Production of Nonlocal Quartets and Phase-Sensitive Entanglement in a Superconducting Beam Splitter

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Three BCS superconductors S_a, S_b, and S and two short normal regions N_a and N_b in a three-terminal $S_aN_aSN_bS_b$ setup provide a source of nonlocal quartets spatially separated as two correlated pairs in S_a and S_b, if the distance between the interfaces N_aS and SN_b is comparable to the coherence length in S. Low-temperature dc transport of nonlocal quartets from S to S_a and S_b can occur in equilibrium, and also if S_a and S_b are biased at opposite voltages. At higher temperatures, thermal excitations result in correlated current fluctuations which depend on the superconducting phases ϕ_a and ϕ_b in S_a and S_b. Phase-sensitive entanglement is obtained at zero temperature if N_a and N_b are replaced by discrete levels.

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Regarding the manipulation of entangled states, quantum nanoelectronics is on the way to address the same fundamental issues with electrons as quantum optics does with photons. An entangled quantum state has a density matrix distinct from that of any ''hidden-variable'' theory. Two-particle entanglement can be probed [1] via the violation of the Bell inequality [2]. Multiparticle entanglement also has a high potential, for instance it [3] can be used to implement error correction codes.

Concerning superconductivity, two-particle entanglement can be generated at normal metal-superconductor N-S-N interfaces, by extracting a split Cooper pair from the BCS condensate of electron pairs [4–7]. We show in this Letter that a nanoscale three-terminal superconducting setup can produce nonlocal quartets separated as two pairs in different electrodes, therefore opening a route for a new generation of entanglers which could be controlled by an electromagnetic field. One must stress that here quartets are absent in the bulk superconductors which instead carry ordinary BCS pairing. This is in contrast with the destruction of the ''ordinary'' pair condensate in certain arrays of Josephson junctions [8], and its relationship with topological quantum computation [9,10]. Microscopically, nonlocal quartet transmission appears here as a generalization of so-called Cooper pair splitting at a double N-S-N interface, and it can be characterized by interference and noise. Recall that an Andreev pair in a normal metal electrode N results from the emission of a charge $2e$ from S at a N-S interface, by the process of Andreev reflection (AR). At a double N-S-N interface, spin-entangled [11] or energy-entangled [12] pairs can be produced through crossed (or nonlocal) Andreev reflection (CAR) [13–15] involving evanescent quasiparticle states in S, on the coherence length ξ_S. CAR coexists with normal transmission through S without electron-hole conversion (elastic cotunneling EC) [16]. CAR or EC can be selected by their different Coulomb interaction energy [17], by their spin sensitivity [6,16], by their distinguishing energy dependence [18], or by their different signature in the nonlocal conductance and in the zero-frequency shot noise cross-correlations [5,11,19,20]. The new effects considered here do not require more advanced technology than the experiments on split pairs already realized with metallic structures [13] or with quantum dots [14].

In this Letter, a route to the production of nonlocal quartets is proposed on the basis of bunching of two Andreev pairs in a superconducting beam splitter made of conventional BCS superconductors. Indeed, two Josephson junctions separated by a distance d_S of the order of the coherence length ξ_S of S can be coupled by nonlocal coherent effects [21]. Here we study microscopically all the possible nonlocal effects and discuss their physical consequences. Remarkably, in a three-terminal $S_aN_aSN_bS_b$ structure, nonlocal quartets can be separately transmitted as two correlated pairs in S_a and S_b. Nonlocal quartet transmission proceeds through double crossed Andreev reflection (dCAR), which coexists with double elastic cotunneling (dEC). The latter process yields Cooper pair transmission between S_a and S_b [22]. The phase-sensitive dCAR is a new coherent nonlocal quantum channel, which has no direct analog for incoherent multiple Andreev reflections [23]. The elementary charges involved in dCAR and dEC are doubled as compared to CAR and EC. The four processes of CAR, EC, dCAR and dEC (see Fig. 1) will be treated on an equal footing [24], as well as AR at each of the S_a-N_a-S or S-N_b-S_b interfaces which transfers Cooper pairs between S and S_a or S_b.

