PHYSICAL REVIEW LETTERS

Detecting Entanglement in Spatial Interference

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(Received 20 November 2010; published 24 May 2011)

We discuss an experimentally amenable class of two-particle states of motion giving rise to nonlocal spatial interference under position measurements. Using the concept of modular variables, we derive a separability criterion which is violated by these non-Gaussian states. While we focus on the free motion of material particles, the presented results are valid for any pair of canonically conjugate continuous variable observables and should apply to a variety of bipartite interference phenomena.

Is it possible to deduce entanglement from an interference pattern, to perform an "entangled Young experiment," following the famous single-particle interference experiments? While the wave-particle duality of single material particles has been a central theme since the early days of quantum mechanics and is impressively confirmed in interference experiments [1,2], a similarly convincing demonstration of quantum nonlocality as implied by entanglement has proven to be much harder to implement with matter waves.

Although recent experimental progress, in particular in controlling ultracold atoms, has rendered experiments conceivable that probe entanglement in the free motion of material particles, a direct implementation of most schemes that have proven successful with other continuous variable degrees of freedom (e.g., field modes) fails due to the restricted possibilities to manipulate and detect material particles. In particular only position measurements are easily doable. Existing proposals therefore rely either on reduced fluctuations in the center of mass and relative motion [3], in the spirit of Einstein, Podolsky, and Rosen (EPR) [4], or on the violation of a Bell inequality [5,6]. Both approaches have drawbacks. The former is based on correlations that appear invitingly easy to explain in terms of a classical (nonquantum) model, and the latter requires interferometers to complete the measurements [6]. If one restricts oneself to elementary position measurements, the states violating a Bell inequality maximally seem to be hard if not impossible to implement experimentally [7].

In view of the great success and the compelling power of single-particle interference experiments, it is natural to ask whether, instead of violating a Bell inequality, it is experimentally easier to establish entanglement in the motion of material particles by means of similarly impressive nonlocal matter wave interference. We discuss an experimentally amenable class of states which provides such nonlocal interference. But conceptually, it is not obvious *a priori* that a nonlocal interference pattern—as intuitively convincing as it may be—can indicate entanglement, thus strictly excluding the possibility to describe the correlations in terms of a separable state. While an extensive state tomography could also supply such a rigorous proof, it would be advantageous to possess an entanglement criterion in terms of observables that can be directly read off the interference pattern, merely complemented by measurements of some likewise accessible "conjugate" observables.

In this Letter we provide such a criterion. To be more specific, suppose we hold a two-particle state $\Psi(x_1, x_2)$ which gives rise to a nonlocal interference pattern when subjected to joint position measurements,

$$|\Psi(x_1, x_2)|^2 = w(x_1 - x_0)w(x_2 + x_0)\cos^2\left(2\pi\frac{x_1 - x_2}{\lambda}\right),$$

(1)

where the envelope $w(x_1 - x_0)$ localizes particle 1 with an uncertainty $\sigma_x \gg \lambda$ in the vicinity of x_0, and similarly particle 2 around $-x_0$. Obviously, the interference pattern describes correlations in the relative coordinate $x_{\rm rel} = x_1 - x_2$ of the two particles. But are these correlations necessarily a signature of entanglement? In the case of EPR states (e.g., squeezed Gaussian states), entanglement can be deduced from the reduced fluctuations in both the relative coordinate $x_{\rm rel}$ and the total momentum $p_{\rm tot} = p_1 + p_2$, since the canonically conjugate operator pairs x_j, p_j ($[\mathsf{x}_j, \mathsf{p}_j] = i\hbar$), $j = 1, 2$, set lower limits to these fluctuations for separable states [8,9]. In the situation described by (1), in contrast, it is not the relative coordinate that is "squeezed," but its value modulo λ.

We show how this observation can be employed to derive an entanglement criterion. The key is to identify *modular variables* [10] as the appropriate pair of conjugate observables. The criterion is rooted in a state-independent additive uncertainty relation (UR) for these variables, which remedies the problems arising from the operator-valued commutator appearing in the Robertson UR. We construct a class of non-Gaussian states, denoted *modular entangled states*, which offer natural and robust generation protocols and violate this criterion. The interference pattern in (1) is shown to represent only the weakest form of nonlocal correlations exhibited by this class.

