All Nonclassical Correlations Can Be Activated into Distillable Entanglement

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We devise a protocol in which general nonclassical multipartite correlations produce a physically relevant effect, leading to the creation of bipartite entanglement. In particular, we show that the relative entropy of quantumness, which measures all nonclassical correlations among subsystems of a quantum system, is equivalent to and can be operationally interpreted as the minimum distillable entanglement generated between the system and local ancillae in our protocol. We emphasize the key role of state mixedness in maximizing nonclassicality: Mixed entangled states can be arbitrarily more nonclassical than separable and pure entangled states.

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The study of quantum correlations has traditionally focused on entanglement [1]. It is generally believed that entanglement is a necessary resource for quantum computers to outperform their classical counterparts. Indeed, it has been shown that, for the setting of pure-state computation, the amount of entanglement present must grow with the system size for an exponential speed-up to occur [2]. In the context of mixed-state quantum information processing, however, there are computational and communication feats which are seemingly impossible to achieve with a classical computer and yet can be attained with a quantum computer using little or no entanglement (e.g., [3,4]). For example, the deterministic quantum computation with one qubit model is believed to estimate the trace of a unitary matrix exponentially faster than any classical algorithm, yet with vanishing entanglement during the computation [5]. A second example is the ability for certain bipartite quantum systems to contain a large amount of "locked" classical correlations, which can then be "unlocked" with a disproportionately small amount of classical communication [4]. This task is impossible classically, yet the quantum states involved are separable, that is, unentangled. This raises the crucial question about what, if not entanglement, is the fundamental resource enabling such feats.

One plausible explanation is associated with the presence in (generic [6]) quantum states of correlations which have nonclassical signatures that go beyond entanglement. Indeed, much attention has recently been devoted to understanding and quantifying such correlations for this very reason [6–16]. In particular, the separable quantum states of the systems involved in deterministic quantum computation with one qubit and the locking protocol have been shown to possess nonzero amounts of such correlations [5,17], as measured by the quantum discord [7]. The latter strives to capture nonclassical correlations beyond entanglement and has recently received operational interpretations in terms of the quantum state merging protocol [18], but is unfortunately not a faithful measure [19]. A more accurate quantification of nonclassical correlations is provided by the so-called relative entropy of quantumness (REQ) [8,10–13], defined as the minimum distance, in terms of relative entropy, between a multipartite quantum state and the closest strictly classically correlated state (see Definition 1). Such a measure is faithful [11], symmetric under permutation of the subsystems, and enables a unified approach to the quantification of classical, separable, and entangled correlations [10].

More generally, the role of nonclassical correlations in quantum information tasks remains unclear. While all entangled states are useful for information processing [20], whether the same holds for all nonclassically correlated (separable) states is an open issue. This raises the question: Is there a setting in which general nonclassical correlations produce a physically relevant effect that distinguishes them from purely classical ones?

In this Letter, we answer the question in the affirmative by demonstrating a protocol which in some sense activates the nonclassicality present in any multipartite quantum system, leading to the creation of entanglement. We then show that the REQ of any system state input to our protocol is precisely the minimum distillable entanglement generated between the system and local ancillae via the protocol. This result renders the REQ both an operational and faithful nonclassicality measure. According to our framework, all and only the quantumly correlated states are shown to possess an entanglement potential that makes them readily useful for better-than-classical information processing. Finally, we prove limits on nonclassical correlations for separable and pure entangled states in any dimension, while, perhaps surprisingly, these bounds can be exceeded by mixed entangled states.

Our results apply to general multipartite states, adopting the following definition of classicality [14].

