Robust Dynamical Decoupling for Quantum Computing and Quantum Memory

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(Received 21 March 2011; published 14 June 2011)

Dynamical decoupling (DD) is a popular technique for protecting qubits from the environment. However, unless special care is taken, experimental errors in the control pulses used in this technique can destroy the quantum information instead of preserving it. Here, we investigate techniques for making DD sequences robust against different types of experimental errors while retaining good decoupling efficiency in a fluctuating environment. We present experimental data from solid-state nuclear spin qubits and introduce a new DD sequence that is suitable for quantum computing and quantum memory.

DOI: 10.1103/PhysRevLett.106.240501 PACS numbers: 03.67.Pp, 03.65.Yz, 76.60.Lz

An obstacle against high-precision quantum control is the decoherence process [1]. The ability to preserve quantum behavior in the presence of noise is essential for the performance of quantum devices, such as quantum memories [2], where one wishes to store a quantum state, and quantum computers, where the quantum information is processed [3].

A promising strategy developed to avoid decoherence is the dynamical decoupling (DD) method [4], which aims to reduce decoherence times by attenuating the system-environment interaction. Since DD does not require auxiliary qubits or measurements, it can be used as an economical alternative to complement quantum error correcting codes [5]. Decoupling schemes were originally developed in the framework of nuclear magnetic resonance (NMR) [6]. In DD, a sequence of control fields is periodically applied to a system in cycles of period τ_c, in order to refocus the system-environment evolution. The delay τ_d between pulses is one of the relevant parameters of a sequence [7]. When τ_d is shorter than the correlation time τ_e of the environment, the preservation of a single qubit state is ideally possible even for the most general dephasing environment [8]. The effectiveness of a decoupling scheme depends crucially on the repetition rate with which the pulses can be applied. Recent experiments have successfully implemented DD methods and demonstrated the resulting increase of the coherence times [7,9,10]. However, systems with fast fluctuating environment, with short τ_e, put enormous demands on the hardware. They may be encountered in a wide range of quantum information processing implementations [11] and represent the most challenging regime for avoiding decoherence.

Apart from preserving the state of a quantum memory as long as possible, DD can also be used to keep a quantum state coherent while logical operations are performed on it. In this case, one needs to refocus the system-environment interactions while not refocusing the desired system-system interactions needed for multiqubit gates, as discussed in Ref. [12]. Suitable DD sequences for quantum computing should therefore keep the number of refocusing pulses small to allow the computational operations and limit power deposition on the sample. Since the precision of any real operation is finite, the performance of experimentally accessible DD sequences is limited also by the pulse errors [7,10,13]. This effect must be minimized by choosing a decoupling scheme that is robust against pulse imperfections.

As the spacing between the pulses is reduced, the refocusing gets more effective, since the state of the environment appears more static on the time scale of the pulse spacing. Ideally, this can be extended indefinitely, until the system decouples completely from the environment in the limit of infinitely short pulse spacing. In practice, however, the observed decay rate goes through a minimum and subsequently increases again. In this regime, the signal is destroyed primarily by imperfections of the refocusing pulses.

We first consider this limiting case, where the effect of the environment is negligible and we need to discuss only the effect of control errors. The dominant cause of these errors is, in most cases, a deviation between the actual and the ideal amplitude of the control field. The result of this amplitude error is that the rotation angle deviates from π, typically by a few percent. A possible approach for compensating these errors is the use of composite pulses [14], which generate rotations that are close to the target value even in the presence of amplitude errors. In this case, the error correction is done ‘‘inside’’ the pulse. Alternatively, it is possible to design the sequence in such a way that the error introduced by one pulse is compensated by subsequent pulses. We refer to the former approach as using robust pulses and to the second approach as using robust or self-correcting sequences.

Apart from reducing the coherence time in a certain parameter range, pulse imperfections also make some DD sequences asymmetric with respect to the initial conditions. We illustrate this in Fig. 1, where we plot the signal of the carbon nuclear spin magnetization in the adamantane molecule [7] measured for two different DD sequences and initial conditions. Considering first the

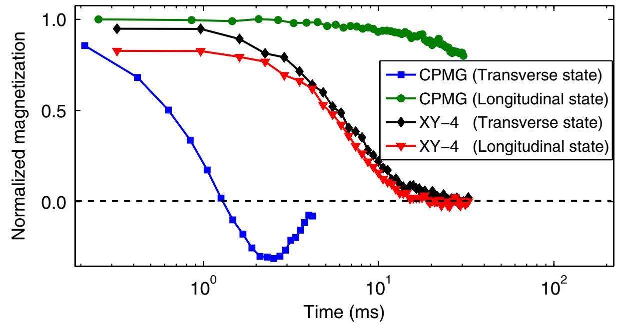

FIG. 1 (color online). Normalized magnetization obtained experimentally for two basic NMR sequences: one nonrobust against errors, CPMG, and one self-correcting sequence, XY-4.