Cooper pair splitting in a $S_aN_aSN_bS_b$ structure with arbitrary interface transparency can be described by generalizing the Andreev–Kulik–Saint-James bound states (ABSs) [25–28]. Those states are coherent superpositions of electrons and holes, forming in a short single channel

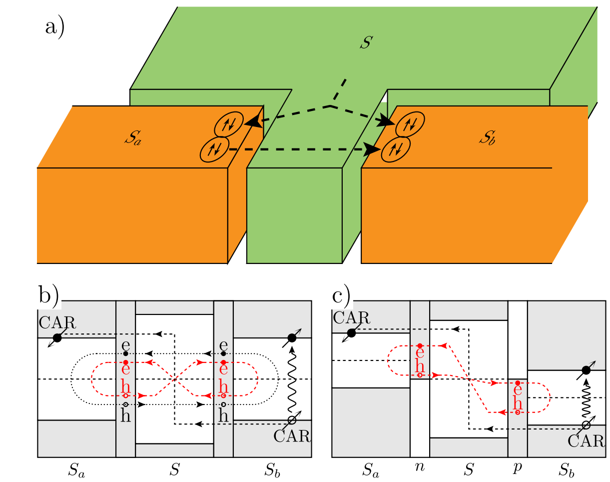

FIG. 1 (color online). Panel (a) shows a S_aSS_b structure, where N_a and N_b have not been represented for clarity. The processes taking place in three-terminal $S_aN_aSN_bS_b$ [panel (b)] and S_anSpS_b [panel (c)] structures are as follows: double crossed Andreev reflection (dCAR, red long dashed lines) producing a nonlocal quartet (a spatially separated pair of pairs), double elastic cotunneling (dEC, dotted black lines) exchanging pairs between S_a and S_b, and crossed Andreev reflection (CAR), thermally activated above the gaps of S_a and S_b. In addition, elastic cotunneling (EC) and local Andreev reflection (AR), not shown in the figure, also take place in $S_aN_aSN_bS_b$. With suitable gate voltages, the n- and p-doped semiconductors on panel (b) have a vanishingly small density of states at negative and positive energies, respectively. They filter CAR and dCAR, and eliminate EC, dEC, and AR.

junction a doublet at opposite energies (E_-, E_+), and carrying Josephson currents in opposite directions. In the $S_aN_aSN_bS_b$ setup, the phases of S_a, S_b, and S being ϕ_a, ϕ_b, and ϕ_S, and taking the bias voltages V_a, V_b to be zero, the total currents I_a, I_b are given at zero temperature $T = 0$ by $I_a(\phi_a, \phi_b) \approx (2e/\hbar)\sum_{n=1,2}(\partial E_n/\partial \phi_a)$, $I_b(\phi_a, \phi_b) \approx (2e/\hbar)\sum_{n=1,2}(\partial E_n/\partial \phi_b)$. The E_n are the energies of the ABSs formed by hybridizing the ABSs of both junctions through dCAR and dEC. They depend on both phase differences $\delta\phi_a = \phi_a - \phi_S$ and $\delta\phi_b = \phi_b - \phi_S$. If $d_S \lesssim \xi_S$, this leads (at lowest order in the dCAR and dEC processes) to $I_a = I_a^0(\delta\phi_a) + I^{\mathrm{dEC}}(\delta\phi_a - \delta\phi_b) + I^{\mathrm{dCAR}}(\delta\phi_a + \delta\phi_b)$ and $I_b = I_b^0(\delta\phi_b) - I^{\mathrm{dEC}}(\delta\phi_a - \delta\phi_b) + I^{\mathrm{dCAR}}(\delta\phi_a + \delta\phi_b)$. Production of nonlocal quartets (dCAR) and pair transmission (dEC) couple the coherent dc Josephson currents in S_a and S_b by the inverse crossed *inductances* $(L^{-1})_{a,b} = \partial I_a(\phi_a, \phi_b)/\partial\phi_b$ and $(L^{-1})_{b,a} = \partial I_b(\phi_a, \phi_b)/\partial\phi_a$, which is an extension of the concept of crossed *conductances* in a N_aSN_b structure.