Multislit interference.—To discuss the prerequisites of particle interference and its relation to the modular variables it is instructive to recapitulate the single-particle case first. Ideally, the (transverse) state immediately after passing an aperture of N slits is described by a superposition of N spatially distinct state components determined by the shape of the slits,

$$\langle x|\psi_{\mathrm{MS}}\rangle = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\langle x + nL|\psi\rangle, \qquad (2)$$

where L denotes the slit separation. The particular shape of the single-slit wave function $|\psi\rangle$ is irrelevant for our discussion provided its spatial width σ_x satisfies $\sigma_x \ll L$. This guarantees that the envelope of the resulting fringe pattern varies slowly on the scale of a single fringe period and thus encloses a large number of interference fringes. Note that the state (2) can equally be read as a longitudinal superposition of comoving wave packets.

The subsequent dispersive spreading of the N wave packets during the free propagation to the screen results in their overlap and interference, yielding in the far-field limit the characteristic interference pattern on the screen. In terms of the initial state (2), this position measurement at asymptotic times corresponds to a formal momentum measurement of $p = m(x - \langle\psi_{\mathrm{MS}}|\mathsf{x}|\psi_{\mathrm{MS}}\rangle)/t$. The probability distribution

$$|\langle p|\psi_{\mathrm{MS}}\rangle|^2 = |\langle p|\psi\rangle|^2 F_N\!\left(\frac{pL}{h}\right) \qquad (3)$$

exhibits the fringe pattern $F_N(x) = 1 + (2/N)\sum_{j=1}^{N-1}(N-j)\cos(2\pi jx)$. In case of $N = 2$ Eq. (3) reduces to the sinusoidal fringe pattern of the double slit, whereas for $N > 2$ one obtains the sharpened main maxima and suppressed side maxima characteristic for multislit interference. This reflects a tradeoff between the number of superposed wave packets N and the uncertainty of the phase of the interference pattern, in analogy to the tradeoff between the variances of a conjugate variable pair. A similar tradeoff exists between the number of fringes $M \approx \sigma_p L/h \approx L/\sigma_x$ covered by the envelope of the interference pattern and the width-to-spacing ratio $\sigma_x/L \approx 1/M$.

Modular variables.—These mutual relationships between the multislit state (2) and the resulting interference pattern (3) are captured best by splitting the position (momentum) operator into an integer component N_x (N_p) and a modular component $\bar{\mathsf{x}}$ ($\bar{\mathsf{p}}$) [10],

$$\mathsf{x} = \mathsf{N}_x\ell + \bar{\mathsf{x}}, \qquad \mathsf{p} = \mathsf{N}_p\frac{h}{\ell} + \bar{\mathsf{p}}, \qquad (4)$$

where $\bar{x} = (x + \ell/2)\mathrm{mod}\,\ell - \ell/2$ and $\bar{p} = (p + h/2\ell)\mathrm{mod}(h/\ell) - h/2\ell$. (For convenience, we define the modular variables symmetrically with respect to the origin.) Recent applications of the modular variables are discussed in [11–13].

For the multislit state (2) the adequate choice of the partition scale is given by $\ell = L$. The probability distribution (3) can then be written as $|\langle p = N_p h/L + \bar{p}|\psi_{\mathrm{MS}}\rangle|^2 \approx |\langle p = N_p h/L|\psi\rangle|^2 F_N(\bar{p}L/h)$, which indicates that the modular variables isolate different characteristic aspects of interference: the periodic fringe pattern is described by the modular momentum $\bar{p}$, its envelope by the integer momentum N_p. Similarly, N_x describes the distribution of wave packets in (2) and $\bar{x}$ their (common) shape.