Definition 1 (strictly classically correlated quantum state).—Given a set of n d-dimensional qudit systems, let $\mathcal{B}_i$ denote an orthonormal basis in $\mathbb{C}^d$ for the ith system consisting of vectors $|\mathcal{B}_i(k)\rangle$ for $0 \leq k \leq d-1$, and let $\mathcal{B}$ denote an orthonormal basis $\{|\mathcal{B}(\boldsymbol{k})\rangle = |\mathcal{B}_1(k_1)\rangle|\mathcal{B}_2(k_2)\rangle \ldots |\mathcal{B}_n(k_n)\rangle\}$ for the entire space $(\mathbb{C}^d)^{\otimes n}$ formed by taking tensor products of all elements in bases $\{\mathcal{B}_i\}_{i=1}^n$. Then, an n-qudit state ρ is "strictly classically correlated"—or simply "classical"—if there exists such a basis $\mathcal{B}$ with respect to which ρ is diagonal. Such states correspond to the embedding of a multipartite classical probability distribution into the quantum formalism.

Activation protocol.—We now describe our protocol for activation of nonclassical correlations. The scheme is somewhat inspired by the quantum optics setup of Ref. [21], where one attempts to quantify nonclassicality of a single field mode (defined there as the state deviation from a mixture of coherent states) by reducing the problem to quantifying the two-mode entanglement that can be generated from the field by using linear optics, auxiliary classical (coherent) states, and ideal photodetectors. Similarly, we may expect that mapping the (still not-well-understood) nonclassicality of multipartite correlations into "more familiar" bipartite entanglement allows one to employ tools from entanglement theory [1] to interpret and quantify general nonclassical correlations.

Our activation protocol can be thought of as a game between an adversary and n players, where the n players together aim to generate an entangled state between a system $\boldsymbol{A}$ they control and an ancillary system $\boldsymbol{A}'$, and the adversary's goal is to thwart their efforts by locally rotating each subsystem of $\boldsymbol{A}$ before the system and ancilla undergo a predefined interaction. More precisely, the protocol proceeds as follows (see Fig. 1). We consider n players $\mathcal{P}_i$, each controlling a system-ancilla pair of qudits (A_i, A_i'). We indicate by $\boldsymbol{A}$ the joint register $A_1, \ldots, A_n$ ("system") and by $\boldsymbol{A}'$ the joint register $A_1', \ldots, A_n'$ ("ancilla"). The initial state of the total $2n$ qudits is a

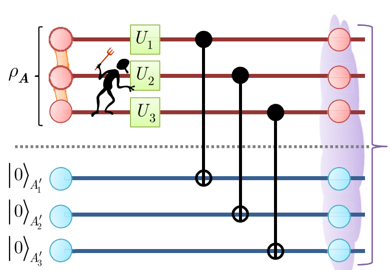

FIG. 1 (color online). Activation protocol for $n = 3$.

tensor product $\rho_{\boldsymbol{A}:\boldsymbol{A}'} = \rho_{\boldsymbol{A}} \otimes |0\rangle\langle 0|_{\boldsymbol{A}'}^{\otimes n}$. For a given $\rho_{\boldsymbol{A}}$, an adversary is first allowed to apply a local unitary U_i of his choice to each A_i. With the adversary's turn complete, each player $\mathcal{P}_i$ now lets their subsystem A_i (control qudit) interact with the corresponding ancillary party A_i' (target qudit) via a CNOT gate $C_{A_i:A_i'}$, whose action on the computational basis states $|j\rangle|j'\rangle$ of $\mathbb{C}^d \otimes \mathbb{C}^d$ is defined as $C|j\rangle|j'\rangle = |j\rangle|j' \oplus j\rangle$, with $\oplus$ denoting addition modulo d. The final state of system plus ancilla is

$$\tilde{\rho}_{\boldsymbol{A}:\boldsymbol{A}'} = V(\rho_{\boldsymbol{A}} \otimes |0\rangle\langle 0|_{\boldsymbol{A}'}^{\otimes n})V^\dagger, \tag{1}$$

with $V = C_{\boldsymbol{A}:\boldsymbol{A}'} \cdot (U_{\boldsymbol{A}} \otimes \mathbb{1}_{\boldsymbol{A}'})$, $U_{\boldsymbol{A}} = \otimes_{i=1}^n U_i$, and $C_{\boldsymbol{A}:\boldsymbol{A}'} = \otimes_{i=1}^n C_{A_i:A_i'}$. We ask: At the end of the protocol, have the n players succeeded in generating bipartite entanglement across the split $\boldsymbol{A}:\boldsymbol{A}'$, and, if so, how much entanglement was created? It is natural to expect that the answer will depend on the initial state $\rho_{\boldsymbol{A}}$ of the n-qudit system. From a physical perspective, our aim is to understand precisely how the nature and amount of correlations between the parts A_i of the system $\boldsymbol{A}$ affects the entanglement that can be created with an ancilla $\boldsymbol{A}'$ via the paradigmatic entangling operation—the CNOT; we consider here the worst case scenario with respect to the choice of the control bases. We then find the following.

