Resummation of Massive Gravity

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We construct four-dimensional covariant nonlinear theories of massive gravity which are ghost-free in the decoupling limit to all orders. These theories resum explicitly all the nonlinear terms of an effective field theory of massive gravity. We show that away from the decoupling limit the Hamiltonian constraint is maintained at least up to and including quartic order in nonlinearities, hence excluding the possibility of the Boulware-Deser ghost up to this order. We also show that the same remains true to all orders in a similar toy model.

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Introduction.—Whether there exists a consistent extension of general relativity by a mass term is a basic question of a classical field theory. A small graviton mass could also be of a significant physical interest, notably for the cosmological constant problem.

A ghost-free linear theory of massive spin-2—the Fierz-Pauli model [1]—had been notoriously hard to generalize to the nonlinear level [2]: In addition to the general relativity momentum constraint, also the Hamiltonian constraint gets lost in a typical massive theory; as a result, the sixth degree of freedom—the Boulware-Deser (BD) ghost—emerges as a mode propagating on otherwise physically meaningful local backgrounds (e.g., on a background of a lump of matter). This can be explicitly seen in the effective field theory (EFT) approach to massive gravity [3] in the decoupling limit [3,4], where the problem manifests itself in the Lagrangian for the helicity-0 component of the massive graviton. This Lagrangian generically contains nonlinear terms with more than two time derivatives. The latter give rise to the sixth degree of freedom on local backgrounds, while, in general, these terms lead to the loss of well-posedness of the Cauchy problem for the helicity-0 field theory [3,4].

A step forward has been made recently in [5] where it was shown that (a) the coefficients of the EFT can be chosen so that the decoupling limit Lagrangian is ghost-free—this involves choosing the ''appropriate coefficients'' order by order, and an algorithm was set for this procedure to an arbitrary order—and (b) once these coefficients are chosen in the effective Lagrangian, only a few terms up to the quartic order survive in the decoupling limit; all the higher-order terms vanish identically. Moreover, the surviving terms are unique as their structure is fixed by symmetries [5,6]. The above results enable one to define a classical EFT that is consistent in the decoupling limit [5]. This theory was not considered in [2].

In the present work we build on the above findings, and go far beyond them. In particular, we note the following. (i) We construct Lagrangians that *automatically* produce the ''appropriate coefficients'' once expanded in powers of the fields; these give rise to theories that are ghost-free automatically to all orders in the decoupling limit. (ii) Using the obtained Lagrangians we study the issue of the BD ghost away from the decoupling limit; we show that the appropriately modified Hamiltonian constraint is maintained at least up to and including quartic order, hence excluding the possibility of the BD ghost up to this order. We also discuss an analogous (1 + 1)-dimensional model and show explicitly how the Hamiltonian constraint is preserved to all orders.

To emphasize, the requirement that the theory be ghost-free in the decoupling limit leads to resummation of an infinite number of terms of the classical EFT away from the decoupling limit. Because of this resummation, it becomes straightforward, but still technically involved, to address a more ambitious question of the sixth mode away from the decoupling limit [7].

Formalism.—Define the tensor $H_{\mu\nu}$ as the covariantization of the metric perturbation, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} = H_{\mu\nu} + \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$, where the four Stückelberg fields ϕ^a transform as scalars, and $\eta_{ab} = (-1, 1, 1, 1)$ [3]. The helicity-0 mode π of the graviton can be extracted by expressing $\phi^a = (x^a - \eta^{a\mu}\partial_\mu\pi)$, such that

$$H_{\mu\nu} = h_{\mu\nu} + 2\Pi_{\mu\nu} - \eta^{\alpha\beta}\Pi_{\mu\alpha}\Pi_{\beta\nu}, \qquad \Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\pi. \tag{1}$$