Voltages V_a, V_b are applied now on the electrodes S_a, S_b ($V_S = 0$ is the reference voltage). Yet a dc Josephson current can flow from S to S_a and S_b if $d_S \sim \xi_S$ [29], in addition to the standard ac Josephson currents. The intensity of this current can be seen as a synchronization of the phases of the ac oscillations. Indeed, considering for simplicity low transparency contacts, the double Josephson junction is described by the Hamiltonian $\mathcal{H} = \mathcal{H}_{\mathrm{loc}} + \mathcal{H}_{\mathrm{dCAR}} + \mathcal{H}_{\mathrm{dEC}} + 2e(\hat{n}_S V_S + \hat{n}_a V_a + \hat{n}_b V_b)$, with $\mathcal{H}_{\mathrm{loc}} = -E_J\{\cos[\delta\phi_a(t)] + \cos[\delta\phi_b(t)]\}$, and $\mathcal{H}_{\mathrm{dCAR}} = -E_J^{\mathrm{dCAR}}\cos[\delta\phi_a(t) + \delta\phi_b(t)]$, $\mathcal{H}_{\mathrm{dEC}} = -E_J^{\mathrm{dEC}}\cos[\delta\phi_a(t) - \delta\phi_b(t)]$. The currents are obtained from Hamilton equations, for instance:

$$\left(\frac{\hbar}{2e}\right)I_a(t) = E_J \sin[\delta\phi_a(t)] + E_J^{\mathrm{dCAR}}\sin[\delta\phi_a(t) + \delta\phi_b(t)] + E_J^{\mathrm{dEC}}\sin[\delta\phi_a(t) - \delta\phi_b(t)]. \tag{1}$$

Applying opposite voltages $V_a = -V_b \equiv V$ leads to a dc Josephson effect for nonlocal quartets because the phase combination $\delta\phi_a(t) + \delta\phi_b(t) = e(V_a + V_b)t/\hbar + \delta\phi_a + \delta\phi_b$ is then time independent. The Josephson effect for nonlocal quartets becomes ac only if the energy $eV_a + eV_b$ acquired by the quartet when separately transmitted into S_a and S_b is finite. This result holds for any transparency.

The dc Josephson effect for nonlocal quartets is further considered for a S_anSpS_b junction biased at opposite voltages, where the previous N_a and N_b metals have been replaced by n- and p-doped semiconductors. The conduction band edge on one side (n type) and the valence band edge on the other (p type) are at zero energy. The gaps in the density of states of the n- and p-doped semiconductors filter the processes with positive energies in n, and with negative energies in p [18] [Fig. 1(c)]. This excludes both the local Josephson effect and the nonlocal dEC, thus leaving at $T = 0$ only the nonlocal dCAR as a coherent coupling between the condensates [30]. In this ideal situation, one obtains a perfect superconducting beam splitter operating at the scale of the coherence length, and producing correlated pairs of Cooper pairs flowing in the leads S_a and S_b. Notice again that the biases V_a and V_b should be *opposite* in the coherent Josephson regime, while they are equal in a normal beam splitter N_aNN_b or a Cooper pair splitter N_aSN_b, where quasiparticles are emitted instead of pairs.

Let us now discuss the noise cross-correlations in a $S_aN_aSN_bS_b$ structure. Zero-frequency thermal noise is present in the absence of applied voltage for sufficiently transparent single junctions [31,32]. Finite values are obtained for all the components of the correlators $S_{i,j}(t) = \langle \delta I_i(t + t')\delta I_j(t)\rangle$, where δI_k is the current fluctuation in S_k ($k = a, b$). This equilibrium noise, due to thermally activated fluctuations between ABSs carrying opposite currents, is phase sensitive because the population of quasiparticles in thermal equilibrium exchanges charge with the condensate. The thermal noise can be very large for a long inelastic lifetime of the ABSs [31,32]. CAR and EC lead to components $S_{a,b}^{\mathrm{CAR}}$ and $S_{a,b}^{\mathrm{EC}}$ of $S_{a,b}$, which are independent of $\delta\phi_a$ and $\delta\phi_b$. They correspond to CAR and EC assisted by thermal activation over the gap Δ (Fig. 1). On the contrary, dCAR and dEC result in thermal fluctuations between hybridized ABSs. dCAR corresponds to random emission and absorption of nonlocal quartets between S and S_a, S_b. dEC corresponds to random transmission of pairs between

