

Influence of Shear Viscosity of Quark-Gluon Plasma on Elliptic Flow in Ultrarelativistic Heavy-Ion Collisions

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We investigate the influence of a temperature-dependent shear viscosity over entropy density ratio η/s on the transverse momentum spectra and elliptic flow of hadrons in ultrarelativistic heavy-ion collisions. We find that the elliptic flow in $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions at RHIC is dominated by the viscosity in the hadronic phase and in the phase transition region, but largely insensitive to the viscosity of the quark-gluon plasma (QGP). At the highest LHC energy, the elliptic flow becomes sensitive to the QGP viscosity and insensitive to the hadronic viscosity.

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Ultrarelativistic heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC) produce a hot and dense system of strongly interacting matter [1]. The subsequent expansion of the created matter has been shown to exhibit a strong degree of collectivity which reveals itself in the transverse momentum (p_T) spectra of finally observed hadrons. In particular, the observed large azimuthal anisotropy of the spectra, quantified by the so-called elliptic flow coefficient v_2 , has been interpreted as a signal for the formation of a quark-gluon plasma (QGP) with very small viscosity in heavy-ion collisions at RHIC [2].

A first indication for the small viscosity of the QGP was the agreement between RHIC data and hydrodynamical calculations in the perfect-fluid limit, i.e., with zero viscosity [3]. An analysis of the elliptic flow at RHIC in the framework of relativistic dissipative hydrodynamics was performed in Refs. [4–6]. These works indeed indicate that the shear viscosity to entropy density ratio, η/s , has to be small in order to keep the agreement between the hydrodynamic simulations and experimental data.

Presently, most hydrodynamical simulations assume a constant, i.e., temperature-independent η/s . It has been claimed [6] that, in order to describe elliptic flow data, this value cannot be larger than 2.5 times the lower bound $\eta/s = 1/4\pi$ conjectured in the framework of the AdS/CFT correspondence [7]. A constant η/s is, however, in sharp contrast to the behavior observed in common liquids and gases, where η/s has a strong temperature dependence and, typically, a minimum near phase transitions. A similar behavior of η/s is expected for finite-temperature matter described by quantum chromodynamics (QCD) near the transition from hadronic matter to the QGP (the QCD phase transition) [8].

A natural question then is whether the temperature dependence of η/s has an effect on the collective flow of

hadrons in heavy-ion collisions. In this work, we investigate this question in the framework of relativistic hydrodynamics. We consider a temperature-dependent η/s with a minimum near the QCD phase transition, and compare the results with those obtained for a constant η/s in either the hadronic phase, or the QGP phase, or both phases. Note that we do not attempt a detailed fit to the data in order to extract η/s . Rather, we are interested in the qualitative effects of different parametrizations for η/s on hadron spectra and elliptic flow.

Concerning the elliptic flow in Au + Au collisions at RHIC, we find little difference whether η/s is constant in the QGP phase or strongly increasing with temperature. In contrast, the elliptic flow values are highly sensitive to whether we use a constant or temperature-dependent η/s in the hadronic phase, corroborating the findings of Refs. [9,10]. On the other hand, we find that the sensitivity of the elliptic flow to the values of η/s in the high-temperature QGP increases with increasing collision energy, while the sensitivity to the hadronic viscosity decreases. At the highest LHC energy, the above conclusion for RHIC energies is reversed: the finally observable elliptic flow is dominated by the viscosity of the QGP and largely insensitive to that of the hadronic phase.