We may therefore define the following quantity:

$$\mathcal{K}^\mu_\nu(g, H) = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu} = -\sum_{n=1}^{\infty} \bar{d}_n (H^n)^\mu_\nu, \tag{2}$$

with

$$\bar{d}_n = \frac{(2n)!}{(1-2n)(n!)^2 4^n}. \tag{3}$$

Here $H^\mu_\nu = g^{\mu\alpha}H_{\alpha\nu}$, and $(H^n)^\mu_\nu = H^\mu_{\alpha_1}H^{\alpha_1}_{\alpha_2}\ldots H^{\alpha_{n-1}}_\nu$ denotes the product of n tensors H^α_β. Below, unless stated otherwise, all the contractions are made by using the metric $g_{\mu\nu}$. The tensor $\mathcal{K}_{\mu\nu} = g_{\mu\alpha}\mathcal{K}^\alpha_\nu$ is defined in such a way that

$$\mathcal{K}_{\mu\nu}(g, H)|_{h_{\mu\nu}=0} \equiv \Pi_{\mu\nu}. \quad (4)$$

We use the same notation as in [4] where square brackets $[\ldots]$ represent the trace of a tensor contracted by using the Minkowski metric, e.g., $[\Pi] = \eta^{\mu\nu}\Pi_{\mu\nu}$ and $[\Pi^2] = \eta^{\alpha\beta}\eta^{\mu\nu}\Pi_{\alpha\mu}\Pi_{\beta\nu}$, while angle brackets $\langle\ldots\rangle$ represent the trace with respect to the physical metric $g_{\mu\nu}$, so that $\langle H\rangle = g^{\mu\nu}H_{\mu\nu}$ and $\langle H^2\rangle = g^{\alpha\beta}g^{\mu\nu}H_{\alpha\mu}H_{\beta\nu}$.

We are first interested in the decoupling limit [11]. For that, let us define the canonically normalized variables $\hat{\pi} = \Lambda_3^3\pi$ with $\Lambda_3^3 = m^2M_{\rm Pl}$ and $\hat{h}_{\mu\nu} = M_{\rm Pl}h_{\mu\nu}$. The limit is then obtained by taking $M_{\rm Pl} \to \infty$ and $m \to 0$ while keeping $\hat{\pi}$, $\hat{h}_{\mu\nu}$, and the scale Λ_3 fixed. First, we construct an explicit example of a nonlinear theory that bears no ghosts in the decoupling limit and then give a general formulation and show the absence of the BD ghost beyond the decoupling limit in quartic order.

Massive gravity.—The consistency of the Fierz-Pauli term (h^2-$h^2_{\mu\nu}$) relies on the fact that the Lagrangian

$$\mathcal{L}^{(2)}_{\rm der} = [\Pi]^2 - [\Pi^2] \quad (5)$$

is a total derivative. To ensure that no ghost appears in the decoupling limit, it is sufficient to extend $\mathcal{L}^{(2)}_{\rm der}$ covariantly away from $h_{\mu\nu} = 0$; i.e., replace $[\Pi]$ and $[\Pi^2]$ by $\langle\mathcal{K}\rangle$ and $\langle\mathcal{K}^2\rangle$, respectively, so that the total Lagrangian reads as

$$\mathcal{L} = \frac{M^2_{\rm Pl}}{2}\sqrt{-g}\left(R - \frac{m^2}{4}\mathcal{U}(g, H)\right), \quad (6)$$

with the potential $\mathcal{U}$ expressed as an expansion in H as

$$\mathcal{U}(g, H) = -4(\langle\mathcal{K}\rangle^2 - \langle\mathcal{K}^2\rangle) = -4\left(\sum_{n\geq 1}\bar{d}_n\langle H^n\rangle\right)^2 - 8\sum_{n\geq 2}\bar{d}_n\langle H^n\rangle. \quad (7)$$