The modular variables $\bar{\mathsf{x}}$, $\bar{\mathsf{p}}$ have the remarkable property that they commute, $[\bar{\mathsf{x}}, \bar{\mathsf{p}}] = 0$, despite originating from conjugate observables [10,14]. The common eigenstates $|\bar{x}, \bar{p}\rangle$ of $\bar{\mathsf{x}}$ and $\bar{\mathsf{p}}$ with eigenvalues $\bar{x}$ and $\bar{p}$ read $|\bar{x}, \bar{p}\rangle = \sqrt{\ell/h}\sum_{n\in\mathbb{Z}}\exp(i\bar{p}n\ell/\hbar)|n\ell + \bar{x}\rangle_x$, or, equivalently, $|\bar{x}, \bar{p}\rangle = \sqrt{1/\ell}\exp(-ip\bar{x}/\hbar)\sum_{m\in\mathbb{Z}}|mh/\ell + \bar{p}\rangle_p$. The tradeoff between the number of superposed wave packets N and the phase of the interference pattern is now reflected by a conjugate relationship between the integer position N_x and the modular momentum $\bar{\mathsf{p}}$,

$$[\mathsf{N}_x, \bar{\mathsf{p}}] = \frac{i\hbar}{\ell}\left(\mathbb{1} - \frac{h}{\ell}\int_{-\ell/2}^{\ell/2}d\bar{x}|\bar{x}, \bar{p} = h/2\ell\rangle\langle\bar{x}, \bar{p} = h/2\ell|\right). \qquad (5)$$

Similarly, the tradeoff between the width-to-spacing ratio σ_x/L and the number of covered fringes is described by the commutator of the modular position $\bar{\mathsf{x}}$ and the integer momentum N_p,

$$[\bar{\mathsf{x}}, \mathsf{N}_p] = \frac{i\ell}{2\pi}\left(\mathbb{1} - \ell\int_{-h/2\ell}^{h/2\ell}d\bar{p}|\bar{x} = \ell/2, \bar{p}\rangle\langle\bar{x} = \ell/2, \bar{p}|\right). \qquad (6)$$

The projection operators on the right-hand side of (5) and (6) result from the boundedness of the modular variables and are indispensible to ensure the validity of the Robertson UR. This is similar to the relationship between an angular position operator and its conjugate angular momentum [15].

Squeezed modular position states.—The multislit states (2) display their interference in momentum. In view of the symmetry between the two pairs $(\mathsf{N}_x, \bar{\mathsf{p}})$ and $(\bar{\mathsf{x}}, \mathsf{N}_p)$, one can construct another class of states where the modular variables exchange their roles. This is achieved by superposing wave packets that are distinct in momentum (instead of position),

$$|\psi_{\mathrm{SMP}}\rangle = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}|\psi_{x_0,(N_0+n)h/\lambda}\rangle, \qquad (7)$$

where $\langle x|\psi_{x_0,p_0}\rangle = \phi(x - x_0)\exp[ip_0(x - \bar{x}_0)/\hbar]$ denotes a (well-behaved) wave packet that is localized in phase space around (x_0, p_0). N_0 represents an arbitrary base integer momentum. Distinctness of the wave packets requires that their momentum width σ_p is smaller than their separation in momentum space, $\sigma_p \ll h/\lambda$, or, equivalently, $\sigma_x \gg \lambda$. A hypothetical "momentum grid" with

slit width h/d, $d \gg \lambda$, could, e.g., prepare a state with $\phi(x) = \mathrm{sinc}[\pi x/d]/\sqrt{d}$.

A position measurement of the states (7) reveals an interference pattern with periodicity λ, $|\langle x|\psi_{\mathrm{SMP}}\rangle|^2 = |\phi(x - x_0)|^2 F_N((x - \bar{x}_0)/\lambda)$. Note that $\bar{x}_0$ determines the phase of the fringe pattern. Increasing N results in the formation of sharp main maxima, which is reflected by the decreasing variance of the modular position variable (now with $\ell = \lambda$),

$$\langle(\Delta\bar{\mathsf{x}})^2\rangle_{\mathrm{SMP}} = \frac{\lambda^2}{12}[1 - S_1(N)]. \qquad (8)$$

In this sense one may denote the states (7) as squeezed modular position states. The monotonically increasing squeezing function $S_1(N) = -(12/\pi^2)\sum_{j=1}^{N-1}(-1)^j \times (N - j)/Nj^2 < 1$ (for $\bar{x}_0 = 0$) is evaluated in Table I for representative N. Notably, in the limit $N \to \infty$ the variance (8) vanishes, indicating perfect squeezing.