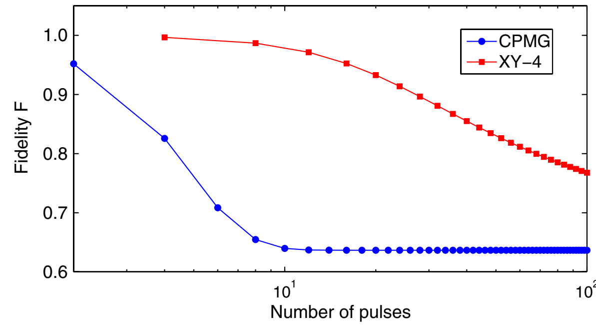

FIG. 2 (color online). Simulation of fidelity decay due to pulse errors. The blue circles represent the decay of the fidelity due only to pulse errors for the CPMG sequence and the red squares for the XY-4 sequence. The fidelity is calculated as an average over a Gaussian distribution of flip angles; the width was 10% of the nominal flip angle.

Carr-Purcell-Meiboom-Gill (CPMG) sequence, which consists of identical π pulses. It was originally designed to preserve a single component of the magnetization [15]. Here, the decoherence time is ≈ 100 ms if the Bloch vector of the qubit is initially oriented parallel to the pulse axis (longitudinal state). Under these conditions, pulse errors do not affect the coherence [15]—they even provide additional stabilization [16]. In contrast, if the initial condition is perpendicular to the direction of the pulses (transverse state; blue squares in Fig. 1), the errors of the individual pulses accumulate and lead to a rapid decay: the signal completely vanishes after ≈ 50 pulses (≈ 5 ms). A similar behavior is found for the UDD sequence [17], which also uses rotations around a single axis [18].

The second DD sequence represented in Fig. 1 is the XY-4 sequence [19], which consists of π pulses applied along the x and y axes. It performs much more symmetrically with respect to the initial condition: both initial states decay on a time scale of ≈ 10 ms. The observed time scale also shows that this sequence has a built-in partial error compensation.

For preserving an unknown quantum state, the appropriate performance measure should not depend on the initial condition. A common choice for quantifying the performance of quantum operations is then the fidelity

$$F = \frac{|\mathrm{Tr}(AB^\dagger)|}{\sqrt{\mathrm{Tr}(AA^\dagger)\mathrm{Tr}(BB^\dagger)}}. \quad (1)$$

Here, A is the target propagator (unity for the examples discussed here) and B is the actual propagator generated by the real pulse sequence. In Fig. 2, we numerically simulate the fidelity decay during the two DD sequences discussed above due only to errors in the flip angles of the pulses, neglecting environmental effects. For the CPMG sequence (blue circles), we observe a rapid decay to the limiting fidelity of ≈ 0.65. At this point, the system reaches the completely disordered state $\rho \propto \mathbf{1}$, with the order dephased in the inhomogeneous field distribution. The fast decay for CPMG is experimentally manifested by the large asymmetry shown in Fig. 1. The XY-4 sequence, however, represented by the red squares in Fig. 2, causes a slower decay for the same parameters, indicating a partial error compensation over the cycle.

In order to correct errors of nonrobust sequences or even to improve the error tolerance of self-correcting sequences, it is possible to replace the simple π pulses by composite pulses [14] that are more robust against errors. We tested different class-A composite π pulses, which produce compensated rotations for any initial condition, and found that the pulse used in Ref. [10]

$$(\pi)_{\pi/6+\phi} - (\pi)_{\phi} - (\pi)_{\pi/2+\phi} - (\pi)_{\phi} - (\pi)_{\pi/6+\phi}, \quad (2)$$

to which we refer here as the Knill pulse, is the most robust against flip-angle errors and off-resonance errors, which are the leading errors in many experimental situations. The Knill pulse is equivalent to a robust π rotation around the axis defined by ϕ followed by a $-\pi/3$ rotation around the z axis. For cyclic sequences, which always consist of even numbers of π rotations, the effect of the additional z rotation vanishes if the flip-angle errors are sufficiently low. We inserted these composite pulses into different DD sequences to test their performance.

