

Quantum Quench of an Atomic Mott Insulator

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We study quenches across the Bose-Hubbard Mott-insulator-to-superfluid quantum phase transition by using an ultracold atomic gas trapped in an optical lattice. Quenching from the Mott insulator to the superfluid phase is accomplished by continuously tuning the ratio of Hubbard tunneling to interaction energy. Excitations of the condensate formed after the quench are measured by using time-of-flight imaging. We observe that the degree of excitation is proportional to the fraction of atoms that cross the phase boundary and that the quantity of excitations and energy produced during the quench have a power-law dependence on the quench rate. These phenomena suggest an excitation process analogous to the Kibble-Zurek mechanism for defect generation in nonequilibrium classical phase transitions.

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The study of nonequilibrium phase transitions is wide ranging, touching on topics as diverse as the formation of structures in the early Universe [1] and the practicality of adiabatic quantum computing [2]. The so-called “Kibble-Zurek” (KZ) mechanism has been used to understand some universal features—principally the rate of topological defect formation—of quenches across classical phase transitions [1,3]. “Quench” in this context refers to rapidly varying a thermodynamic parameter in order to drive the system across the critical point of a phase transition and out of equilibrium for a finite time. The KZ theory has recently been extended to quantum phase transitions [4–12]. In contrast to the classical case, quantum phase transitions involve closed quantum mechanical evolution at zero temperature, for which quenches are accomplished by varying a parameter in the Hamiltonian in order to tune between different quantum phases. While the KZ mechanism has successfully been tested for classical transitions (e.g., on liquid crystals [13]), and spontaneous vortex formation has been observed during cooling an atomic gas through the Bose-Einstein condensation transition [14], experimental examination of quantum quenches in the context of the KZ mechanism has been scant. Notably, there is evidence that the formation of ferromagnetic domains in a spin-1 Bose-Einstein condensate can be attributed to a quantum quench [15]. In this work, we probe quantum quenches for a paradigm of quantum phase transitions—the Bose-Hubbard model—by using atoms confined in an optical lattice. In contrast to previous experiments [16], we quench from the Mott insulator (MI) to the superfluid (SF) state, and we systematically investigate the formation of excitations as the quench amplitude and rate are varied.

In our experiment, a cubic optical lattice formed from three intersecting pairs of 812 nm laser beams is superimposed on a parabolically confined ⁸⁷Rb Bose-Einstein condensate; details of our apparatus can be found in Ref. [17] and references therein. The atoms in the lattice are described by the inhomogeneous Bose-Hubbard model

with Hubbard tunneling energy t and interaction energy U , the ratio of which is controlled by tuning the lattice laser intensity to adjust the lattice potential depth s . By changing s , MI and SF phases can be sampled inhomogeneously in the gas [18]: For $s \geq 13E_R$ nested Mott-insulator and superfluid layers exist in the lattice, and for $s \leq 13E_R$ the gas is purely superfluid, as shown in Fig. 1 ($E_R = h/8md^2$, where m is the atomic mass, $d = 406$ nm is the lattice spacing, and h is Planck’s constant).

Quenching across the SF-MI phase transition is accomplished by adjusting s dynamically in such a way to transform the gas between equilibrium configurations with and without atoms in the MI phase present (Fig. 1). While quenches are possible on all relevant time scales, in this Letter we explore quenches that occur at rates $1/\tau_Q = d(t/U)/d\tau$ (τ is time) that are too slow to excite atoms into higher vibrational states in the lattice potential. How $1/\tau_Q$ compares with U/h and t/h is complicated because the phase boundary is crossed at a range of densities and t/U in the trap, and therefore the Hubbard energies U_c and t_c at the phase transition change during the quench. Despite this, the quenching rate is always slow compared with U_c , since $1/\tau_Q$ varies from $1 \times 10^{-3} - 0.2U_c/h$. The quench rate is not consistently fast or slow compared with t_c [19] or the confining trap frequencies, the geometric mean of which varies (because of confinement from the lattice beams) from 43 ± 2 Hz at $s = 0E_R$ to 82 ± 6 Hz at $s = 25E_R$. The trap aspect ratio changes from 1:1.3:1.8 to 1:1:1.2 across the same range.