Expanding this expression to quintic order,

$$\begin{aligned}\mathcal{U}(g, H) = {}& (\langle H^2\rangle - \langle H\rangle^2) - \tfrac{1}{2}(\langle H\rangle\langle H^2\rangle - \langle H^3\rangle) \\ & - \tfrac{1}{16}(\langle H^2\rangle^2 + 4\langle H\rangle\langle H^3\rangle - 5\langle H^4\rangle) \\ & - \tfrac{1}{32}(2\langle H^2\rangle\langle H^3\rangle + 5\langle H\rangle\langle H^4\rangle - 7\langle H^5\rangle) + \cdots,\end{aligned} \quad (8)$$

we recover the decoupling limit presented in [5] with the special indices $c_3 = d_5 = f_7 = 0$.

The Lagrangian (6) with (7) can be obtained from the Lagrangian with a finite number of terms

$$\begin{aligned}\mathcal{L}_\lambda = {}& \frac{M^2_{\rm Pl}}{2}\sqrt{-g}[R - m^2(\mathcal{K}^2_{\mu\nu} - \mathcal{K}^2)] \\ & + \sqrt{-g}\lambda^{\mu\nu}(g^{\alpha\beta}\mathcal{K}_{\mu\alpha}\mathcal{K}_{\beta\nu} - 2\mathcal{K}_{\mu\nu} + H_{\mu\nu}),\end{aligned} \quad (9)$$

where $\mathcal{K}_{\mu\nu}$ is an independent tensor field that gets related to $H_{\mu\nu}$ as in (2) due to the constraint enforced by the Lagrange multiplier λ^μ_ν. Note that the expression (2) can be rewritten as $\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{\partial^\mu\phi^a\partial_\nu\phi^b\eta_{ab}}$, which gives a square root structure in the full Lagrangian.

Decoupling limit.—It is straightforward to notice that the leading contribution to the decoupling limit

$$\sqrt{-g}\,\mathcal{U}(g, H)|_{h_{\mu\nu}=0} = -4[(\Box\pi)^2 - (\partial_\alpha\partial_\beta\pi)^2] \quad (10)$$

is a total derivative. The resulting interaction Lagrangian in the decoupling limit is then given by [5]

$$\mathcal{L}_{\rm int} = \hat{h}_{\mu\nu}\bar{X}^{\mu\nu}, \quad (11)$$

with

$$\bar{X}^{\mu\nu} = -\frac{M^2_{\rm Pl}m^2}{8}\frac{\delta}{\delta h_{\mu\nu}}[\sqrt{-g}\,\mathcal{U}(g, H)]|_{h_{\mu\nu}=0}. \quad (12)$$

The expression for $\bar{X}$ simplifies to

$$\begin{aligned}\bar{X}_{\mu\nu} = {}& \tfrac{1}{2}\Lambda_3^3[\Pi\eta_{\mu\nu} - \Pi_{\mu\nu} + \Pi^2_{\mu\nu} - \Pi\Pi_{\mu\nu} \\ & + \tfrac{1}{2}(\Pi^2 - \Pi^2_{\alpha\beta})\eta_{\mu\nu}].\end{aligned} \quad (13)$$

The tensor $\bar{X}_{\mu\nu}$ is conserved and gives rise to at most second-order derivative terms in the equations of motion. This tensor can be expressed as the product of two epsilon tensors appropriately contracted with powers of $\Pi_{\mu\nu}$ [6]. For the potential (7), the Lagrangian in the decoupling limit is then given by (see Ref. [5])

$$\mathcal{L}^{\rm lim}_{\Lambda_3} = -\tfrac{1}{2}\hat{h}^{\mu\nu}(\hat{\mathcal{E}}\hat{h})_{\mu\nu} + \hat{h}_{\mu\nu}\bar{X}^{\mu\nu}, \quad (14)$$

where $\hat{\mathcal{E}}$ denotes the standard Einstein operator normalized as in [5], and this result is exact (i.e., no higher-order corrections). Notice that this is also in agreement with the results of [5] up to quintic order, for the special case $c_3 = d_5 = f_7 = 0$, but we explicitly demonstrate here that this result remains valid to all orders.

