# A New Tolman Test of a Cosmic Distance Duality Relation at 21 cm

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Under certain general conditions in an expanding universe, the luminosity distance ($d_L$) and angular diameter distance ($d_A$) are connected by the Etherington relation as $d_L = d_A(1 + z)^2$. The Tolman test suggests the use of objects of known surface brightness, to test this relation. In this Letter, we propose the use of a redshifted 21 cm signal from disk galaxies, where neutral hydrogen (HI) masses are seen to be almost linearly correlated with surface area, to conduct a new Tolman test. We construct simulated catalogs of galaxies, with the observed size-luminosity relation and realistic redshift evolution of HI mass functions, likely to be detected with the planned Square Kilometer Array. We demonstrate that these observations may soon provide the best implementation of the Tolman test to detect any violation of the cosmic distance duality relation.

*Introduction.*—Observed fluxes and apparent sizes of cosmologically distant objects determine their $d_L$ and $d_A$, respectively, which are tied by the Etherington [1] relation as $d_L = d_A(1 + z)^2$. This relation holds true for photons traveling on unique null geodesics in an expanding universe described by a metric theory of gravity and is guaranteed by phase space conservation of photons along with Lorentz invariance. Testing the Etherington relation is important because any violation of it would be clear evidence of new physics. At the same time, it is important to distinguish between violations occurring due to new physics and astrophysical effects; often the signatures of these effects may not be distinguishable. Among the important astrophysical effects are dimming due to the intergalactic dust and gravitational lensing effects [2], while signatures of new physics could arise, for example, from interactions of photons with the dark sector.

Tolman [3] suggested that measurements of angular extension, luminosity, and redshift of objects with a standard surface brightness may be used to test this cosmic distance duality relation (CDDR) [4]. In this Letter, we propose 21 cm observations of disk galaxies as a new Tolman test that promises a systematics-free and clean probe of the CDDR. The advantage of our test over many of the existing ones is that it is immune to many astrophysical uncertainties. We show that future observations like those with the planned Square Kilometer Array (SKA) would easily be able to probe this relation up to $z \sim 1$. We also discuss possible strategies to obtain significantly tighter constraints on any possible violation of this relation.

*Existing tests.*—Sandage and Lubin [5] performed optical photometry of nearby galaxies and compared them with a sample of early-type galaxies at high redshift [6], performing the Tolman surface brightness test. The surface brightness was seen to be falling at a rate significantly slower than the $\propto (1 + z)^{-4}$ expected from the Etherington relation. However, the authors suggested that this discrepancy

may be reconciled with an expanding universe by invoking luminosity evolution of early-type galaxies.

Another way to test the CDDR is to make independent measurements of the cosmic distances $d_L$ and $d_A$ using different objects. References [2,7,8] provide upper limits on the cosmic transparency by observing the flux dimming of standard candles [9] along with independent measurements of a cosmic distance scale through baryon acoustic oscillation measurements [10] and $H(z)$ [11] data. However, all these methods suffer from uncertainties due to absorption of optical photons by dust, and also the possibility of being affected by lensing magnification effects.

Holanda *et al.* [12] have also examined the consistency of the CDDR through combined x-ray and Sunyaev-Zel'dovich effect observations of galaxy clusters. Although this method has the advantage of being unaffected by lensing magnification bias, their results depend largely on the cluster model and could suffer from unknown systematics.

*Our proposed test at 21 cm.*—The Arecibo Dual-Beam Survey (ADBS) covered ~430 deg² of the sky using the Arecibo main beam with a velocity coverage of $-654$ to $7977 \mathrm{~km \, s^{-1}}$ [13] to detect the emission at 21 cm from galaxies. Out of the 265 identified galaxies, only a few were resolved in the Arecibo observations. Rosenberg and Schneider [14] made spatially resolved D-array follow-up observations with the Very Large Array (VLA) for 84 of these galaxies. Of these, 50 well resolved sources were used [13] to determine their neutral hydrogen (HI) masses and surface areas. In this study we use this sample to calibrate the size-luminosity correlations to be used in the generation of our mock catalogs.

HI masses and physical sizes of ADBS [13] galaxies are found to be consistent with a nearly constant average HI surface density of the order $\sim 10^7 M_\odot \mathrm{~kpc}^{-2}$ [14]. The HI masses of individual ADBS galaxies, spanning more than 3

orders of magnitude, deviate by only $\sim 0.13$ dex ($1\sigma$) from those expected from a constant HI surface density. This surprising relation has recently been explained by [15] as the result of self-regulated star formation, driven by the competition between gravitational instabilities in a rotationally supported disk and mechanical feedback from supernovae. Silk [16] shows that supernovae drive up the porosity of the gas disk and stabilize it by increasing the gas dispersion velocity. Analytic [16] and simulation [17] results suggest that the gas dispersion velocity driven by supernova feedback is insensitive to the star formation rate (SFR)[18] as well as metallicity [19]. We find the redshift dependent effects in the evolution of the luminosity-area relation to be much lower than the intrinsic scatter in the HI masses.

