when the inputs are reset before the output, dissipationless operation is possible when the output is reset before the inputs. This is an example of Bennett clocking and is a promising route for performing computations at the ultimate efficiency limits. Note that while the following analysis is for a single MLG, the approach can be scaled to a nanomagnetic logic circuit of arbitrary complexity.

Examples of reversible and irreversible computation cycles are shown in Fig. 3(a) for a nanomagnetic logic circuit containing a single MLG. Both cycles begin and end in the same state, in which all nanomagnets in the circuit are forced into their null (hard-axis) state by a magnetic field. Logic execution is carried out by slowly removing



(a)Logic execution cycle (b)X-axis hysteresis loop (c)Energy dissipation vs. coupling energy

FIG. 3 (color online). (a) Two possible computation cycles for a nanomagnetic logic circuit containing a single majority logic gate (MLG) and (b),(c) their corresponding hysteresis loops and energy dissipation. In (a), the circuit computes the majority vote of three inputs (leftmost nanomagnets) and passes the result to an output (rightmost nanomagnet). After logic execution, the circuit is reset to its initial state irreversibly (top branch) or reversibly (lower branch). In (b), the hysteresis loops for the magnetic field applied to the nanomagnets to the right of the inputs is plotted. The nearest neighbor coupling energy was set to $50kT$ in this simulation. In (c), the energy dissipation of both computation cycles is plotted as a function of the nearest neighbor coupling energy between nanomagnets.

the hard-axis magnetic field after setting the inputs using localized magnetic fields. Magnetostatic coupling between nanomagnets causes the final state of the circuit to be determined by the state of the three inputs. The output (rightmost) nanomagnet contains the result of the majority logic calculation and can be detected electronically or passed on to another portion of the circuit. Further details on signal propagation and thermal effects in nanomagnetic logic circuits can be found in [12].

To complete the computation cycle, the MLG circuit is reset to its null state by restoring the hard-axis field. In the irreversible cycle, the inputs are reset before the outputs, as is characteristic of information propagation in the forward direction. A pipelined nanomagnetic logic circuit must be reset irreversibly because the inputs of each stage are the outputs of a previous stage that must be reset first. Alternatively, it is possible to reset the circuit reversibly; i.e., the outputs are reset before the inputs. Past experimental implementations of the MLG have (perhaps unintentially) been operated reversibly because the inputs are hard coded into the circuit and persist before and after the application of the hard-axis field [9].

To calculate the average energy dissipation for both cases, we employ the thermal equilibrium calculations described in the supplementary material [17]. The hysteresis loops we obtain correspond to the X- and Y-axis fields applied locally to each input and the X-axis field driving the remainder of the gate (all nanomagnets to the right of the inputs). Each of the eight possible input permutations are calculated separately and averaged. In Fig. 3(b), the average hysteresis loops for the X-axis magnetic field applied to the magnets to the right of the inputs are plotted for the irreversible and reversible cases. In the reversible cycle (solid line), no hysteresis is observed, indicating zero energy dissipation. This result can be attributed to the temporal symmetry of the reversible cycle; because the computation proceeds in the same manner forwards and backwards, all of the hysteresis loops exactly retrace their path as the applied magnetic fields are lowered and raised. On the other hand, there is no such symmetry in the irreversible case. As a result, the hysteresis loop [dotted line in Fig. 3(b)] opens and the energy dissipation is nonzero. The amount of energy dissipation is a function of the magnetostatic coupling energy, as observed in Fig. 3(c). The coupling-energy dependent dissipation mechanism for irreversible nanomagnet operations has been discussed previously in Ref. [15].

Note that there is a difference between the reversible MLG operation considered here and conservative logic schemes, which are also logically reversible but do not require the inputs to be stored for the duration of the computation [18]. A nanomagnet-based implementation of a conservative logic gate, while desirable, has not been demonstrated at this time.

Summary.—Nanomagnetic logic typifies much of the theoretical work that has been carried out to date on the thermodynamic limits of computation. We have found that the combined area of the hysteresis loops of a nanomagnet during bit erasure approaches Landauer's thermodynamic limit of $kT\ln(2)$ with high precision in the damped switching mode. In addition, we showed that nanomagnets behave according to generalized formulations of Landauer's principle that hold for small systems and generic logic operations. Finally, we calculated the energy dissipation in a nanomagnetic majority logic gate, finding that reversible, dissipationless computation can be achieved when the outputs are reset before the inputs. We conclude that nanomagnetism is an attractive platform for experimentally investigating the thermodynamics limits of computation and suggest possible applications for this work in energy-efficient magnetic memory technologies and write-erase procedures thereof.

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