The effect of variations in the fraction of atoms crossing the SF-MI phase boundary is investigated by quenching s linearly in 5 ms from a variable initial value s_0 to $s = 4E_R$ (corresponding to $t/U \approx 1$, i.e., the purely SF regime), as shown in the inset in Fig. 2 [20]. The ratio t/U changes nonlinearly during this quench, and therefore $1/\tau_Q$ (according to our definition) varies during the quench; in the large s limit, $t/U \propto e^{-2\sqrt{s}}$ [21]. The fraction of atoms crossing the phase boundary is varied by adjusting s_0 . The

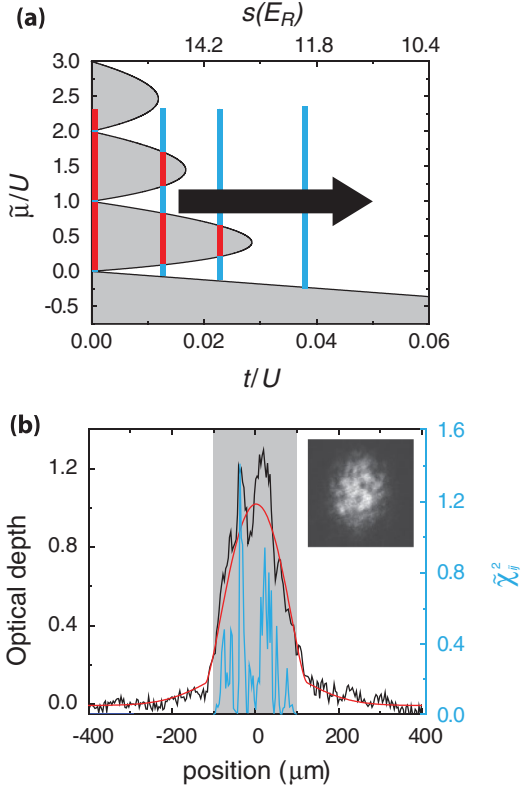


FIG. 1 (color). Quench across the MI-SF boundary (a) and excitation measure (b). (a) As shown by the vertical lines, the trapped gas samples a range of densities and effective chemical potentials $\tilde{\mu}$ (in the local-density approximation) [18]; MI regions in the gas are colored red and SF blue. Given the overall confining potential in our experiment and atom number $[(161 \pm 13) \times 10^3]$, averaged across all measurements, the maximum $\tilde{\mu}$ is roughly fixed at $2U$ for the measurements described here, which corresponds to a central filling of 4 atoms per site at low s and a central MI with 3 atoms per site at high s . A quench is accomplished by rapidly reducing s and thereby increasing t/U , as shown by the arrow. (b) A slice (black line) through a typical image (inset) taken after a quench for $s_0 = 25E_R$ is displayed. The image is fit to a smooth profile (red line), which is used to determine the deviation $\tilde{\chi}_{ij}^2$ (blue line) at each pixel in a masked region (gray).

data shown in Fig. 2 sample a range such that at high s_0 nearly all of the atoms start in the MI phase (with fillings ranging from 1 to 3 particles per site) and therefore cross the phase boundary and at low enough s_0 so that all of the atoms are in the SF phase, and consequently no atoms cross the phase boundary. The fraction of atoms in the MI phase before the quench, which is identical to the overall fraction of atoms traversing the phase boundary, is shown as a red line in Fig. 2 and is determined according to the measured atom number and a zero-temperature mean-field calculation in the local-density approximation [18]. After the quench, the lattice is turned off in $200 \mu\text{s}$. This “band-mapping” step [17]—which maps quasimomentum in the lattice to free momentum and suppresses atom