Fluid dynamics is determined by the conservation of energy, momentum, and charges like baryon number. Here, we are interested in the collective flow at midrapidity in heavy-ion collisions at RHIC and LHC energies. Consequently, we may neglect baryon number and assume longitudinal boost invariance [11]. We also need the constitutive relations for the dissipative currents. Here, we only consider the shear stress tensor $\pi^{\mu\nu}$, the evolution of which we describe in the approach of Israel and Stewart [12], $\langle D\pi^{\mu\nu} \rangle = \frac{1}{\tau_\pi} (2\eta\sigma^{\mu\nu} - \pi^{\mu\nu}) - \frac{4}{3}\pi^{\mu\nu}\partial_\lambda u^\lambda$, where $D = u^\mu\partial_\mu$, $\sigma^{\mu\nu} = \nabla^{(\mu}u^{\nu)}$, and the angular brackets $\langle \rangle$ denote the symmetrized and traceless projection,

orthogonal to the fluid four-velocity u^μ . We have also taken the coefficient of the last term in the massless limit. For details, see Ref. [13].

We solve the conservation equations numerically by using the SHASTA algorithm, see, e.g., Ref. [13]. The relaxation equations for the components of $\pi^{\mu\nu}$ are solved by discretizing spatial gradients using centered second-order finite differences. We found that, in contrast to SHASTA, this method produces numerically stable solutions also for low-density matter at the edges of the system.

With longitudinal boost invariance, we need to specify the values of the energy-momentum tensor in the transverse plane at some initial time τ_0 . We assume that the initial energy density profile is proportional to the density of binary nucleon-nucleon collisions as calculated from the optical Glauber model (model eBC in Ref. [14]). The initial transverse velocity and $\pi^{\mu\nu}$ are set to zero. The maximum energy densities ε_0 in central collisions (impact parameter $b = 0$) are chosen to reproduce the observed multiplicity in the 0%–5% most central $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions at RHIC [15] and $\sqrt{s_{NN}} = 2.76$ TeV Pb + Pb collisions at LHC [16]. For the $\sqrt{s_{NN}} = 5.5$ TeV Pb + Pb collisions at LHC we use the multiplicity predicted by the minijet + saturation model [17]. The initialization parameters are collected in Table I.

Our equation of state (EoS) is a recent parametrization of lattice-QCD data and a hadron resonance gas (s95p-PCE of Ref. [18]), with chemical freeze-out at a temperature $T_{\text{chem}} = 150$ MeV implemented as in Ref. [19].

Hadron spectra are calculated by using the Cooper-Frye freeze-out description [20] with constant decoupling temperature $T_{\text{dec}} = 100$ MeV, which will be shown below to give reasonable agreement with both the p_T spectrum and the elliptic flow coefficient for pions at RHIC. For the sake of simplicity, we include viscous corrections to the equilibrium distribution function f_0 as for Boltzmann particles, even though f_0 obeys the appropriate quantum statistics [21]:

$$f(x, p) = f_0 + \delta f = f_0 \left[1 + \frac{P_\mu P_\nu \pi^{\mu\nu}}{2T^2(\varepsilon + p)} \right], \quad (1)$$

where p is pressure and p^μ is the hadron four-momentum. Two- and three-body decays of unstable hadrons are included as described in Ref. [22]. We include resonances up to mass 1.7 GeV.

The shear viscosity to entropy density ratio is parametrized as follows. For the hadronic phase, it reproduces the results of Ref. [23]. In the QGP phase, η/s follows the

lattice-QCD results of Ref. [24]. Then, η/s has to assume a minimum value at a certain temperature; in our case we take $\eta/s = 0.08$ at $T = 180$ MeV. This is the same parametrization as used in Ref. [25]. In total we have four cases, see Fig. 1: (*LH-LQ*) $\eta/s = 0.08$ for all temperatures, (*LH-HQ*) $\eta/s = 0.08$ in the hadron gas, and above $T = 180$ MeV η/s increases according to lattice-QCD data, (*HH-LQ*) below $T = 180$ MeV, η/s is that of a hadron gas, and above we set $\eta/s = 0.08$, (*HH-HQ*) we use a realistic parametrization for both the hadron gas and the QGP. For the relaxation time we use a result motivated by kinetic theory $\tau_\pi = 5\eta/(\varepsilon + p)$ [26].