Correspondingly, the variance of the integer momentum N_p increases with N, $\langle(\Delta\mathsf{N}_p)^2\rangle_{\mathrm{SMP}} = (\mathsf{N}^2 - 1)/12$. For $N = 1$, however, $\langle(\Delta\mathsf{N}_p)^2\rangle_{\mathrm{SMP}}$ vanishes (since the $|\psi_{x_0,p_0}\rangle$ are localized on the scale of the integer momentum), while $\langle(\Delta\bar{\mathsf{x}})^2\rangle_{\mathrm{SMP}}$ remains finite according to (8). Validity of the Robertson UR, $\langle(\Delta\mathsf{N}_p)^2\rangle_{\mathrm{SMP}}\langle(\Delta\bar{\mathsf{x}})^2\rangle_{\mathrm{SMP}} \geq |\langle[\bar{\mathsf{x}}, \mathsf{N}_p]\rangle_{\mathrm{SMP}}|^2/4$, thus requires that the projector on the right-hand side of (6) renders the Robertson UR trivial for $N = 1$. Indeed, we find $|\langle[\bar{\mathsf{x}}, \mathsf{N}_p]\rangle_{\mathrm{SMP}}| = \ell/(2\pi)[1 - (1 + (-1)^{N+1})/2N]$, which vanishes for $N = 1$. This irrelevance of the Robertson UR in the case $N = 1$ impedes its employment in the separability criterion presented below. Note that $|\langle[\bar{\mathsf{x}}, \mathsf{N}_p]\rangle_{\mathrm{SMP}}|$ converges towards the canonical constant value, while the projector term in (6) is still relevant for $N = 3$. Its alternating structure can be traced back to either minima or (side) maxima of the fringe pattern coinciding with $\bar{x} = \lambda/2$.

As an advantage of the modular position squeezed states (7) compared to the modular momentum squeezed states (2), they exhibit interference by immediate position measurements, while to determine the integer momentum one must only distinguish the macroscopically distinct components $|\psi_{x_0,p_0}\rangle$, which is easy once they are sufficiently separated by free propagation. At the same time, as superpositions of different velocities, they are genuine matter wave states without photonic analogue.

Modular entangled states.—We are now prepared to move on to entangled states of two material particles.

TABLE I. Evaluation of the squeezing functions $S_1(N)$ and $S_2(N)$ for several superposition ranks N. $S_1(N)$ and $S_2(N)$ describe the squeezing of the modular position $\bar{x}$ in the single-particle case (8) and of the modular relative position $\bar{x}_1 - \bar{x}_2$ in the two-particle entangled case (11), respectively.

N	1	2	3	4	10	100
$S_1(N)$	0.0	0.61	0.71	0.79	0.92	0.99
$S_2(N)$	0.0	0.30	0.46	0.55	0.76	0.96

Ultimately, we are interested in states that reveal their entanglement by a nonlocal interference pattern similar to (1). To this end, we introduce two-particle modular position entangled (MPE) states, which are defined by superposing correlated pairs of (counterpropagating) wave packets of different velocities,

$$|\Psi_{\mathrm{MPE}}\rangle = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} |\psi_{x_0,(N_0+n)h/\lambda}\rangle_1 |\psi_{-x_0,-(N_0+n)h/\lambda}\rangle_2. \qquad (9)$$

Only for clarity we assume that the particles are spatially separated, positioned at $\pm x_0$. Moreover, it is clear that one could equally define modular momentum entangled (MME) states. Such states could be generated (to good approximation) by the sequential coherent dissociation of a diatomic molecule [16]. For convenience we consider a superposition of product states; correlated components $|\Psi_{x_0,p_0;-x_0,-p_0}\rangle$ would not modify our conclusions, since the latter are based on entangled integer momenta $\mathsf{N}_{p,j}$, i.e., distinctive "bulk" properties of the particles. This is in contrast to EPR states, where the relevant correlations reside in the microscopic fluctuations of the center of mass and relative variables. Performing position measurements on each side, the nonseparable structure of (9) gives rise to an interference pattern in the relative position x_{rel}, $|\langle x_1, x_2|\Psi_{\mathrm{MPE}}\rangle|^2 = |\phi(x_1 - x_0)|^2|\phi(x_2 + x_0)|^2 F_N((x_1 - x_2)/\lambda)$ (with $\bar{x}_{0,1} = \bar{x}_{0,2}$), or, equivalently, to a squeezing in the modular relative position $\bar{x}_{\mathrm{rel}} = \bar{x}_1 - \bar{x}_2$.