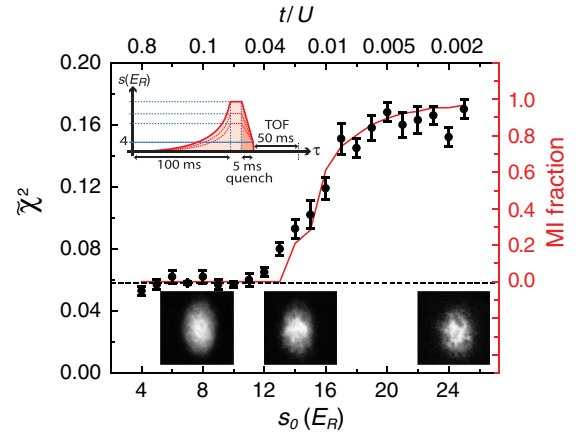


FIG. 2 (color online). Amount of excitation produced by quenching as the fraction crossing the phase SF-MI boundary is varied. The top left inset shows the experimental timeline; a magnetic field gradient is applied to support the atoms against gravity during TOF. Characteristic TOF images are shown as insets for $s_0 = 9, 15,$ and $25E_R$, and the error bars in this and the next figure are the standard deviation for the average taken over 5 images. There is a 7% systematic uncertainty to s_0 , which is calibrated by using Kapitza-Dirac diffraction. The overall uncertainty in the MI fraction (red line) ranges from 30% at $s_0 = 16E_R$ to 10% at $s_0 = 20E_R$; below $s_0 = 12E_R$ and above $s_0 = 22E_R$, the uncertainty in the MI fraction is zero.

diffraction—is necessary to improve the imaging signal-to-noise ratio given the long expansion times employed for these measurements.

The amount of excitation produced during the quench is determined by measuring the deviation from a smooth profile of time-of-flight (TOF) images taken after release of the trapped gas immediately following band mapping. The absorption image is taken after a relatively long 50 ms of free expansion, so that vortices, if present, are visible [22], and phase gradients related to other topological or wavelike excitations are converted into large density fluctuations [23]. We fit the image to a smooth function f that is a combination of a Thomas-Fermi profile and a Gaussian and measure the amount of excitation $\tilde{\chi}^2$ as the deviation from the smooth profile: $\tilde{\chi}^2 = \sum_{ij} \tilde{\chi}_{ij}^2 = \alpha \sum_{ij} \frac{(\mathcal{O}_{ij} - f_{ij})^2}{f_{ij}} / \sum_{ij} \mathcal{O}_{ij}$, where i and j index the pixels in the image within a mask defined by an imaging signal-to-noise ratio greater than 5, \mathcal{O} is the measured optical depth, and α is a proportionality constant that is determined by using a numerical simulation. We find that all of the images used in this work are well described by this fit—a condensate is present after the quench under all circumstances, and the condensate fraction varies from 0.35 to 0.6 across all of the data. While it was suggested in Ref. [24] that the condensate fraction may oscillate after the quench, we find no evidence for such behavior.

The measure $\tilde{\chi}^2$ is chosen such that it is related to the fraction of atoms in excited states for the trapped, weakly

interacting gas present before band mapping. The physical meaning of $\tilde{\chi}^2$ can be understood most straightforwardly for a one-dimensional noninteracting gas. In this case, the density profile after a sufficiently long TOF is the momentum distribution $n(q) = |\psi(q)|^2 = |\psi_0(q) + \delta\psi(q)|^2$, where $\delta\psi$ are plane-wave excitations, $\psi_0(q) = \sqrt{n_0(q)}$ is the ground-state condensate wave function, and we work in the momentum representation. After averaging over random excitation phases, the number of atoms in excited states is $\int dq |\delta\psi(q)|^2 = \int dq \{[n(q) - n_0(q)]^2\} / 2n_0(q)$. Given that $\int dq n_0(q)$ is the total number of atoms, $\tilde{\chi}^2$ is naturally interpreted as proportional to the number of excited atoms in the noninteracting limit.