Theorem 1.—The (initial) state $\rho_{\boldsymbol{A}}$ of an n-qudit system is strictly classically correlated if and only if there exists some adversarial choice of local unitaries $U_{\boldsymbol{A}}$ such that the (final) state $\tilde{\rho}_{\boldsymbol{A}:\boldsymbol{A}'}$ output by the activation protocol is separable across the system-ancilla split.

In other words, the system always becomes (for any choice of $U_{\boldsymbol{A}}$) entangled with the ancilla as a result of the activation protocol, if and only if the input state of the system is nonclassically correlated. This establishes a qualitative equivalence between multipartite nonclassical correlations among components of a quantum system, and bipartite entanglement between the system and an ancilla, and settles the issue of the usefulness of nonclassical correlations in (even separable) quantum states for quantum primitives: Any kind of multipartite nonclassicality initially present in $\boldsymbol{A}$ is a resource for information processing that can always be activated or mapped into bipartite entanglement across the $\boldsymbol{A}:\boldsymbol{A}'$ split. While a direct proof of this result is quite straightforward (see Appendix A [22]), in the following we show a more powerful result that promotes the equivalence between nonclassicality and entanglement to a quantitative relationship.

Quantifying nonclassicality.—Having run the activation protocol, we proceed to quantify the entanglement generated in the $\boldsymbol{A}:\boldsymbol{A}'$ split whenever $\boldsymbol{A}$ is initially in a nonclassically correlated state. The present framework is general enough to allow us to uncover a full zoology of nonclassicality measures, as each choice of a different entanglement monotone [23] we adopt (at the output) leads in principle to a unique nonclassicality measure (for the input state), the association stemming exactly from the activation protocol. More precisely, let E denote some entanglement measure of choice and $\tilde{\rho}_{A:A'}$ the final system-ancilla state as in Eq. (1), and define by

$$Q_E(\rho_A) := \min_{U_A} E_{A:A'}(\tilde{\rho}_{A:A'}) \qquad (2)$$

the minimum entanglement generated across the $A:A'$ split over all choices of adversarial local unitaries U_A. We call $Q_E(\rho_A)$ the minimum entanglement potential of ρ_A with respect to E. As a consequence of Theorem 1, Q_E is a measure of nonclassical correlations in the multipartite state ρ_A, for every entanglement monotone E.