Since the Knill pulse consists only of π rotations, it can also be used as a DD sequence: instead of concatenating the pulses directly, we inserted delays between them and obtained thus a new DD sequence with the rotation axes given by Eq. (2): $\mathrm{KDD}_\phi = \tau/2 - (\pi)_{\pi/6+\phi} - \tau - (\pi)_\phi - \tau - (\pi)_{\pi/2+\phi} - \tau - (\pi)_\phi - \tau - (\pi)_{\pi/6+\phi} - \tau/2$. To further improve the robustness of the sequence, we also extended it by combining 5-pulse blocks shifted in phase by $\pi/2$, such as $[\mathrm{KDD}_\phi - \mathrm{KDD}_{\phi+\pi/2}]^2$, where the lower index gives the overall phase of the block. We will refer to the cyclic repetition of these 20 pulses as the KDD sequence.

Figure 3 summarizes how sensitive the different DD sequences are to the two main pulse imperfections (no dephasing environment): each panel shows the simulated

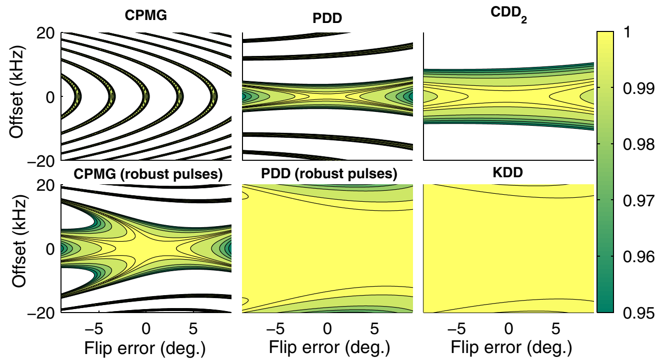

FIG. 3 (color online). Error tolerance of different DD sequences. The upper row shows the calculated fidelity F for standard DD sequences, while the lower row shows the results for the CPMG and PDD $= XY$-4 sequences when the π pulses are replaced by Knill pulses. The last panel corresponds to the KDD sequence, also based on the Knill pulse. Each panel shows the fidelity after 100 pulses as a function of flip-angle error and offset errors. The regions where the fidelity is lower than 0.95 are shown in white. The highest contour level is $F = 0.999$.

fidelity F after 100 pulses with a combination of frequency (vertical axis) and amplitude errors (horizontal axis). Fidelities >0.95 are color coded. For CPMG (first panel), the fidelity after 100 pulses drops to <0.95 even for very small flip-angle errors or offsets. The concatenated sequences (CDD-n) were constructed as $[\mathrm{CDD}_{n-1} - X - \mathrm{CDD}_{n-1} - Y]^2$ and $[\sqrt{\mathrm{CDD}_{n-1}} - X - \mathrm{CDD}_{n-1} - Y - \sqrt{\mathrm{CDD}_{n-1}}]^2$ for the standard and symmetrized versions, respectively. For $n = 1$, we have $\mathrm{CDD}_1 = XY\text{-}4 = \mathrm{PDD}$. The basic sequence XY-4 is defined as $[\tau_d - X - \tau_d - Y]^2$ for the standard case. The symmetrized sequence is $[\tau_d/2 - X - \tau_d - Y - \tau_d/2]^2$. The second and third panel in Fig. 3 show the corresponding results for the standard (asymmetric) XY-4 and CDD_2 sequence. Clearly, they are much less susceptible to flip-angle errors than the CPMG sequence. As shown in the lower panels, a further significant improvement is achieved if the π rotations are replaced by Knill pulses, at the expense of increased power deposition (by a factor of 5). The best performance, with similar power deposition as for the panels in the upper row, is achieved with the KDD sequence.

Now we start looking at the effect of imperfect DD pulses on a fluctuating environment. We tested the sequences discussed above in an experimental setting. As the system qubit, we used ^{13}C nuclear spins in the CH_2 groups of a polycrystalline adamantane sample. The natural abundance carbon spins are surrounded by ^{1}H nuclear spins acting as a rapidly fluctuating environment [7]. The bath correlation time was $\approx 100\ \mu$s, the pulse length was 10.6 μs, and the delays between the pulses were varied from 5 to 150 μs. Under our conditions, the interaction between the carbon nuclei can be neglected and the decoherence mechanism is a pure dephasing process [7].