General formulation.—As mentioned in [5], at each order in the expansion there exists a total derivative

$$\mathcal{L}^{(n)}_{\rm der}(\Pi) = -\sum_{m=1}^{n}(-1)^m\frac{(n-1)!}{(n-m)!}[\Pi^m]\mathcal{L}^{(n-m)}_{\rm der}(\Pi), \quad (15)$$

with $\mathcal{L}^{(0)}_{\rm der}(\Pi) = 1$ and $\mathcal{L}^{(1)}_{\rm der}(\Pi) = [\Pi]$. These total derivatives generalize the "Fierz-Pauli" structure used previously to all orders; only the $n \leq 4$ terms are nonzero [5]. Then, the potential of any theory of massive gravity with no ghosts in the decoupling limit can be expressed nonlinearly as

$$\mathcal{U}(g,H) = -4\sum_{n\geq 2}\alpha_n \mathcal{L}_{\rm der}^{(n)}(\mathcal{K}), \qquad (16)$$

where $[\Pi^m]$ in (15) should be replaced by $\langle \mathcal{K}^m\rangle$ and expressed in terms of g and H using (2).

Here again this specific structure ensures that the leading contribution to the decoupling limit is manifestly a total derivative by construction:

$$\sqrt{-g}\,\mathcal{U}(g,H)|_{h_{\mu\nu}=0} = \text{total derivative}, \qquad (17)$$

and the resulting interaction Lagrangian can be derived by noticing the general relation

$$\frac{\delta}{\delta h^{\mu\nu}}\langle \mathcal{K}^n\rangle|_{h_{\mu\nu}=0} = \frac{n}{2}(\Pi^{n-1}_{\mu\nu} - \Pi^n_{\mu\nu}), \qquad (18)$$

so that

$$\frac{\delta}{\delta h^{\mu\nu}}[\sqrt{-g}\,\mathcal{L}_{\rm der}^{(n)}(\mathcal{K})]|_{h_{\mu\nu}=0} = \sum_{m=0}^{n}\frac{(-1)^m n!}{2(n-m)!}(\Pi^m_{\mu\nu} - \Pi^{m-1}_{\mu\nu})\mathcal{L}_{\rm der}^{(n-m)}(\Pi), \qquad (19)$$

by using the notation $\Pi^0_{\mu\nu} = \eta_{\mu\nu}$ and $\Pi^{-1}_{\mu\nu} = 0$. The decoupling limit Lagrangian is then given by (14) with the same definition (12) for the tensor $X_{\mu\nu}$, giving here

$$\bar{X}_{\mu\nu} = \frac{1}{2}\Lambda_3^3\sum_{n\geq 2}\alpha_n(X^{(n)}_{\mu\nu} + nX^{(n-1)}_{\mu\nu}), \qquad (20)$$

with

$$X^{(n)}_{\mu\nu} = \sum_{m=0}^{n}(-1)^m\frac{n!}{2(n-m)!}\Pi^m_{\mu\nu}\mathcal{L}_{\rm der}^{(n-m)}(\Pi). \qquad (21)$$

The special theory found in [8,9] corresponds to the specific choices of coefficients $\alpha_2 = 1$ and $\alpha_3 = -1/2$; see Ref. [12]. However, we emphasize that the results in this Letter are now valid to all orders in nonlinearities.

Furthermore, at each order the tensors $X^{(n)}_{\mu\nu}$ are given by the recursive relation $X^{(n)}_{\mu\nu} = -n\Pi^\alpha_\mu X^{(n-1)}_{\alpha\nu} + \Pi^{\alpha\beta}X^{(n-1)}_{\alpha\beta}\eta_{\mu\nu}$, with $X^{(0)}_{\mu\nu} = 1/2\eta_{\mu\nu}$. So since $X^{(4)}_{\mu\nu} \equiv 0$, all these tensors vanish beyond the quartic one, $X^{(n)}_{\mu\nu} \equiv 0$ for any $n \geq 4$, and the decoupling limit therefore stops at that order, as previously implied in [5].