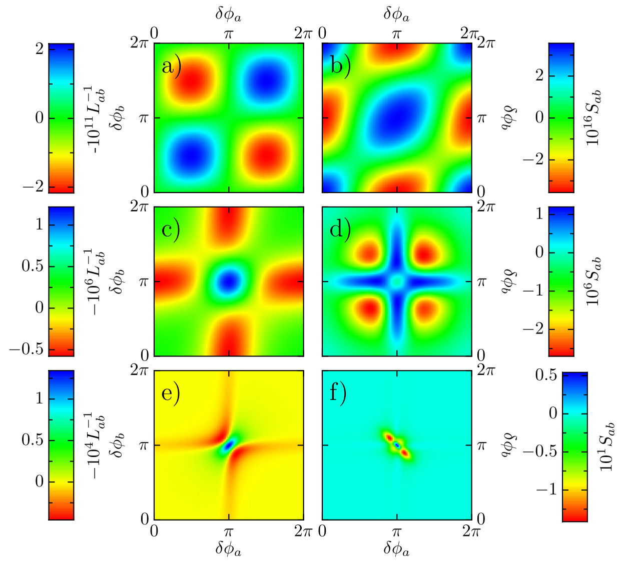

FIG. 2 (color online). The plots show $-(L^{-1})_{a,b}$ [panels (a), (c), (e)] and $S_{a,b}$ [panels (b), (d), (f)] as a function of $(\delta\phi_a, \delta\phi_b)$ for $T/\Delta = 0.1$, $T_N = 4 \times 10^{-4}$ [panels (a),(b)], $T_N = 0.64$ [panels (c), (d)] and $T_N = 1$ [panels (e), (f)]. The ratio $\Delta/\Delta_S = 0.1$ is used.

S_a and S_b. The contributions $S_{a,b}^{\mathrm{dCAR}}$ and $S_{a,b}^{\mathrm{dEC}}$ of dCAR and dEC to $S_{a,b}$ depend, respectively, on the phase combinations $\delta\phi_a + \delta\phi_b$ and $\delta\phi_a - \delta\phi_b$. Generalizing Refs. [31–33], the zero-frequency noise cross-correlations $S_{a,b}$ in the absence of applied voltage is written as

$$S_{a,b} = \frac{e^2}{\eta\hbar^2}\sum_n \frac{1}{\cosh(E_n/2k_BT)}\frac{\partial E_n}{\partial\phi_a}\frac{\partial E_n}{\partial\phi_b}, \qquad (2)$$

where the lifetime of the Andreev states $1/\eta$ shows the relevance of pair-breaking effects in the superconductor [21]. Such noise correlations could be large at temperatures T^* comparable to the ABS energy level difference. The crossover temperature T^* is strongly reduced as the interface transparency increases.