Using a numerical simulation of the 3D time-dependent Gross-Pitaevskii equation, we determined that $\tilde{\chi}^2$ accurately reproduces the fraction of Bogoliubov excitations for a trapped condensate (even though the density after TOF is not the momentum distribution) and the constant α . We start the simulation with a condensate at equilibrium in a parabolic trap (by using the experimental parameters) and imprint Bogoliubov excitations under the local-density approximation for a range of wave vectors corresponding to $0.8\text{--}3 \mu\text{m}^{-1}$; the Thomas-Fermi radius of the gas is approximately $10 \mu\text{m}$ before release. Images are generated by time evolving the condensate wave function for a free expansion and then integrating through the imaging line of sight. The measure $\tilde{\chi}^2$ is determined for a range of excitation fractions averaged over 10 relative phases. We determine that $\tilde{\chi}^2$ is equal to the fraction of excited atoms for $\alpha = 10$ under simulated conditions. This method does not properly account for long-wavelength (i.e., trap-length-scale) excitations or topological excitations such as vortices, which are evident in the insets in Figs. 1 and 2.

As shown in Fig. 2, we find that $\tilde{\chi}^2$ is proportional to the fraction of atoms crossing the phase boundary. Below the emergence of the unit filling MI phase at $s_0 \approx 13E_R$, $\tilde{\chi}^2$ is constant at $\tilde{\chi}_0^2 \approx 0.06$ (determined by averaging over all images with $s_0 < 13E_R$ and indicated by the dashed line in Fig. 2), a value that is consistent with the combination of photodetection shot noise and technical noise present in our imaging system. Above $s_0 \approx 13E_R$, the degree of excitation grows, until $\tilde{\chi}^2$ saturates to approximately 0.17 at high lattice depth, for which more than 90% of the atoms are in the MI phase.

The behavior evident in Fig. 2 suggests that a Kibble-Zurek-like mechanism is responsible for generating excitations during the quench. In the KZ picture, the diverging relaxation time near the phase boundary “freezes in” fluctuations in the relative phase between atomic wave functions at different lattice sites present in the MI [6]. Some time after crossing the phase boundary, dynamics effectively restart, and the fluctuations develop into superfluid excitations, potentially including sound waves and topological excitations such as vortices. Given that only the

regions of the lattice that cross the SF-MI phase boundary will give rise to excitations, the direct relation between the fraction initially in the MI phase and the degree of excitation is strong evidence for KZ physics.

In the KZ scenario, the quench rate controls the number of excitations generated according to a power law that depends on the critical exponents for the phase transition. We measured this power law, as shown in Fig. 3, across 2 orders of magnitude in quench rate. For this measurement we quench the lattice potential depth starting from a gas composed nearly entirely of the MI phase at $s_0 = 20E_R$ (i.e., $t/U = 0.005$) according to $s(\tau) = 0.25 \ln^2[(\pi a_s / \sqrt{2}d) \frac{\tau}{\tau_Q} + e^{-2\sqrt{s_0}}]$ ($a_s \approx 5 \text{ nm}$ is the scattering length) so that $1/\tau_Q$ is approximately constant. Because $\tilde{\chi}^2$ may misinterpret excitations such as vortices and to estimate the quench-induced heating, we also measure the kinetic energy generated by the quench. The kinetic energy (KE) per particle is measured from TOF images according to $\text{KE} = m\langle r^2 \rangle / 2\tau_{\text{TOF}}^2$, with the second moment of the density distribution