The present galaxy sample lies on the fundamental line [15] even though the sample probes a large range in size and metallicity from dwarf galaxies to giant spirals. These objects with nearly constant surface brightness which is both documented [14] and understood [15], then provide us with astrophysical sources to give a direct measurement of the distance ratio $d_L/d_A$ at various redshifts.

The HI emission starts out at 21 cm radio in the rest frame of the source. It gets redshifted out of the resonance line to longer wavelengths as the universe expands during its travel to the observer. The universe is essentially transparent at these wavelengths as their propagation is not affected by gas and dust. Only HI can reabsorb the 21 cm emission before it is redshifted out of the resonance line. But the absence of the Gunn and Peterson [20] absorption feature for quasars below $z \lesssim 6$, confirms that there are hardly any HI ($n_{HI} \lesssim 6 \times 10^{-11}$ atoms cm$^{-3}$) along these lines of sight. Considering the cross section of the 21 cm hyperfine transition, we get the optical depth as $\sim 10^{-7}$, which is negligible when compared to the intrinsic scatter. This is a clear advantage over other tests of the CDDR at visible wavelengths where the effect of dimming due to dust is inseparable from effects due to new physics. Additionally, the use of the same objects to measure the ratio $d_L/d_A$ also means that there is no risk of lensing magnification bias, since the surface brightness of a single source remains preserved after gravitational lensing [21].

*Simulated galaxy catalogs.*—To assess the potential of future radio observations in probing the CDDR using our proposed Tolman test, we create mock galaxy catalogs with a realistic redshift distribution that would be expected to be seen in these surveys (see Fig. 1). We briefly outline the procedure involved in doing so.

Properties of the Galaxies: The luminosity, $L$ and area, $A$ of disk galaxies are seen to be related almost linearly [14,15],

$$L = 10^B (A/A_0)^\alpha \text{ Jy km s}^{-1} \text{ Mpc}^2 \qquad (1)$$

where we choose $A_0 = 1$ Mpc$^2$. Figure 2 shows the constraints on the parameters $B$ and $\alpha$ with the existing data of the ADBS galaxies.

We use the Schechter mass function from Zwaan *et al.* [22] to model the HI mass distribution at low redshifts,

$$\frac{dN}{dV dM_{HI}} = \frac{\theta^*}{M_{HI}^*}\left(\frac{M_{HI}}{M_{HI}^*}\right)^\beta \exp\left(-\frac{M_{HI}}{M_{HI}^*}\right) \qquad (2)$$

with $\beta = -1.37$ and $\log(M_{HI}^*/M_\odot) = 9.80$. We hold $M_{HI}^*$ constant as it is likely to be regulated by the mass scale beyond which AGN feedback dominates supernova feedback [23]. However, as HI mass is directly related to the star formation rate, we choose a redshift dependent normalization for the HI mass function, $\theta^*(z) = 6 \times 10^{-3}(1 + 7.6z)/[1 + (z/3.3)^{5.3}]$, such that it matches the shape of SFR($z$) from [24]. Any uncertainty in modeling the mass function here, only affects the number of galaxies yielded by the simulated survey and has no effect on the actual Tolman test.

Following Rohlfs and Wilson [25], we relate the HI masses to the flux $f$ as, $M_{HI}/M_\odot = 2.356 \times 10^5 f d_L^2 (1 + z)^{-1}$ [26], where $f$ is in Jy km s$^{-1}$ and $d_L$ is in Mpc.

Corresponding to the flux cut of a given survey, we calculate the limiting luminosities $L(z)$ and corresponding masses $M_{HI}^{(lim)}(z)$. The flux from the galaxy is assumed to be spread out, due to rotation, over a velocity range of $\Delta V = 200$ km s$^{-1}$. If one assumes the weak scaling ($\Delta V \propto M_{HI}^{0.3}$) from Briggs and Rao [27] and inclination effects, one would expect $\sim 30\%$ more sources [28]. In addition to
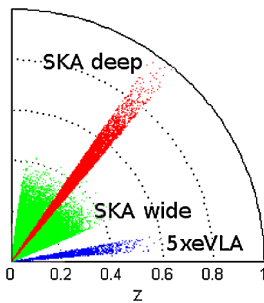