Calculations of the equilibrium nonlocal inverse inductance $(L^{-1})_{a,b}(\delta\phi_a, \delta\phi_b)$ and cross-correlations $S_{a,b}$ $(\delta\phi_a, \delta\phi_b)$ are based on microscopic Nambu-Keldysh Green's functions [29] in which the interfacial hopping amplitude is accounted for by a self-energy. Arbitrary values of the temperature and of the normal-state transmission coefficient T_N can be treated in equilibrium because the time convolutions in the Dyson equations then simplify into products of Green's function depending only on energy. The microscopic calculations carried out for a three-dimensional ballistic superconductor apply to a voltage range in which the proximity effect is negligible. $S_{a,b}$ goes to zero at $T = 0$ because it is thermally activated over the ABS gap. $(L^{-1})_{a,b}$ saturates at low T to the zero-temperature response of the condensate. As seen from perturbation theory in the tunnel amplitudes, both dCAR and dEC contribute with a positive value to $S_{a,b}(\delta\phi_a = 0, \delta\phi_b = 0)$. Thus, for tunnel contacts, $S_{a,b}(\delta\phi_a = 0, \delta\phi_b = 0)$ is positive at the small $T/\Delta = 0.1$ [see Fig. 2(b)]. The oscillations in $-(L^{-1})_{a,b}(\delta\phi_a, \delta\phi_b)$ [Fig. 2(a)] match those of $S_{a,b}(\delta\phi_a, \delta\phi_b)$ [see Fig. 2(b)]. They reflect the distinguishing phase dependences of dCAR and dEC. Increasing T_N has the effect of favoring the transmission of pairs by dEC from S_a to S_b (or from S_b to S_a), and disfavoring their transmission as a pair of holelike quasiparticles by dCAR. The cross-shaped variations of $(L^{-1})_{a,b}(\delta\phi_a, \delta\phi_b)$ and $S_{a,b}(\delta\phi_a, \delta\phi_b)$ for intermediate T_N can be understood from the bound state energy level minima in the $(\delta\phi_a, \delta\phi_b)$ plane for $\delta\phi_a, \delta\phi_b \in \{0, \pi, 2\pi\}$.

This superconducting beam splitter also generates entanglement between pair numbers in the two branches, as pairs are emitted two by two. Indeed a split Josephson current due to nonlocal quartets connects coherently pair number states $|N_a\rangle|N_S\rangle|N_b\rangle$ with states $|N_a + 1\rangle|N_S - 2\rangle|N_b + 1\rangle$, entangling the added pairs in S_a and S_b [29]. To illustrate this, let us replace the N junctions by quantum dots D_a, D_b. Indeed phase-sensitive entanglement is obtained in a $S_aD_aSD_bS_b$ structure biased at voltages $V_a = -V_b \equiv V$ larger than Δ, with $\Delta_S \gg \Delta, eV$. In a first step, D_a and D_b are supposed to carry spin-degenerate orbitals. In addition, the gate voltages are such that D_a (D_b) has a level at eV_a (eV_b) but no level at $-eV_a$ ($-eV_b$). Quasiparticles are transmitted from S_a to D_a (from S_b to D_b) but local Andreev processes between D_a and S (D_b and S) are not possible. In the limit of large gaps, three terms contribute to the effective Hamiltonian of the two coherently coupled levels at energies eV_a (in D_a) and eV_b (in D_b): (i) The exchange of pairs between S_a and D_a (S_b and D_b): $\mathcal{H}_{\mathrm{AR},a(b)} = -\alpha[\exp(i\phi_{a(b)})c^\dagger_{a(b),\uparrow}c^\dagger_{a(b),\downarrow} + \mathrm{H.c.}]$. (ii) Cooper pair splitting: $\mathcal{H}_{\mathrm{CAR}} = -\beta[c^\dagger_{a,\uparrow}c^\dagger_{b,\downarrow} + c^\dagger_{b,\uparrow}c^\dagger_{a,\downarrow} + \mathrm{H.c.}]$. (iii) Production of nonlocal quartets: $\mathcal{H}_{\mathrm{dCAR}} = -\gamma[c^\dagger_{a,\uparrow}c^\dagger_{a,\downarrow}c^\dagger_{b,\uparrow}c^\dagger_{b,\downarrow} + \mathrm{H.c.}]$. Exact diagonalizations of $\mathcal{H}_{\mathrm{AR},a} + \mathcal{H}_{\mathrm{AR},b} + \mathcal{H}_{\mathrm{CAR}} + \mathcal{H}_{\mathrm{dCAR}}$ result in an entangled ground state characterized by a positive concurrence, which depends on the values of $\delta\phi_a$ and $\delta\phi_b$ via the combination $\delta\phi_a + \delta\phi_b$ typical of dCAR [29]. The Coulomb interaction Hamiltonian is $\mathcal{H}_U = U\hat{n}_{a(b),\uparrow}\hat{n}_{a(b),\downarrow}$, with $\hat{n}_{a(b),\sigma}$ the number of spin-σ electrons in dot a (in dot b). As U increases, the zero-temperature concurrence of the ground state remains finite because of virtual excitations coupling to dCAR.

