

Fermi-Liquid Behavior of the Normal Phase of a Strongly Interacting Gas of Cold Atoms

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We measure the magnetic susceptibility of a Fermi gas with tunable interactions in the low-temperature limit and compare it to quantum Monte Carlo calculations. Experiment and theory are in excellent agreement and fully compatible with the Landau theory of Fermi liquids. We show that these measurements shed new light on the nature of the excitations of the normal phase of a strongly interacting Fermi gas.

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In 1956 Landau developed an elegant description of interacting Fermi systems at low temperature relying on the existence of long-lived quasiparticles. While this Fermi-liquid theory (FLT) describes well Helium 3 and many solid-state materials above the superfluid temperature, there exist notable exceptions such as underdoped cuprates [1], where despite tremendous theoretical and experimental efforts, the nature of the normal phase is not yet understood. Similarly to high-critical temperature superconductors, the properties of the normal phase of strongly correlated atomic fermionic gases and the nature of its excitations are still debated. This issue was addressed recently for spin-balanced gases above the superfluid transition, through the measurement of equations of state [2–5], the study of the single-particle excitation spectrum [6,7], or of spin fluctuations [8]. On the one hand, recent photoemission spectroscopy experiments near the critical temperature were interpreted using a pseudogap model [7]. On the other hand, measurement of the temperature dependence of the specific heat displayed a linear behavior compatible with Fermi liquid's prediction [2]. All these experimental probes give access to the properties of the normal phase of the unpolarized normal phase above the critical temperature T_c . This limitation can be overcome by stabilizing the normal state at $T < T_c$ by imposing a spin population imbalance in the trapped gas [9–11] and extrapolating its properties to zero imbalance. Previous works focused on the highly polarized limit where minority atoms behave as impurities: $n_2 \ll n_1$, where n_i is the density for species i [2,12–20]. Here, we interpret the spin imbalance as the application of an effective magnetic field to the unpolarized normal gas at very low temperature and using a combination of Monte Carlo simulations and experimental results, we extract from the equation of state the magnetic spin response of the normal phase in the limit $T \ll T_c$. We show that our results are compatible with a Fermi-liquid description of the normal phase, and we

extract the Fermi-liquid parameters in the universal unitary limit where scattering length is infinite. The relationship between these parameters and the properties of low-lying excitations of the system allow us to quantitatively interpret spectroscopic data from [6,7].

The polarization dependence of the energy E of the system directly reflects the presence of spin-singlet dimers in the sample. Indeed, the presence of a gap in the spin excitation spectrum implies a linear dependence of the energy E with polarization $p = (N_1 - N_2)/(N_1 + N_2)$ at low temperature, and hence a zero spin susceptibility. We have performed quantum Monte Carlo simulations of the partially polarized Fermi gas at $T = 0$ in the BEC-BCS crossover. We make use of the fixed-node diffusion Monte Carlo method that was employed in earlier studies of polarized Fermi gases [14,18]. The state of the system is forced to be in the normal phase by imposing the nodal surface of a many-body wave function incompatible with off-diagonal long-range order. A simple way to implement this requirement is by choosing the trial function of the Jastrow-Slater form

$$\psi_T(\mathbf{R}) = \prod_{i,i'} f(r_{ii'}) D(N_1) D(N_2), \quad (1)$$

where $\mathbf{R} = (\mathbf{r}_1, \dots, \mathbf{r}_N)$ is the spatial configuration vector of the N particles and D denotes the Slater determinant of plane waves in a cubic box of size L with periodic boundary conditions. The positive Jastrow correlation term $f(r)$ is determined as described in Ref. [14]: at short distances it corresponds to the lowest-energy solution of the two-body problem, while it satisfies the boundary condition on its derivative $f'(r = L/2) = 0$.

The results for the canonical equation of state $E(N_1, N_2)$ are shown in Fig. 1. They are well fitted by the energy functional

$$E(p) = \frac{3}{5} N E_F (\xi_N + \frac{5}{9} \tilde{\chi}^{-1} p^2 + \dots), \quad (2)$$