In fact, the condition $Q_E(\rho_A) = 0$ perfectly characterizes the set of classically correlated states ρ_A if E is a faithful entanglement measure (i.e., if E vanishes only for separable states). However, even certain nonfaithful entanglement measures can be plugged in to obtain a faithful measure of nonclassical correlations [19]. The reason is that the output state $\tilde{\rho}_{A:A'}$ has the so-called maximally correlated form [24] between A and A'; namely, $\tilde{\rho}_{A:A'} = \sum_{kl} \rho_{kl}^{\mathcal{B}} |k\rangle\langle l|_A \otimes |k\rangle\langle l|_{A'}$ with $\rho_{kl}^{\mathcal{B}} = \langle \mathcal{B}(k)|\rho_A|\mathcal{B}(l)\rangle$, $|\mathcal{B}(k)\rangle_A = U_A^\dagger |k\rangle$, and $|k\rangle = |k_1\rangle|k_2\rangle \ldots |k_n\rangle$. In particular, let us consider the nonfaithful (as it vanishes on so-called bound entangled states) but physically motivated distillable entanglement E_D [23] as a bipartite entanglement monotone. We find that the $A:A'$ distillable entanglement of $\tilde{\rho}_{A:A'}$ is equal to $E_D(\tilde{\rho}_{A:A'}) = S(\tilde{\rho}_A) - S(\tilde{\rho}_{A:A'}) = S(\rho_A^{\mathcal{B}}) - S(\rho_A)$, where $S(\sigma) = -\mathrm{Tr}(\sigma \log_2 \sigma)$ is the von Neumann entropy of a state σ. In the first equality we used the results of Ref. [25] about distillable entanglement for maximally correlated states—for which it happens to coincide with the relative entropy of entanglement [26]. The second equality is justified by the fact that $\rho_A^{\mathcal{B}}$ is the state resulting from local projective measurements in the local bases $\mathcal{B}$ on ρ_A and is unitarily equivalent to $\tilde{\rho}_A$, while $\tilde{\rho}_{A:A'}$ is obtained from ρ_A via the activation protocol isometry, Eq. (1). Thus, the minimum distillable entanglement potential $Q_{E_D}(\rho_A)$ takes on the form $Q_{E_D}(\rho_A) = \min_{\mathcal{B}}[S(\rho_A^{\mathcal{B}}) - S(\rho_A)]$, where the minimization is over the choice of the bases $\mathcal{B}$. As proven in Ref. [10], this is an equivalent expression for the REQ,

$$Q(\rho_A) = \min_{\text{classical } \sigma_A} S(\rho_A \parallel \sigma_A), \qquad (3)$$

where the relative entropy is defined as $S(\rho \parallel \sigma) = \mathrm{Tr}(\rho \log_2 \rho - \rho \log_2 \sigma)$ and the minimization is over all strictly classically correlated states σ_A. We have thus proven that the REQ quantifying general nonclassical correlations between the n subsystems A_i of A is exactly equal to the minimum bipartite distillable entanglement potential—or, equivalently, to the minimum relative entropy of entanglement potential—generated between the system A and the ancillary register A'.

This finding immediately provides a clear-cut operational interpretation for the REQ, a quantity whose original definition was purely geometric [Eq. (3)], which thus emerges as a mathematically sound and physically motivated measure of nonclassical correlations, quantifying the resource power of such correlations for (distillable) entanglement generation. Incidentally, since the REQ is faithful [11], this yields a proof of Theorem 1.

Other nonclassicality measures can be induced by different entanglement monotones. Choosing, e.g., the "negativity" $\mathcal{N}$ [27] as an entanglement measure, one obtains $Q_{\mathcal{N}}(\rho_A) = (\min_{\mathcal{B}} \sum_{i \neq j} |\rho_{ij}^{\mathcal{B}}|)/2$ as a quantifier of nonclassical correlations (see Appendix B [22] for details), directly related to the off-diagonal coherences of the density matrix of the system, minimized over all local bases.

Nonclassicality versus mixedness and entanglement.—Equipped with a faithful and operational measure of nonclassical correlations, the REQ $Q \equiv Q_{E_D}$, we can investigate the interplay between nonclassicality, entanglement, and mixedness of general states ρ_A. For the sake of simplicity, from now on we restrict to the bipartite case $A_1 = A$, $A_2 = B$. We begin with a few simple but general observations following from the definition of Q.