Figure 2(a) shows the p_T spectrum of positive pions in the 0%–5% most central $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions at RHIC. Our calculations are compared to PHENIX data [15]. All the different parametrizations of η/s give similar agreement with the low- p_T pion spectra. For $p_T \gtrsim 1.0$ GeV, the parametrizations (*LH-HQ*) and (*HH-HQ*) start to give slightly flatter spectra. While the effect of the QGP viscosity on the p_T slopes is small for our comparatively long initialization time $\tau_0 = 1.0$ fm, it becomes more pronounced for smaller values of τ_0 . On the other hand, the slopes of the spectra are almost independent of the hadronic viscosity and this conclusion remains true at least for $\tau_0 = 0.2$ –1.0 fm.

Figures 2(b) and 2(c) show the spectra for $\sqrt{s_{NN}} = 2.76$ TeV and 5.5 TeV Pb + Pb collisions, respectively. Here we observe a much stronger dependence of the p_T spectra on the high-temperature values of η/s , but the main reason for this is the earlier initialization time $\tau_0 = 0.6$ fm. On the other hand, the p_T spectra are independent of the hadronic viscosity also at LHC.

In Figs. 2(d)–2(f) we show the elliptic flow coefficients for charged hadrons in the 20%–30% centrality class for $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions and $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.5$ TeV Pb + Pb collisions, respectively. In Fig. 2(d) the results from the hydrodynamic simulations are compared to STAR 4-particle cumulant data [27] and in Fig. 2(e) to recent data from the ALICE Collaboration [28].

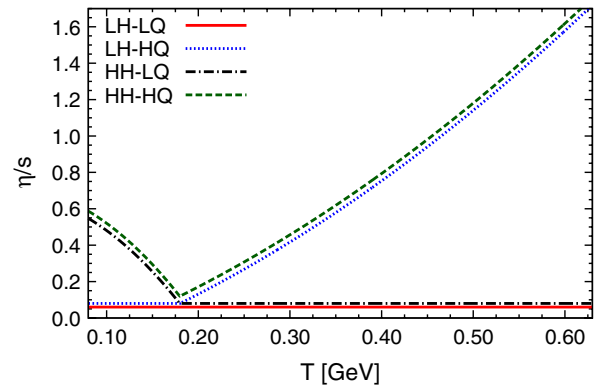


FIG. 1 (color online). Different parametrizations of η/s as a function of temperature. The (*LH-LQ*) line is shifted downwards and the (*HH-HQ*) line upwards for better visibility.

TABLE I. Initialization parameters for different collisions.

$\sqrt{s_{NN}}$ [GeV]	τ_0 [fm]	ε_0 [GeV/fm ³]	T_{max} [MeV]
200	1.0	24.0	335
2760	0.6	187.0	506
5500	0.6	240.0	594