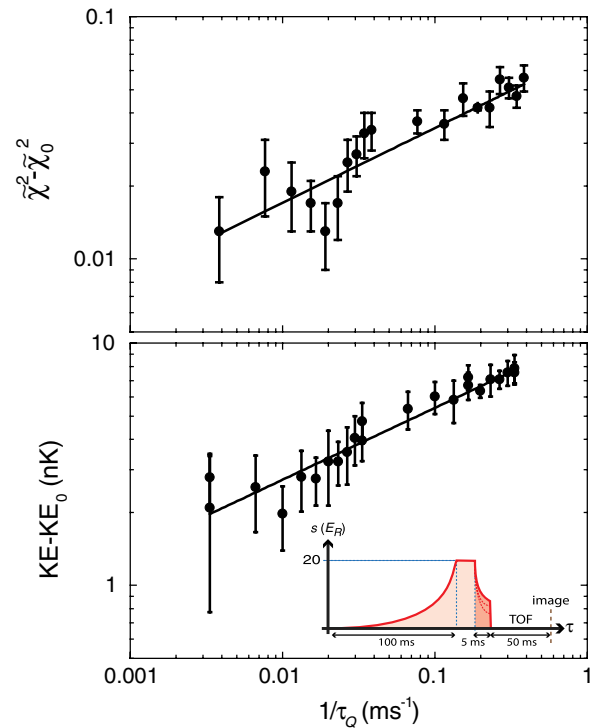


FIG. 3 (color online). Excitation dependence on quench rate. The amount of excitation and kinetic energy determined from TOF images is shown as the quench rate $1/\tau_Q$ is varied. The measured offset $\tilde{\chi}_0^2$ is subtracted from $\tilde{\chi}^2$. Analogously, the measured expansion energy without the quench KE_0 (determined by averaging across images with $s_0 < 13E_R$) is subtracted from KE. For comparison, the critical temperature for condensation in the trap before turning on the lattice and after band mapping is approximately 100 nK. The inset shows the experimental timeline.

$\langle r^2 \rangle = \frac{3}{2} \sum_{ij} \mathcal{O}_{ij} r_{ij}^2 / \sum_{ij} \mathcal{O}_{ij}$ after the TOF τ_{TOF} . The factor of 3/2 arises from assuming the energy is distributed equally among three directions.

A fit (solid lines) to the data in Fig. 3 reveals power laws $1/\tau_Q^r$ for $\tilde{\chi}^2$ and KE consistent within the fit uncertainty: $r = 0.31 \pm 0.03$ and 0.32 ± 0.02 , respectively. While there are numerous detailed theoretical predictions for the number of excitations produced during a quench across the MI-SF phase transition [4,6,8–10], none that we know of are directly applicable to our experiment. Generically, the size of domains associated with an excitation formed during the quench should scale as $\tau_Q^{\nu/(\nu z+1)}$ [25], where z is the dynamical critical exponent and the correlation length diverges as $(t/U - t_c/U_c)^{-\nu}$ near the phase boundary. In three dimensions, the density of excitations is therefore proportional to $1/\tau_Q^{3\nu/(\nu z+1)}$. For our experiment, nearly all of the atoms cross the “generic” phase transition and not the multicritical point at the “tip” of the MI “lobes.” In this case $\nu = 1/2$ and $z = 2$, and therefore the number of excitations should scale as $1/\tau_Q^{3/4}$, which is inconsistent with our data.

This disagreement may be explained by numerous issues that deserve more theoretical attention. For example, the spatially inhomogeneous nature of the gas gives rise to a phase transition “front” that moves through the gas; this has been examined in the context of certain classical and quantum phase transitions [5,26]. Depending on the nature of the excitations, n_{ex} may scale differently with τ_Q —e.g., if the excitations are only vortices, then $n_{\text{ex}} \sim 1/\tau_Q^{1/2}$, which is the areal density of vortex lines. Also, the finite size of the gas will affect quench dynamics, as discussed in Ref. [11] for the Bose-Hubbard model in 1D. Finally, since the data here were taken at a low but finite temperature (the initial condensate fraction was more than 90% before turning on the lattice), thermal effects may play an important role in the quench dynamics [7,27].

In conclusion, the method we have demonstrated provides a window into excited states and dynamics, which are beyond our current theoretical understanding in a wide variety of strongly interacting many-body quantum systems. Quench dynamics may also have significant consequences for thermometry in optical lattice experiments [28]. One commonly employed technique to estimate temperature in a lattice is to slowly turn off the lattice potential, measure temperature, and then infer entropy in the lattice assuming that the turn-off was adiabatic. We find across a wide range of linear lattice quench rates that adiabaticity is violated; for example, for a quench from $s_0 = 20E_R$, $\tilde{\chi}^2$ decreases from 0.17 to only 0.12 for turn-off times varying from 5 to 25 ms.

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