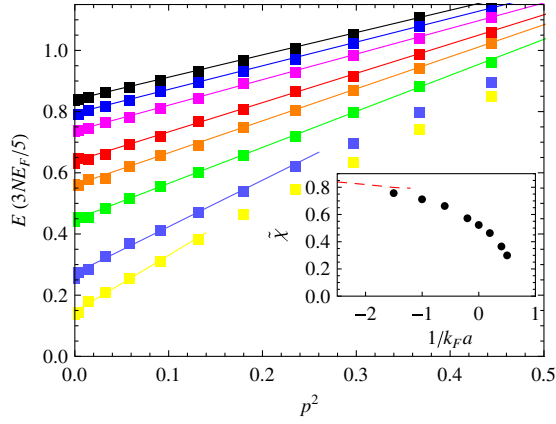


FIG. 1 (color online). Canonical equation of state of a two-component Fermi gas calculated using quantum Monte Carlo simulation, for $1/k_F a = -1.5, -1, -0.6, -0.2, 0, 0.2, 0.4, 0.5$ (from top to bottom). The solid lines are fits of the low-polarization data with Eq. (2). Inset: Extracted values of the susceptibility $\tilde{\chi}$ as a function of $1/k_F a$. The dashed red line is the result of a perturbation expansion valid up to order $(k_F a)^2$.

holding for a spin polarizable system at low temperature, where both ξ_N and the dimensionless spin susceptibility $\tilde{\chi}$ (in units of the susceptibility of an ideal Fermi gas $3n/2E_F$) depend on $1/k_F a$, where $k_F = (3\pi^2 n)^{2/3}$. The Monte Carlo method indicates the absence of spin gap, and thus of preformed molecules in the normal phase for $1/k_F a \lesssim 0.5$. Note that the extracted values of $\tilde{\chi}$ reported in the inset of Fig. 1 show a rapid drop for positive values of a when entering the BEC side of the Feshbach resonance. A likely explanation is the binding of fermions into spin-singlet pairs for some positive value of the interaction strength $1/k_F a$. Monte Carlo calculations for values of $1/k_F a \geq 0.7$ show that $E(p)$ is indeed linear rather than quadratic in p , indicating the emergence of a gap. However, pairing fluctuations play a major role for such values of the coupling and the nodal surface of the Jastrow-Slater state (see the supplemental material [21]) is no longer sufficient to enforce the normal phase. This behavior is reminiscent of the pairing transition investigated in the framework of BCS theory [22], as well as in the normal phase of the attractive Hubbard model, extrapolated to a temperature range below the superfluid transition [23,24], while in our work the extrapolation is made towards a small spin imbalance.

We now compare these simulations with the grand-canonical equation of state (EOS) of a homogeneous system obtained experimentally in Refs. [2,12]. We prepare a deeply degenerate mixture of the two lowest internal states of ${}^6\text{Li}$, held in a cylindrically symmetric hybrid optical-magnetic trap, of radial (axial) frequency ω_r (ω_z , respectively). The bias magnetic field B_0 is chosen between 822 and 981 G, allowing us to tune the strength of interactions $-1 < 1/k_F a < 0.2$. The final atom number is 2 to 10×10^4 atoms per spin state, and the gas temperature is

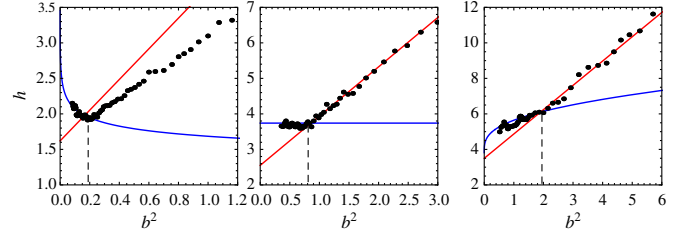


FIG. 2 (color online). Thermodynamic function $h(b)$ measured at different magnetic fields $B_0 = 871, 834, 822$ G. The blue lines correspond to the superfluid equation of state $h_S(\delta)$ measured in [12]. The red line is a linear fit of the data in the normal phase, $b > b_c$. The dashed line indicates the superfluid/normal phase transition ($b = b_c$).

smaller than $0.06T_F$, as measured from the fully polarized wings of a trapped gas [25]. From dimensional analysis, the EOS of a spin-imbalanced Fermi gas can be written as

$$P(\mu_1, \mu_2, a) = P_0(\mu)h\left(\delta = \frac{\hbar}{\sqrt{2m\mu a}}, b = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right),$$

where $\mu = (\mu_1 + \mu_2)/2$ is the mean chemical potential and $P_0(\mu)$ is the pressure of a noninteracting unpolarized Fermi gas. δ is a grand-canonical analog of the interaction parameter $1/k_F a$, and b is a dimensionless number proportional to the “spin-polarizing field” $\mu_1 - \mu_2$.