Entanglement can also be more directly assessed by showing that no classical correlation can account for the crossed noise of nonlocal quartets. Let us consider $S_aD_aSD_bS_b$ or S_apSnS_b setups, biased at opposite V_a and V_b larger than Δ, and with $\Delta_S \gg \Delta$, eV [assumption (A1)]. The notation $\langle N_a(t,\tau)\rangle_\rho$ stands for the average number of electrons transmitted into electrode S_a in the time interval $[t, t+\tau]$. The average over the hidden-variable density matrix $\rho = \int d\lambda \rho_a(\lambda) \otimes \rho_b(\lambda)$ is noted $\langle\cdots\rangle_\rho$ [11]. We make the assumption (A2) that each subsystem a and b is separately described by quantum solid-state physics: only communication through S corresponds to a hidden variable. The assumption (A2) leads to $\langle\delta N_a(t,\tau)\delta N_b(t,\tau)\rangle_{\mathrm{qu}} = \langle\delta N_a(t,\tau)\delta N_b(t,\tau)\rangle_\rho$, with

$$\langle \delta N_a(t,\tau)\delta N_b(t,\tau)\rangle_\rho = \int d\lambda f(\lambda)\langle \delta N_a(t,\tau)\rangle_\lambda \langle \delta N_b(t,\tau)\rangle_\lambda, \tag{3}$$

where $f(\lambda)$ is the probability density of the hidden variable λ. The value of $\delta N_a(t,\tau)$ for the specific value λ of the hidden variable is denoted by $\langle \delta N_a(t,\tau)\rangle_\lambda$.

An additional assumption (A3) concerning the setup is made in order to simplify the discussion: the gap Δ_S is larger than the bandwidth W of the superconductors S_a and S_b, and the linewidth broadening in S is $\eta_S = 0$. The three assumptions (A1), (A2) and (A3) imply that charge transport is blocked at any temperature because the D_a and D_b, or the n and p energy filters [assumption (A1)], suppress all Josephson processes taking place locally within each subsystem a or b for any realization of the hidden variable λ [assumption (A2)]. No quasiparticle is transmitted from S to S_a (from S to S_b) within subsystem a (b) for any value of λ [assumption (A3)]. The equality $\langle \delta N_a(t,\tau)\delta N_b(t,\tau)\rangle_{\rm qu} = 0$ is then obtained because $\langle \delta N_a(t,\tau)\rangle_\lambda = \langle \delta N_b(t,\tau)\rangle_\lambda = 0$. The following two statements are in conflict: (i) The cross-correlations are finite; and (ii) The nonlocal processes is a classical communication (related to some hidden variables) rather than the quantum mechanical CAR, dCAR.

For the S_anSpS_b structure considered above, the cross-correlations are vanishingly small at zero temperature and the Bell-like argument does not imply entanglement in this case. Indeed, the ground state of a Josephson junction is entangled only if the total number of pairs is fixed [29]. However, the denomination *phase-sensitive entanglement* is appropriate at zero temperature for the $S_aD_aSD_bS_b$ structure because the cross-correlations are finite at $T = 0$, and the Bell-like argument is in agreement with the direct calculation of the concurrence, being also finite.

To conclude, we propose to generate nonlocal quartets in the solid state, by producing correlated pairs in a superconducting beam splitter involving three superconductors. The nonlocal inductance and the phase-sensitive thermal cross-correlations may be probed in future experiments. Of particular interest is the possibility to obtain entanglement at zero temperature from current-current cross-correlations in a $S_aD_aSD_bS_b$ structure. Nonlocal Shapiro steplike experiments are also promising for investigating dCAR and dEC, in the sense that resonances due to dCAR or dEC could be obtained if the ac voltage oscillations are synchronized in two incoming channels S_a and S_b.

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