| Survey | area (sq. deg.) | flux density cut ($\mu$Jy) | beam size (arc-sec) | $N_{gal}$ | $\Delta B$ | $\Delta \alpha$ | $\Delta \epsilon$ |
|---|---|---|---|---|---|---|---|
| ADBS + VLA | 430 | *selected follow-up* | $\sim 60$ | 50 | 0.146 | 0.038 | - |
| 5×eVLA | 1 | 20 | 4 | 1280 | 0.072 | 0.019 | 0.095 |
| SKA *deep* | 1 | 6 | 0.2 | 10k | 0.026 | 0.006 | 0.026 |
| SKA *wide* | 100 | 60 | 0.2 | 36k | 0.012 | 0.003 | 0.026 |
| SKA *deep+wide* | - | - | 0.2 | 46k | 0.007 | 0.002 | 0.010 |

FIG. 1 (color online).   The figure on the left represents the expected redshift distribution from some upcoming surveys. All angles in this figure are scaled up by a factor of 4 for representative purposes. The survey specifications and the $1\sigma$ marginalized constraints on the parameters $B$, $\alpha$ and $\epsilon$ are listed in the table on the right.

the flux cutoff we also impose a size cutoff according to the beam size of the survey. The number of galaxies above the limiting mass of the survey with a coverage area $\Delta\Omega$ in a redshift bin $\Delta z$ is given by,

$$\Delta\Omega\Delta z \frac{dV}{dzd\Omega} \int_{M_{\text{HI}}^{(\text{lim})}(z)}^{\infty} \frac{dN}{dVdM_{\text{HI}}} dM_{\text{HI}} \qquad (3)$$

where $V$ is the comoving volume up to a redshift $z$. This gives the luminosity and redshift distribution of galaxies and is used to create the simulated catalogs of galaxy data containing $z$, $L$, $S$ and $\sigma(S)$, where $\sigma(S)$ obtains contribution mostly from the intrinsic scatter in the data as calibrated from the ADBS galaxies.

Surveys at 21 cm: The properties of the simulated surveys are presented in Fig. 1. "ADBS + VLA" represents the constraints on $B$ and $\alpha$ obtained using the real data [14]. "$5 \times$ eVLA" represents results from a simulated catalog, assuming 80 hrs of on-source time using a hypothetical telescope with 5 times the sensitivity of the present Extended VLA (eVLA). This may soon become a reality with upgrades at the eVLA or upcoming pathfinder missions of the SKA. "SKA deep" represents a single snapshot (2 minutes) with the fully operational SKA. "SKA wide" represents the same amount of SKA time, but spent on 100 different fields instead of 1. While the SKA deep is good for constraining any redshift evolution in the CDDR, the SKA wide is good for constraining the actual size-luminosity relation of the galaxies.

*Results.*—We modify the Etherington relation to test for any violation by using a simple parametrized form, $d_L/d_A = (1 + z)^{2+\epsilon}$. Using the definitions $d_A^2(z) = A/S$ and $d_L^2(z) = (1 + z)L/(4\pi f)$ [26], we relate the observed angular area ($S$) to observed flux ($f$) as follows,

$$S(z, f) = \left(\frac{4\pi f(1 + z)^{3+2\epsilon}}{10^B d_A(z)^{2(\alpha-1)}}\right)^{1/\alpha}. \qquad (4)$$

The almost linear relation ($\alpha \simeq 1$) between HI masses and surface area of disk galaxies implies that the residual

cosmological dependence (through $d_A(z)$) in the above equation is only expected to be weak. For the cosmology we assume the standard $\Lambda$CDM model [29] keeping priors (from WMAP7 results [30]) on $h$ and $\Omega_m$. We perform a MCMC likelihood analysis of the simulated data with the parameters—$B$, $\alpha$, $\epsilon$, $h$ and $\Omega_m$. The likelihoods from the data are computed as $\exp(-\chi^2/2)$, where $\chi^2 = \sum_i \{\log[S(z_i, f_i)] - \log(S_i)\}/\sigma_i^2$.

We first analyze the ADBS data keeping the parameter $\epsilon$ to be fixed, since the existing data are at very low redshifts ($z < 0.025$) which is clearly insufficient to test for violations of the CDDR. We find the best fit parameter values as $B = 9.003$ and $\alpha = 1.128$ (see Fig. 2). We use these values for $B$ and $\alpha$ along with the fiducial value of $\epsilon = 0$ to create the mock catalog following the procedure as explained previously.