The theory with (16) has a well-posed Cauchy problem on arbitrary backgrounds (some of which could flip the sign of the π kinetic term and be unstable [6]).

Boulware-Deser ghost.—The previous argument ensures the absence of a ghost in the decoupling limit, but it is feasible that the ghost reappears beyond the decoupling limit and is simply suppressed by a mass scale larger than Λ_3. Certain arguments have hinted towards the existence of a BD ghost [4]. We reanalyze the arguments here and show the absence of ghosts within the regime studied. To compute the Hamiltonian, we fix the unitary gauge for which $\pi = 0$, such that

$$\langle H^n\rangle = \sum_{\ell\geq 0}(-1)^\ell C_\ell^{\ell+n-1}[h^{\ell+n}], \qquad (22)$$

where the C^n_m are the Bernoulli coefficients. We also focus on the case where $\alpha_2 = 1$ and $\alpha_n = 0$ for $n \geq 3$. Below, we work in terms of the Arnowitt-Deser-Misner variables [13] $g^{00} = -N^{-2}$, $g_{0i} = N_i$, and $g_{ij} = \gamma_{ij}$, with the lapse $N = 1 + \delta N$, and the three-dimensional metric $\gamma_{ij} = \delta_{ij} + h_{ij}$. In terms of these variables, the potential is then of the form

$$\sqrt{-g}\,\mathcal{U} = \mathcal{A} + \delta N\mathcal{B} + N_iN_j[-2\delta^{ij} + \mathcal{C}^{ij} + \delta N(\delta^{ij} + \mathcal{D}^{ij}) - \tfrac{1}{2}\delta N^2\delta^{ij} - \tfrac{1}{8}\delta^{ij}N_k^2], \qquad (23)$$

where $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}^{ij}$, and $\mathcal{D}^{ij}$ are functions of h_{ij}, at least first order in perturbations, and $\mathcal{C}^{ij} + 2\mathcal{D}^{ij} = -\frac{1}{2}h^{ij} + \mathcal{O}(h^2_{ij})$, and in this section we raise and lower the spacelike indices using δ_{ij}. Notice that this is in agreement with the analysis performed in [4] and corresponds to setting the coefficients in Eq. (43) of [4] to $A = B = D = E = 0$, while $C = -1/2$. However, we emphasize here that the presence of the term $CN_i^2\delta N^2$ does not signal the presence of a ghost despite the fact that the equations for δN and N_j naively appear to determine δN and N_j: To see this explicitly, one can solve the equation for N_j and substitute the solution obtained order by order into the equation for δN; then, in the latter equation there is a cancellation of the cubic order term containing δN. Hence, to that order δN disappears from that equation, which ends up being a constraint for h_{ij}. The cancellation of δN and the resulting constraint are consequences of the no-ghost condition in the decoupling limit.

The existence of the constraint can be shown more directly in the Hamiltonian formalism in the quartic order (corresponding to the cubic order in the equations) by using a redefined shift n_i:

$$N_j = (\delta^i_j + \tfrac{1}{2}\delta N\delta^i_j - \tfrac{1}{8}\delta Nh^i_j)n_i \equiv L^i_jn_i; \qquad (24)$$

then, the Hamiltonian is of the form

$$\mathcal{H} = \frac{M^2_{\rm Pl}}{2}\sqrt{\gamma}(NR^0 + N_jR^j) + \frac{m^2M^2_{\rm Pl}}{8}(\mathcal{A} + \mathcal{B}\delta N) - \frac{m^2M^2_{\rm Pl}}{4}L^{ij}\left(n_in_j - \frac{1}{2}\mathcal{C}^k_in_jn_k + \frac{1}{16}n_k^2n_in_j\right), \qquad (25)$$

