Strong Local-Field Effect on the Dynamics of a Dilute Atomic Gas Irradiated by Two Counterpropagating Optical Fields: Beyond Standard Optical Lattices

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We study a recent experiment [K. Li *et al.*, Phys. Rev. Lett. **101**, 250401 (2008)] on diffracting a Bose-Einstein condensate by two counterpropagating optical fields. Including the local-field effect, we explain the asymmetric momentum distribution and self-imaging of the Bose-Einstein condensate self-consistently. Moreover, we find that the two counterpropagating optical fields could not produce a perfect optical lattice, which is actually deformed by the local-field effect. Our work implies that the local-field effect could be essential for getting a better quantitative analysis of other optical lattice experiments. In particular, the intensity imbalance of the two optical fields could act as a new means to tailor both cold atom dynamics and light propagation.

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Intuitively, two counterpropagating optical fields within a dilute atomic gas will induce a dipole potential, which is now called an "optical lattice" [1], and the atomic gas trapped within the lattice could function as a photonic crystal [2]. Currently, optical lattices are playing an important role in many aspects of cold atom [3–7] and molecule [8] research, such as splitting, reflecting, and diffracting matter waves [3], generating matter-wave mixing [4] and atom entanglement [5], as well as studying strongly correlated quantum phenomena [6].

The interaction between light and a cold atom or molecule gas is an interesting research topic. Many theoretical papers on this issue have shown that the interaction includes two processes [9]. First, the gas density will be modulated by the light-induced dipole potential and interatomic interaction. Second, the atomic motion will produce a back influence on the light propagation through the local modulation of the density-dependent refraction index. The latter processing is called the local-field effect. However, the local modulation of refraction index by a far-off resonance optical lattice is very small in a dilute atomic/molecular gas, and thus the local-field effect is usually ignored [3–8].

Now we show that in some situations, a very small local-field modulation could produce a strong influence on the dynamics of a cold atom gas irradiated by two counterpropagating optical fields, and deform an optical lattice. Our conclusion is drawn from analyzing a recent experiment on diffracting a Bose-Einstein condensate (BEC) by two counterpropagating optical fields with unequal intensities [10]. We show that the local-field effect is the key for successfully explaining the asymmetric momentum distribution of the BEC. We also find that self-imaging could occur when the two fields have the same intensity, but a longer light-condensate interaction time is required.

Our scheme is shown in Fig. 1. An atomic BEC is irradiated by two counterpropagating optical fields E_1 and E_2 with the same far-off resonant frequency. E_3 and E_4 are scattered optical fields. When the optical frequency is far-off resonant from the electronic transition frequency of the atoms, the wave function $\Psi(\mathbf{r}, t)$ for the condensate satisfies the Gross-Pitaevskii equation [9],

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + \frac{|\mathbf{d}\cdot\mathbf{E}|^2}{\hbar\Delta} + g|\Psi|^2\right]\Psi, \qquad (1)$$

where $\mathbf{d}$ is the induced dipole moment of the atom by the total optical field $\mathbf{E}$, Δ is the detuning, and g is the pseudopotential. Equation (1) shows that the nonuniform optical field will disturb the matter wave. On the other hand, the refraction index of the condensate will be modified, i.e., $n(\mathbf{r}, \mathbf{t}) = \sqrt{1 - d^2/(\varepsilon_0\hbar\Delta)|\Psi(\mathbf{r}, t)|^2}$. As a result, the atomic motion will have a back influence on the propagation of the optical field $\mathbf{E}(\mathbf{r}, t) = \tilde{\mathbf{E}}(\mathbf{r}, t)e^{-i\omega t}$, with the amplitude $\tilde{\mathbf{E}}(\mathbf{r}, t)$ controlled by the Helmholtz equation with wave number $k_L = \omega/c$,

$$\nabla^2\tilde{\mathbf{E}}(\mathbf{r}, t) - [n^2(\mathbf{r}, t)k_L^2]\tilde{\mathbf{E}}(\mathbf{r}, t) \approx 0. \qquad (2)$$

When the incident optical fields and the matter wave have a very large transverse width, the spatial derivatives of the electric field and the wave function in the transverse direction could be neglected in our further

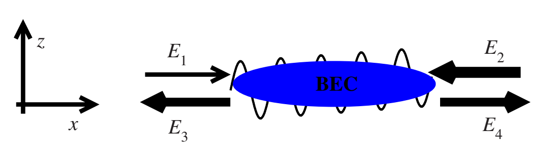

FIG. 1 (color online). A Bose-Einstein condensate is irradiated by two counterpropagating optical fields, E_1 and E_2. Outside the condensate, E_3 and E_4 are scattered optical fields.

analysis. We now use Eqs. (1) and (2) to study a recent experiment in Ref. [10] with the initial function $\Psi(\mathbf{r},0) = [\sqrt{N}/(w_\perp\sqrt{w_x}\sqrt[4]{\pi^3})]\exp[-x^2/(2w_x^2)]$, $w_x = 75\ \mu$m, $w_\perp = 14\ \mu$m. The intensity for the optical field E_1 is $I_1 = 215$ mW/cm^2; the atom number is $N = 2\times10^5$.