Modular entanglement criterion.—The correlations in $\bar{x}_{\mathrm{rel}}$ and the total integer momentum $\mathsf{N}_{p,\mathrm{tot}} = \mathsf{N}_{p,1} + \mathsf{N}_{p,2}$ exhibited by the MPE states (9) can be exploited to demonstrate the underlying entanglement. In analogy to [8], we consider the sum of variances, $\langle(\Delta\mathsf{N}_{p,\mathrm{tot}})^2\rangle_\rho + \langle(\Delta\bar{\mathsf{x}}_{\mathrm{rel}})^2\rangle_\rho/\ell^2$. Using the Cauchy-Schwarz inequality one can show that a separable state of motion, $\rho = \sum_i p_i \rho_{1i} \otimes \rho_{2i}$, implies $\langle(\Delta\mathsf{N}_{p,\mathrm{tot}})^2\rangle_\rho + \langle(\Delta\bar{\mathsf{x}}_{\mathrm{rel}})^2\rangle_\rho/\ell^2 \geq \sum_{i,j} p_i\{\langle(\Delta\mathsf{N}_{p,j})^2\rangle_i + \langle(\Delta\bar{\mathsf{x}}_j)^2\rangle_i/\ell^2\}$, with $j = 1, 2$. In contrast to [8], we cannot use the Robertson UR to estimate the remaining sums of variances, since the expectation value of the state-dependent commutator (6) vanishes when evaluated for an MPE state (9). However, one can establish a state-independent additive uncertainty relation for the modular variables, $\langle(\Delta\mathsf{N}_{p,j})^2\rangle + \langle(\Delta\bar{\mathsf{x}}_j)^2\rangle/\ell^2 \geq \mathcal{C}_{\mathsf{N}_p,\bar{x}} > 0$. Using this, we immediately get the desired criterion,

$$\langle(\Delta\mathsf{N}_{p,\mathrm{tot}})^2\rangle_\rho + \frac{1}{\ell^2}\langle(\Delta\bar{\mathsf{x}}_{\mathrm{rel}})^2\rangle_\rho \geq 2\mathcal{C}_{\mathsf{N}_p,\bar{x}}, \qquad (10)$$

which must be satisfied by any separable state. The constant $\mathcal{C}_{\mathsf{N}_p,\bar{x}}$ is given by the smallest eigenvalue μ_0 of the operator $\mathsf{A}_j = \mathsf{N}_{p,j}^2 + \bar{\mathsf{x}}_j^2/\ell^2$. The corresponding differential equation in the common eigenbasis of $\bar{\mathsf{x}}_j$ and $\bar{\mathsf{p}}_j$ is solved by $\psi(\bar{x}_j, \bar{p}_j) = \exp(-\pi\bar{x}_j^2/\ell^2)M(-\pi\mu/2 + 1/4, 1/2, 2\pi\bar{x}_j^2/\ell^2)\chi(\bar{p}_j)$, with $M(a, b; x)$ the Kummer function and $\chi(\bar{p})$ arbitrary. Continuity requires a vanishing first derivative at $\bar{x}_j = \ell/2$, which implicitly determines the

(discrete) spectrum $\{\mu_\nu\}$ of A_j. Its smallest eigenvalue evaluates numerically as $\mathcal{C}_{\mathsf{N}_p,\bar{\mathsf{x}}} \cong 0.078\,235$; second order perturbation theory (with A_j expressed in the common eigenbasis of $\mathsf{N}_{p,j}$, $\bar{\mathsf{p}}_j$) yields a reasonable analytic approximation, $\mathcal{C}_{\mathsf{N}_p,\bar{\mathsf{x}}} \approx 1/12(1{-}1/15)$. We note that a criterion similar to (10) can be established for $\mathsf{N}_{x,\mathrm{tot}}$ and $\bar{\mathsf{p}}_{\mathrm{rel}}$.