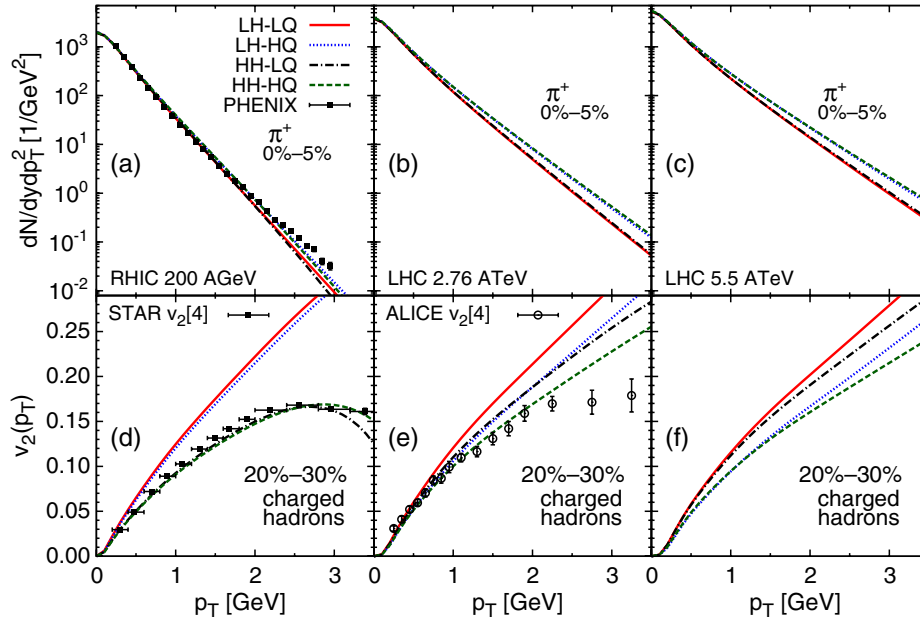


FIG. 2 (color online). Transverse momentum spectra of positive pions in the 0%–5% most central collisions and elliptic flow coefficients in the 20%–30% centrality class at RHIC and LHC. Different curves correspond to the different parametrizations of the temperature dependence of η/s . Data in panel (a) are from Ref. [15] and in panels (d) and (e) from Refs. [27,28].

We immediately see that, for RHIC, the four parametrizations for η/s produce values for the elliptic flow that fall into two classes. The curves are largely insensitive to the values of η/s in the QGP phase and follow the value of the viscosity in the hadron gas: the parametrizations (*LH-LQ*) and (*LH-HQ*) with constant η/s in the hadron gas result in larger $v_2(p_T)$ than the parametrizations (*HH-LQ*) and (*HH-HQ*) with realistic η/s in the hadron gas. We have confirmed the insensitivity to the values of η/s in the high-temperature QGP phase by decoupling the system at $T_{\text{dec}} = 170$ MeV. In that case, $v_2(p_T)$ is largely independent of the η/s parametrization. The separation of curves occurs in the subsequent evolution in the hadronic phase. This shows that, within this model and at RHIC, viscous effects from the hadron gas dominate over viscous effects from the QGP, see also Refs. [9,10]. Because of the strong longitudinal expansion, the initial shear stress enhances the transverse pressure and thus the buildup of the flow anisotropy, but this is counteracted by the viscous suppression of anisotropies. Our simulations suggest that at RHIC these two effects cancel each other in the QGP phase.

The main reason for the hadronic suppression of $v_2(p_T)$ are the viscous corrections δf to the particle distribution function. Thus, the values of $\pi^{\mu\nu}$ on the decoupling boundary are significantly larger in the case with large hadronic η/s . On the other hand, the azimuthal anisotropies of the hydrodynamic flow field are quite similar in all cases. This is demonstrated in Fig. 3, where we plot $v_2(p_T)$ of pions at RHIC without δf . All curves are much closer to each other, indicating that the space-time evolution in the hadron gas is similar in all four cases.

We have tested that these conclusions are unchanged if we use different $\tau_0 = 0.2$ –1.0 fm, different EoS,

e.g., with or without chemical freeze-out, use nonequilibrium initial conditions (the same nonzero initial $\pi^{\mu\nu}$ for all four cases), or shift the η/s parametrizations up by a constant value, such that η/s at $T = 180$ MeV is 5 times the AdS/CFT lower bound. Although $v_2(p_T)$ and the slopes of the p_T spectra change when we change the setup, the observed sensitivity of $v_2(p_T)$ on the viscosity around $T \sim 180$ MeV and below, rather than on the high-temperature QGP viscosity is quite generic at RHIC. If we increase η/s above $T = 200$ MeV by a factor of 10 in parametrization (*HH-LQ*), the elliptic flow is practically the same as shown in Fig. 2(d). This confirms that the value of η/s in the high-temperature QGP phase has no effect on the final observable $v_2(p_T)$ at RHIC, even though during the evolution the system spends approximately equal times above $T \sim 200$ MeV and between $T \sim 170$ and 200 MeV.