We show the momentum distribution $\Phi(p,t) = \int\Psi(x,t)\exp(ipx/\hbar)dx/\sqrt{2\pi}$ calculated with the local-field effect (dashed line) in Fig. 2. The left and right panels, respectively, correspond to the intensity balance case ($I_2 = I_1$) and the intensity imbalance case in which $I_2 = 14$ mW/cm^2 is much weaker than I_1. To highlight the role of the local-field effect, the results calculated without the local-field effect are shown with "+" symbols for comparison. Without including the local-field effect in our calculation, the distributions are symmetric in both cases. However, once the local-field effect is included, the asymmetric distributions appear in the intensity imbalance case. The distribution in the right panel at 25 μs is nearly the same as the initial distribution, implying the matter-wave self-imaging. The self-imaging time calculated by us agrees with the experimental result in Ref. [10]. Moreover, we show that in the intensity balanced case, it takes a longer time (52.3 μs) to induce the self-imaging, which was not demonstrated in Ref. [10].

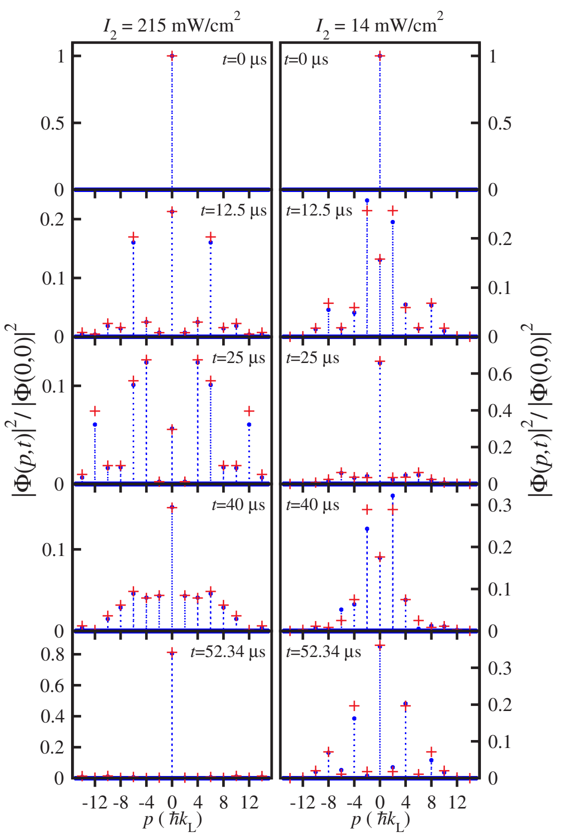

FIG. 2 (color online). The momentum distributions $|\Phi(p,t)|^2$ calculated, respectively, with the local-field effect (dotted line) and without the local-field effect (''+'' symbols) are shown.

Figure 2 shows that self-imaging is irrelevant to the intensity difference, and not much affected by the local-field effect.

Furthermore, we work out the spatial distributions of atomic density (solid line) and optical dipole potential (dashed line) in Figs. 3 and 4. In the intensity balance case, the condensate is symmetrically localized in optical potential wells. In contrast, in intensity imbalance case, the condensate spreads wider due to the lower depth of the dipole potential wells and loses symmetry in the potential wells. Thus, in view of the optical fields, the condensate acts as an imperfect photonic crystal with defects. On the other hand, although Figs. 3 and 4 show that in a small region the optical field looks like a crystal for the atomic gas, the detailed plots of the optical intensity in a larger region, as shown in Fig. 5, display the deformed optical lattice. The left panel of Fig. 5 shows that the optical intensity in the intensity balance case is symmetric, but nonuniform, and varies with time. The right panel of Fig. 5 shows that the optical field in the intensity imbalance case behaves as a tilted board whose tilting direction varies with the time. In this sense, the atoms are like riding a seesaw, which forces the gas to adjust its momentum distribution alternately with time.

To get a more transparent physical picture of interaction between the light and the atomic gas, we now turn to a modified coupled mode theory, i.e., $E_z = [A^+e^{i\varphi(x,t)} + A^-e^{-i\varphi(x,t)}]/\sqrt{n}$ [11] where $\varphi(x,t) = k_L\int_0^x n(x',t)dx'$, and A^+ and A^-, respectively, are the right- and left-propagating optical field. Neglecting the term $\partial^2A^\pm/\partial x^2$, from Eq. (2) we obtain $\partial A^\pm/\partial x = S^\mp A^\mp$ where $S^\pm = dn/dx/(2n)\exp[\pm2i\varphi(x)]$ [11]. The results obtained with this theory agree well with those obtained by directly solving the Gross-Pitaevskii equation and Eq. (2).