For pure states $\rho_{AB} = |\psi\rangle\langle\psi|$, the quantumness Q reduces to the von Neumann entropy of entanglement $S(\rho_A) = S(\rho_B)$ [13] and is thus at most equal to $\log_2 d$. On the other hand, for arbitrary mixed ρ_{AB}, we have that $Q(\rho_{AB})$ is at most $2\log_2 d$, since from Eq. (3) one has $Q(\rho_{AB}) \leq S(\rho_{AB} \parallel \rho_A \otimes \rho_B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \equiv I(\rho_{AB})$, where I denotes the mutual information, a measure of total correlations. From this and the results of Ref. [28], one realizes that for a separable state a bound $Q(\rho_{AB}^{\text{sep}}) \leq \log_2 d$ holds. In Appendix C [22], we prove in fact that this inequality is always sharp for separable states; i.e., the bound $\log_2 d$ cannot be exactly saturated for separable nonclassical states, while it is instead trivially reached by pure maximally entangled states $|\psi\rangle = d^{-1/2} \sum_{j=0}^{d-1} |j\rangle|j\rangle$. Almost all separable states thus possess nonclassical correlations [6] but not to a maximal extent (as already observed in the particular cases of two-qubit [29] and two-mode Gaussian states [16]). However, with increasing $d \to \infty$ we find quite surprisingly that the upper bound on the REQ of separable states becomes asymptotically tight, in the sense that separable states exist such that $Q(\rho_{AB}^{\text{sep}})/\log_2 d \to 1$. Even more intriguingly, we can show that the upper bound on general mixed bipartite states ρ_{AB} is also asymptotically tight, in the sense that families of mixed states exist such that in the limit $d \to \infty$, their quantumness converges to the maximum, $Q(\rho_{AB})/\log_2 d \to 2$. More precisely, in Appendix D [22] we prove the following two results by using techniques from Refs. [30,31]. Let $m = \lceil (\log_2 d)^4 \rceil$.

Theorem 2.—Define the following random separable state: $\sigma_{AB} = \frac{1}{dm} \sum_{\substack{i=1,\ldots,d \\ j=1,\ldots,m}} |i\rangle\langle i|_A \otimes (U_j |i\rangle\langle i| U_j^\dagger)_B$, with unitaries U_j drawn independently from the Haar measure. Then, $S(\sigma_{AB}) \leq \log_2 d + \log_2 m$, while, on the other hand, for d sufficiently large and with high probability, $S(\sigma_{AB}^{\mathcal{B}}) \geq 2\log_2 d - \text{const}$, for all $\mathcal{B}$. Hence, $Q(\sigma_{AB}) \geq \log_2 d - O(\log_2 \log_2 d)$.

Theorem 3.—Define the following random state: For C a system of dimension m, let $\rho_{AB} = \mathrm{Tr}_C|\psi\rangle\langle\psi|_{ABC}$, where $|\psi\rangle \in \mathbb{C}^d \otimes \mathbb{C}^d \otimes \mathbb{C}^m$ is uniformly distributed (with probability induced by the Haar measure). Then, $S(\rho) \leq \log_2 m$, while, on the other hand, for d sufficiently large and with high probability, $S(\rho^{\mathcal{B}}) \geq 2\log_2 d - \mathrm{const}$, for all $\mathcal{B}$. Hence, $Q(\rho_{AB}) \geq 2\log_2 d - O(\log_2 \log_2 d)$.

These results show that, first, there are separable states that asymptotically (in d) are as nonclassical as the most nonclassical pure state (which is the maximally entangled state); second, mixed entangled states can be twice as nonclassical as pure entangled states. Both entanglement and mixedness are required to "break the barrier" of $\log_2 d$, thus showing that entanglement by itself is not the strongest form of nonclassicality.

Conclusions.—The study of general nonclassical correlations is currently a burgeoning area, but in many ways such correlations are still not well understood. Our activation protocol lends new insight into the nature of these correlations by furnishing them, in full generality, with a new operational meaning in terms of resources for entanglement generation. Furthermore, we have reduced the problem of quantifying nonclassicality to the more familiar setting of quantifying entanglement, for which a multitude of tools are already known (see, e.g., [1]). As an added bonus, we have obtained an alternative operational interpretation for the relative entropy of quantumness [8,10]. With respect to the latter, we have demonstrated that, remarkably, there exist mixed entangled quantum states whose nonclassical correlations are stronger than those of pure entangled states. Further investigation on the nature and the structure of nonclassical correlations, following the program laid out by this Letter, may trigger novel developments in quantum technology and shed light on quantum foundations.

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Note added.—After completion of this Letter, we became aware of some related results by Streltsov, Kampermann, and Bruss [32], who showed that the quantumness of correlations (as measured, e.g., by the quantum discord) is also related to the minimum entanglement generated between the system and apparatus in a partial measurement process. In light of those results, our findings can be understood also as dealing with the interplay between system-apparatus entanglement and nonclassicality of correlations when realizing local measurements.

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