We then analyze the simulated catalogs for some of the upcoming 21 cm observations like $5 \times$ eVLA and the SKA. All of these surveys would throw up a large number of galaxies in detection, which means that we may simply self-calibrate the scaling relation of 21 cm luminosity with area [see Eq. (1)]. We discuss the prospects of the surveys to probe the CDDR by obtaining marginalized constraints on $\epsilon$, the parameter for violation of this relation.

The marginalized constraints on $\epsilon$ from $5 \times$ eVLA would be comparable to some of the present constraints [31], with $\Delta\epsilon = 0.095$. These constraints can be significantly improved by the next generation telescopes like the SKA. The SKA deep survey with a very good flux sensitivity is expected to detect galaxies up to a high redshift of $z \sim 1$. The 100 sq. deg. wide SKA survey would have many more objects; however, because of its higher flux cut, it would mostly detect low mass galaxies at lower redshifts, $z \sim 0.4$ (see Fig. 1). Both the deep and wide surveys from the SKA would give $\Delta\epsilon = 0.026$; however, because these surveys probe the relation in Eq. (1) at a very different range of masses they place constraints along different directions. This fact can be used to a great
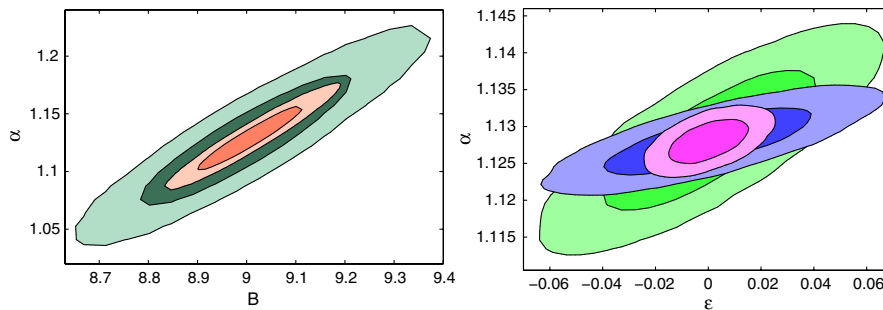


FIG. 2 (color online).    $1\sigma$ and $2\sigma$ marginalized constraints from 21 cm galaxy surveys. (Left) The green [(dark gray) and (medium-light gray)] regions indicate parameter constraints obtained from observed ADBS galaxies (in which we fix $\epsilon = 0$), while the orange [(light gray) and (medium gray)] regions represents constraints (marginalized over $\epsilon$) from a simulated galaxy catalog expected from the $5 \times$ eVLA survey. (Right) The green (medium gray)/blue (dark gray) regions correspond to the expected constraints from simulated galaxies in the SKA deep/wide surveys, respectively. Note, how the degeneracy is broken (magenta [medium-light gray]) by combining a deep and wide survey.

advantage by using the deep and the wide surveys in conjunction; as shown in Fig. 2, this is excellent at breaking the degeneracy between $\epsilon$ and $\alpha$. This wedding cake [32] survey strategy would strongly constrain $\epsilon$ to 0.01 while also constraining the relation in Eq. (1) to a high precision with $\Delta B = 0.007$ and $\Delta \alpha = 0.002$.

*Discussions.*—The planned SKA will allow the detection of 21 cm emission from distant ($z \sim 1$) galaxies with small observing times. Apart from being useful in performing the Tolman test, the same data will be useful for cosmology and galaxy evolution studies. The deepest observations may allow one to reach $z \sim 6$ and start probing the HI content of the first galaxies, which would have played a vital role in the epoch of reionization.

Self-regulation of mechanical feedback from supernovae and gravitational instability of the star forming disks keeps the surface density nearly constant, irrespective of the global star formation rates. The almost constant observed surface density of HI in disk galaxies makes them ideal sources for the Tolman test—first, by using the same objects to directly probe the ratio $d_L/d_A$ at various redshifts, the results are not affected by many possible astrophysical model uncertainties. Second, our test also promises to be immune to bias from effects of gravitational lensing as it preserves the surface brightness of such sources. Lastly, our proposed test does not fix the scaling relation between luminosity and the area of HI regions in galaxies; the upcoming surveys would simply detect enough number of these galaxies to permit a self-calibration of this relation.

We have shown that observations at the 21 cm wavelength opens up the exciting possibility of testing the CDDR in a completely independent manner. Probing this relation with our proposed method, offers several advantages over all of the existing tests. At radio frequencies the universe is essentially transparent. Thus one can cleanly distinguish between violations of distance duality relation arising from astrophysical effects and those that may arise from new physics. Our proposed test of the CDDR may soon enable us to probe the possibility of photons coupling to the dark sector.

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