At all values of the scattering length addressed in this work, the equation of state exhibits a clear discontinuity of its derivative at the critical field $b_c(\delta)$ (See Fig. 2), indicating a first-order phase transition from a superfluid state for $b < b_c$ to a normal state for $b > b_c$, where h is linear in b^2 . [10,12]. The equation of state of the superfluid phase has been discussed in a previous work [12] and we focus here on the properties of the normal phase. We write

$$h(\delta, b) = h_N(\delta)(1 + \frac{15}{8}\tilde{\chi}^{\text{GC}}(\delta)b^2 + O(b^4)). \quad (3)$$

$h_N(\delta)$ is the grand-canonical equation of state in the normal state, extrapolated to a spin-symmetric configuration. $\tilde{\chi}^{\text{GC}}(\delta)$ is a grand-canonical magnetic susceptibility. For an ideal two-component Fermi gas, the functions h_N and $\tilde{\chi}^{\text{GC}}$ are equal to 1. Fitting our data in the normal phase with (3), we obtain the parameters $h_N(\delta)$ and $\tilde{\chi}^{\text{GC}}(\delta)$ in the BEC-BCS crossover shown in Fig. 3 where we compare their values to the predictions of the Monte Carlo simulations. To this end, we fit the dependence with $1/k_F a$ of the parameters ξ_N and $\tilde{\chi}$ determined by Monte Carlo simulations, and perform a Legendre transform to obtain the grand-canonical EOS $h_N(\delta)$ of the normal phase and magnetic susceptibility $\tilde{\chi}^{\text{GC}}(\delta)$ measured experimentally. In the investigated parameter range, the agreement between theory and experiment is excellent. We also remark that our value for the susceptibility of the normal phase at unitarity is about twice larger than the value measured in [8] on a gas with a 35% condensate fraction, confirming a significant

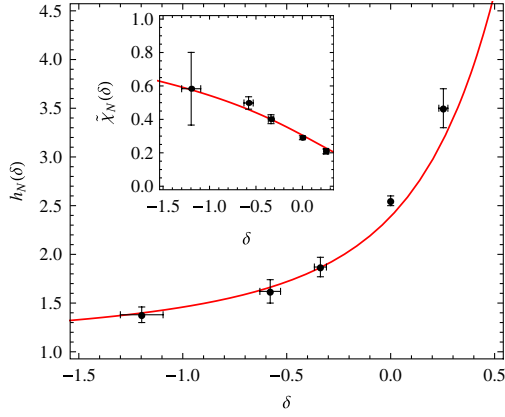


FIG. 3 (color online). Fermi-liquid equation of state extrapolated to a spin-symmetric configuration $h_N(\delta)$. The black dots are the experimental data, and the red line is calculated from the Monte Carlo data. Inset: Grand-canonical susceptibility $\tilde{\chi}^{\text{GC}}(\delta)$ of a Fermi gas in the BEC-BCS crossover.

suppression of the spin susceptibility in the superfluid phase.

Our findings demonstrate that for $1/k_F a \lesssim 0.5$, the spin excitations of the system are not gapped in the normal phase which therefore does not support “true” molecules. However, a certain class of theories predicts a reminiscence of this gap in the form of a dip in the density of states over a range Δ^* around the Fermi level [26]. Δ^* is often called the pseudogap, in relationship to some features of high-critical temperature superconductors. These theories predict a departure of $E(p)$ from its quadratic behavior when the Fermi levels of the two spin species reach the edges of the dip, $\mu_2 - \mu_1 \simeq \Delta^*$. (see Auxiliary Materials). The absence of such an anomaly in Fig. 1 and 2, and in the whole range $-1 < 1/k_F a < 0.5$ thus suggests that the dip is either extremely narrow or very broad: the density of state remains flat over the range of polarizations and interaction strength studied in our work. For instance, at unitarity this range covers $0 < b^2 < 3$. If a sizeable dip existed, then its width cannot be smaller than $\simeq (\mu_1 + \mu_2)\sqrt{3} \simeq 1.4E_F$ where we have used the unitary equation of state, $\mu = 0.41E_F$ [12]. Such a large pseudogap is not compatible with the photoemission data of [7] (See below). Furthermore, we would expect on physical grounds that Δ^* becomes smaller on the BCS side of the resonance. This is observed neither in the experimental data of Fig. 2 nor in the quantum Monte Carlo results of Fig. 1.

On the contrary, Landau’s theory of Fermi liquids is fully compatible with our observations. This theory assumes the existence of long-lived fermionic excitations above the Fermi surface. Combining the measurement of the low-temperature compressibility κ and specific heat C_v of [2] with the data presented here, we can fully characterize the parameters of the theory at the unitary limit. From the magnetic response of the $T = 0$ gas, we obtain here its

magnetic susceptibility and another determination of κ . The two determinations of κ coincide within 5%, showing that the two approaches indeed probe the same Fermi liquid. From this set of thermodynamic quantities we derive, according to Landau’s Fermi-liquid theory, a complete characterization of the low-lying excitations of the unitary gas: besides their effective mass $m^* = 1.13m$ and Landau parameters $F_0^s = -0.42$, $F_1^s = 0.39$ found in [2], we recover here $F_0^s = -0.40$ and obtain the new parameter $F_0^a = m^*/m\tilde{\chi}(0)^{-1} - 1 = 1.1(1)$. Note that $F_0^a > 0$ corresponds to magnetic correlations which do favor the singlet configuration.