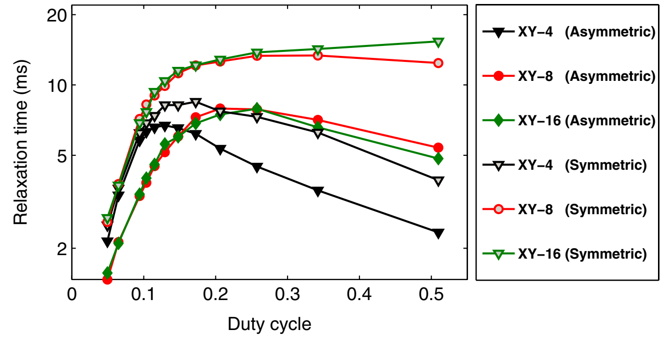

FIG. 4 (color online). Comparison of the decoupling performance of symmetric and asymmetric XY sequences.

The experiments were performed on a homebuilt 300 MHz solid-state NMR spectrometer.

Figures 4 and 5 summarize the experimental results by plotting the relaxation times as a function of the duty cycle (total irradiation time divided by total time). The relaxation times are defined as the $1/e$ decay time of the magnetization. In the first set of experiments, Fig. 4, we compare different members of the XY family. The sequences were constructed, as explained in [19], using the standard definition of XY-4 and the its symmetric definition, introduced above. For all sequences, the symmetric version performs significantly better, with relaxation times ≈ 5 times longer than for the asymmetric version, irrespective of the initial condition. This can be attributed to the fact that in the symmetric sequences, all odd-order terms of the Magnus expansion vanish [20]. This is a significant advantage, considering that the power deposition and the complexity of the sequences are identical.

In the second set of experiments, Fig. 5, we compare standard CDD sequences against CDD sequences with robust pulses and symmetric timing. For low duty cycles, standard CDD sequences perform better. This is due to the shorter cycle time of the standard sequence if constant duty cycles are compared. Using nonrobust pulses may therefore be the preferred option if DD sequences are applied

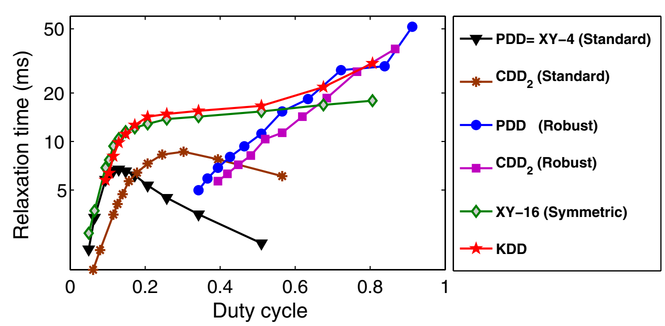

FIG. 5 (color online). Comparison of the decoupling performance of standard DD sequences (PDD, CDD_2) with the same sequences using composite pulses and the self-correcting sequences XY-16 and KDD.

parallel to gate operations. However, their performance always saturates or decreases with increasing duty cycle under the present experimental conditions, while the performance of sequences with robust pulses continuous to improve. Thus, if only the preservation of a quantum state is required, without considering power deposition, the sequences with robust pulses provide the best performance. The performance of the sequences with robust pulses shows no significant dependence on the concatenation level. Apparently, the additional corrections due to the concatenation are not required in this case.

For a large range of duty cycles, we find that KDD provides the best performance. At low duty cycles the performance of KDD is comparable to that of self-correcting sequences without robust pulses, indicating that the errors of the individual pulses compensate over a cycle. For high duty cycles, instead of saturating, the relaxation time continues to increase, as in the case of sequences with robust pulses. This shows that KDD has suitable properties for computing and memory applications.

In summary, we have considered the problem of protecting a qubit in the presence a fast fluctuating spin-bath and pulse imperfections. Different strategies for correcting pulse errors were investigated and verified experimentally in DD experiments. We observed that the symmetrization of sequences is an important feature since this leaves the power deposition and the complexity of the sequence constant but always leads to better performance. The best sequences that are suitable for parallel application of quantum gate operations are the symmetric self-correcting sequences. However, their performance saturates at higher duty cycle, while the performance of sequences with robust pulses continues to improve under our experimental conditions. Thus, if the objective is only to preserve a quantum state, the best performance is obtained at high duty cycles, using robust pulses. We also introduced a new DD sequence, which combines the useful properties of robust sequences with those of robust pulses and can thus be used for both quantum computing and state preservation. This new sequence contains rotations around different axes in the xy plane, not only around the more conventional directions x and y. We believe that mixing nontrivial directions will be a helpful ingredient for developing new robust dynamical decoupling sequences for future quantum information applications.

We acknowledge useful discussions with Daniel Lidar. This work is supported by the DFG through Su 192/24-1. G. A. A. thanks the Alexander von Humboldt Foundation.

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