Interestingly, the sensitivity of $v_2(p_T)$ to the QGP viscosity increases with increasing collision energy, while the sensitivity to the hadronic viscosity decreases. This can be

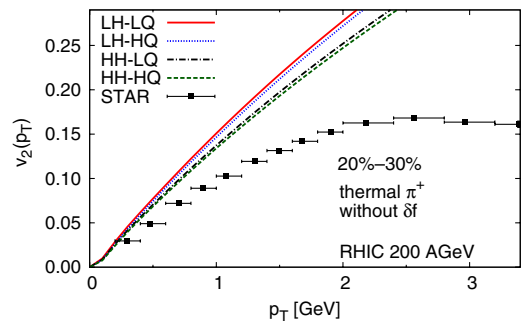


FIG. 3 (color online). Same as Fig. 2(d), but for elliptic flow of thermal (i.e., without decays) pions with $\delta f = 0$.

seen in Figs. 2(e) and 2(f), which show $v_2(p_T)$ for $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.5$ TeV Pb + Pb collisions, respectively.

At the highest LHC energy, the behavior of $v_2(p_T)$ is completely opposite to that at RHIC. It is almost independent of the hadronic viscosity, but sensitive to the QGP viscosity. In contrast to the RHIC case, at LHC the differences in $v_2(p_T)$ are mostly due to the difference in the transverse flow profiles (caused by the different QGP viscosities) and not due to the viscous corrections to the distribution function at freeze-out. The latter are much smaller than at RHIC: the magnitude of δf is the difference between the curves ($LH-LQ$) and ($HH-LQ$) or ($LH-HQ$) and ($HH-HQ$) in Fig. 2(f). We have also checked that $v_2(p_T)$ at low- p_T remains insensitive to the hadronic viscosity, even if we increase the hadronic η/s in such a way that it reaches $\eta/s = 1.0$ at $T = 100$ MeV, but keep the minimum of η/s fixed. The collisions at $\sqrt{s_{NN}} = 2.76$ TeV are between these two extreme behaviors, as elliptic flow depends both on the hadronic and the QGP η/s .

There are several reasons why the effect of η/s on the elliptic flow at LHC is so different from that at RHIC: first, the longer lifetime of the QGP phase, which results in a stronger dependence of the transverse flow on the viscous properties of the QGP. Second, once the system decouples, it has much larger transverse size and velocity gradients are smaller. Subsequently, dissipative effects from the hadronic stage are smaller and have less effect on the observed $v_2(p_T)$.

In conclusion, we have investigated the effects of a temperature-dependent η/s on the hadron spectra and elliptic flow coefficients at $\sqrt{s_{NN}} = 200$ GeV Au + Au collisions at RHIC and $\sqrt{s_{NN}} = 2.76$ TeV and $\sqrt{s_{NN}} = 5.5$ TeV Pb + Pb collisions at LHC. We found that in all cases the slopes of the p_T spectra of pions depend mainly on the high-temperature η/s , and hardly at all on the hadronic viscosity.

The effect of η/s on the differential elliptic flow $v_2(p_T)$ is more subtle. At RHIC energies, $v_2(p_T)$ is highly sensitive to the viscosity in hadronic matter and almost independent of the viscosity in the QGP phase. In contrast, at the highest LHC energy the opposite holds: elliptic flow is almost independent of the hadronic viscosity, but depends strongly on the QGP viscosity. Thus the extraction of an η/s value for the QGP, except for its value at the expected minimum around T_c , is basically impossible using the elliptic flow data at RHIC alone. On the other hand, a determination of the temperature dependence of η/s in the QGP phase from elliptic flow data seems to be possible at LHC. This could allow the observation of a possible transition from the strongly coupled plasma near T_c ; see, e.g., Ref. [29], to the weakly coupled QGP.

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