We can finally test FLT on the single-particle photoemission spectrum obtained at the unitary limit and at the onset of superfluidity from Ref. [7]. The experimental signal $\bar{A}(k, \omega)$ is directly proportional to the spectral function $A(k, \omega - \mu)$ averaged over the trap that we estimate using the following procedure: In the vicinity of the Fermi surface, the dispersion relation of the Fermi-liquid quasiparticles reads $\hbar\omega_k = \mu + \hbar^2(k^2 - k_F^2)/2m^*$ where $m^* = 1.13m$. Assuming long-lived quasiparticles, we approximate $A(k, \omega)$ by $\delta(\omega - \omega_k)$ and perform the integration over the trap to obtain $\bar{A}(k, \omega)$ given by [7]

$$\bar{A}(k, \omega) = \frac{48k^2}{\pi^2} \int d^3r \frac{A(k, \omega - \mu(\mathbf{r})/\hbar)}{1 + \exp\frac{\hbar\omega - \mu(\mathbf{r})}{k_B T}}, \quad (4)$$

where $\mu(\mathbf{r})$ is the local chemical potential at position \mathbf{r} . In order to calculate the integrated spectral function $\bar{A}(k, \omega)$ of a Fermi liquid, we replace the spectral function by $\delta(\omega - \omega_k)$, and perform the integration in Eq. (4). $k_F(r)$ is calculated from the equation of state of the unitary gas determined in [2]. The temperature is chosen at the onset of superfluidity $k_B T/\mu^0 = 0.32$ [2,27]. In order to make a direct comparison with the experimental data, we finally convolve our result with the experimental resolution in ω [7], equal to $0.25E_F/\hbar$ and results for various values of k are shown in Fig. 4.

With no free parameter in the theory, FLT well reproduces the experimental spectra for $\bar{A}(k, \omega)$ in the region $k < k_F$, with an excellent agreement in the region $0.3k_F \leq k \leq k_F$ close to the most probable Fermi level in the trap ($\simeq 0.7k_F$) where FLT is expected to be more accurate. Interestingly, we observe that the width of the peak at $k/k_F = 0.6$ is well reproduced by our model meaning that the broadening of the line is not limited by the lifetime of the quasiparticles, but rather by trap inhomogeneity and measurement resolution. Significant deviations between experiment and FLT appear for $k > 1.1k_F$, far from the most probable Fermi wave vector. However, in this region the energy spectrum signal is very broad and weak, corresponding to an incoherent background in the spectral function. Our Fermi-liquid description thus accounts for the coherent part of the excitation spectrum from [7].

In conclusion we have shown that the magnetic and thermal responses of the unitary Fermi gas support a

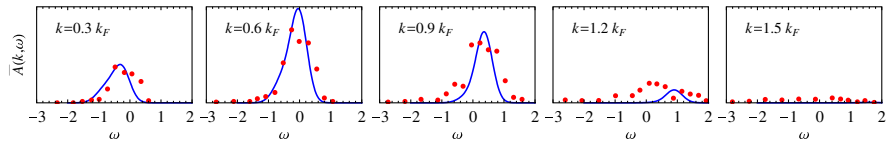


FIG. 4 (color online). Energy distribution data $\bar{A}(k, \omega)$ from [7] (red dots), compared with the prediction of Fermi-liquid theory (blue lines), for $k/k_F = 0.3, 0.6, 0.9, 1.2, 1.5$.

description of the normal phase in terms of Fermi-liquid theory despite the fact that this system exhibits a high-critical temperature for superfluidity. This behavior is in contrast with underdoped cuprate high T_c materials displaying anomalous magnetic susceptibility or pseudogap physics in the normal phase. Recent quantum oscillation experiments on cuprates in high magnetic fields, aiming at studying the incipient normal state (somewhat analogously to the present work) do suggest long-lived quasiparticles [28]. The drop of the susceptibility on the BEC side of the resonance for $1/k_F a \gtrsim 0.5$ indicates the appearance of a spin gap in this regime that deserves further investigations. Finally, the magnetic susceptibility could be a key observable for characterizing the onset of itinerant ferromagnetism in a repulsive Fermi gas [29,30].

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