up to quartic order in the metric perturbations. One can check that the variation of the Hamiltonian (25) with respect to the shift n_i gives an equation which is independent of N and serves to determine n_j. Moreover, the lapse remains a Lagrange multiplier even after integration over the shift, hence giving rise to a Hamiltonian constraint on the physical variables. Whether this constraint gives rise to a secondary constraint, and whether the system should be quantized as a first- or second-class system, is a separate interesting question. The mere existence of the Hamiltonian constraint is sufficient to claim the absence of the BD ghost to that order [14]. This remains true in the presence of sources coupled covariantly to $g_{\mu\nu}$; the redefinition (24) does not involve the canonical momenta and does not lead to any complications.

The Hamiltonian evaluated on the constraint surface is proportional to m^2, and whether or not it is positive semidefinite is determined by $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}^{ij}$, and $\mathcal{D}^{ij}$. Thus, in general, certain backgrounds could have slow tachyonlike instabilities; however, this is a separate issue from that of the BD ghost that we clarified above.

(1 + 1)-D massive gravity.—Proving the absence of the BD ghost in complete generality beyond the quartic order is a grand task, which we save for a separate study. However, we can analyze here a similar issue in a (1 + 1)-D toy model, where we consider the Hamiltonian

$$\mathcal{H} = \sqrt{\gamma}\left[NR^0 + \gamma^{11}N_1R_1 + \frac{m^2}{4}N\mathcal{U}(g, H)\right], \quad (26)$$

with R^0 and R_1 arbitrary functions of the spacelike metric γ_{11} and its conjugate momentum, and the potential $\mathcal{U}$ is given in (7). In 1 + 1 dimensions, it is relatively easy to check that the Hamiltonian then takes the exact form

$$\mathcal{H} = \sqrt{\gamma}[NR^0 + \gamma^{11}N_1R_1 - 2m^2N] - 2m^2[1 - \sqrt{(\sqrt{\gamma} + N)^2 - \gamma^{11}N_1^2}] \quad (27)$$

and seemingly includes terms quadratic in the lapse when working at quartic order and beyond:

$$\mathcal{H} \sim \mathcal{H}_0 + \mathcal{H}_1 N + m^2 N_1^2 N^2 + \cdots. \quad (28)$$

By stopping the analysis at this point, one would infer that the lapse no longer enforces a constraint. However, in terms of the redefined shift n_1, $N_1 = n_1(\gamma_{11} + N\sqrt{\gamma})$, the Hamiltonian takes the much more pleasant form

$$\mathcal{H} = \sqrt{\gamma}NR^0 - 2m^2(1 + \sqrt{\gamma}N) + (\sqrt{\gamma} + N)(n_1R_1 + 2m^2\sqrt{1 - n_1^2}), \quad (29)$$

which remains linear in the lapse, even after integration over the shift. It is again straightforward to see that the lapse does enforce a constraint and does so for an "arbitrary background."

Outlook.—We have given a covariant nonlinear realization of massive gravity in 4D which (i) is automatically free of ghosts in the decoupling limit, to all orders in nonlinearities, and (ii) keeps the lapse as a Lagrange multiplier away from the decoupling limit, at least up to quartic order in nonlinearities. These findings constitute what we believe is a very significant step forward and strongly suggests the existence of an entirely ghost-free classical theory of massive gravity. However, to prove this statement in complete generality, two important ingredients are yet missing: (a) proving that the lapse remains a Lagrange multiplier to all orders; (b) checking whether the secondary constraint is generated or not and whether the theory could be canonically quantized as a first- or second-class system. For the consistency of the theory at the quantum loop level, one would have to establish the existence of a symmetry which protects this theory against quantum corrections that could revive the ghost. These points will be explored in a further study.

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