The MPE states (9) violate the separability criterion (10) for any $N \geq 2$. Indeed, the resulting variances read $\langle(\Delta\mathsf{N}_{p,\mathrm{tot}})^2\rangle_{\mathrm{MPE}} = 0$ and

$$\langle(\Delta\bar{\mathsf{x}}_{\mathrm{rel}})^2\rangle_{\mathrm{MPE}} = \frac{\lambda^2}{6}[1 - S_2(N)], \qquad (11)$$

(again with $\ell = \lambda$) where the monotonically increasing squeezing function $S_2(N) = (6/\pi^2)\sum_{j=1}^{N-1}(N-j)/Nj^2 < 1$ is evaluated in Table I for several representative N. This proves the possibility to deduce entanglement from a nonlocal interference pattern. Again, one can achieve perfect squeezing in the limit $N \to \infty$; the interference pattern (1) corresponds to $N = 2$.

The MPE states (9) (and MME states alike) generalize single-particle interferometric schemes such as double-slit or grid experiments to the case of two entangled particles. Aside from the additional requirement to provide the correlations between the particles, the MPE states thus inherit both the advantages and the challenges of such schemes. Similar to any interference experiment, the phase $\bar{x}_0$ of the superposed components $|\psi_{x_0,p_0}\rangle$ must be well controlled, and also all components should share the same shape $\phi(x)$ [see (7)]. (On the other hand, the particular shape is to a large extent irrelevant, which leaves it to the experimenter to choose easily producible states.) Deviations from these conditions result in a visibility-reducing blurring of the fringe pattern and thus in an attenuation of the squeezing of the modular variable. However, a simple robustness check, where the MPE states are mixed with merely classically (integer momentum) correlated states, reveals that for $N = 2$ a classically correlated admixture of up to 79% would sustain the violation of the separability criterion, corresponding to a fringe visibility of merely 21%. This robustness, which even improves with increasing N, underlines the appropriateness of the separability criterion (10) to capture entanglement in spatial interference, and it should leave sufficient freedom to cope with possible experimental limitations.

A realistic generation protocol for MPE states would, e.g., gradually dissociate an ultracold diatomic Feshbach molecule such that subsequent dissociation instants produce wave packets with staggered kinetic energies [16]. Appropriate dissociation pulses can achieve that all of these consecutive wave packets meet simultaneously on each side. This constitutes an approximate MPE state, where the superposed $|\psi_{x_0,p_0}\rangle$ then realistically differ by different stages of dispersion. We checked for $N = 2$ and lithium atoms that this dispersion-induced shape difference can easily be kept under control with realistic parameters,

yielding an experimentally resolvable fringe pattern with $\lambda \approx 100~\mu$m and a visibility of 85%. On the other hand, a "grid state preparation" of transversal MME states, starting, e.g., with an EPR correlated particle pair and then each particle passing a grating, would provide the identity in phase and shape of the $|\psi_{x_0,p_0}\rangle$ for any N by means of the state preparation.

Conclusion.—We presented a scheme to provide and detect entanglement in the motion of two free material particles. Elementary position measurements at macroscopically distinct sites give rise to a nonlocal interference pattern; the nonseparability then follows from reduced fluctuations in adapted modular variables. In this sense, the scheme allows one to "deduce entanglement from interference," and hence to illustrate the wave-particle duality on a new level including quantum nonlocality.

We emphasize that the modular variables are merely a matter of interpretation in our scheme and can be deduced from ordinary position and momentum measurements. Finally, it is clear that the entanglement criterion is applicable to any bipartite continuous variable system with conjugate operator pairs, e.g., quadrature amplitudes of field modes, and could thus offer a valuable alternative to existing entanglement detection schemes. Homodyning entangled coherent states [17] may serve as an immediate example.

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