The right-propagating (left-propagating) optical field is reflected by the atomic medium and thus has loss of its intensity; however, it also has gain from the reflection of the left-propagating (right-propagating) field. The local-field effect will lead to both time- and position-dependent loss and gain, and thus has a strong influence on optical propagation, as shown in Figs. 6 and 7, where the spatial distributions of the optical intensities $I_+ = |A_+|^2$ and

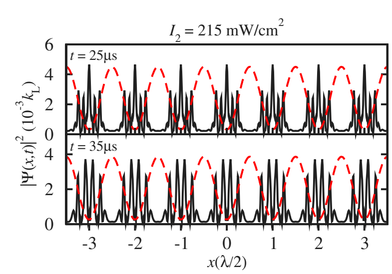

FIG. 3 (color online). The spatial density distributions (solid line), $|\Psi(x,t)|^2$, and the optical potential (dashed line) at time $t = 25\ \mu$s and 35 μs in the intensity balance case are shown.

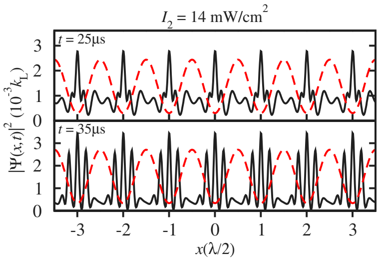

FIG. 4 (color online). The spatial density distributions (solid line), $|\Psi(x, t)|^2$, and the optical potential (dashed line) at time $t = 25$ μs and 35 μs in the intensity imbalance case are shown.

$I_- = |A_-|^2$ are plotted, respectively, for the intensity balance case (I_- is not plotted in this case, due to its being identical to I_+) and intensity imbalance case. In the intensity balance case (Fig. 6), the loss and gain is symmetric in space; consequently the optical intensity is always symmetric, which leads to the symmetric intensity distribution in Fig. 5 and the symmetric momentum distribution in Fig. 2. Because of the time-dependent refraction index through the local atom density, the optical intensity "breathes" with the time, which accounts for the height change of the central peak in the left panel of Fig. 5. In the intensity imbalance case (Fig. 7), the loss and gain is asymmetric such that the optical fields at the initial stage lose spatial symmetry and the asymmetry later becomes even remarkable. This asymmetric optical field creates a deformed interference pattern (see the right panel of Fig. 5) and leads to the asymmetric optical dipole potential, which results in the asymmetry of both the atomic density in Fig. 4 and the momentum distribution in Fig. 2. The density asymmetry further leads to the change of the reflectance for the right- and left-propagating light. Thus, Fig. 7 shows that when the time increases, the asymmetry of the optical intensities is further enhanced, and the right-propagating light is amplified $[I_+(L, t)/I_1 > 1]$ or reduced $[I_+(L, t)/I_1 < 1]$ with the time. Meanwhile the left-propagating optical field is correspondingly reduced or amplified.

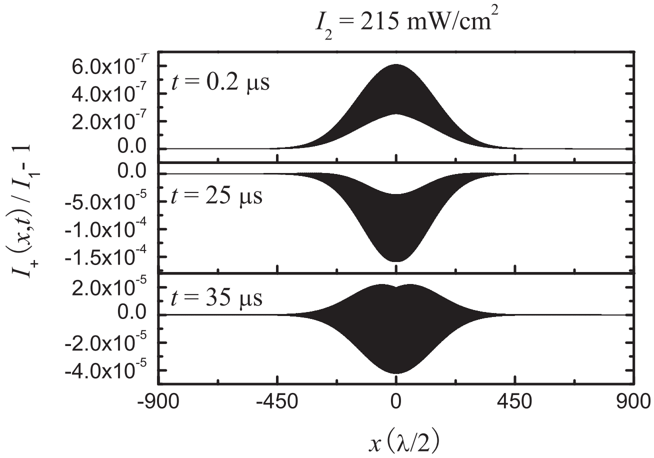

FIG. 6. The intensity of the right-propagating optical field in the intensity balance case is shown.

Based on the above physical picture, we are now ready to establish a straightforward connection of the local-field effect to the asymmetric momentum distribution of the BEC. Making the transformation $\Psi(x, t) = \sum_n \Phi_n(t) e^{2ink_L x}$, we approximately obtain the equation, $i\hbar\dot{\Phi}_n \approx J_1 \exp(-i\delta_+ t)\Phi_{n+1} + J_0\Phi_n + J_{-1}\exp(-i\delta_- t)\Phi_{n-1}$, in which $J_l = \int |\mathbf{d}\cdot\mathbf{E}|^2/(L\Delta) \times \exp(i2lk_L x)dx$ ($l = 0$, ± 1, L is the lattice length), and $\delta_\pm = 2(1 \pm 2n)\hbar k_L^2/m$. Without the local-field effect, $|\mathbf{d}\cdot\mathbf{E}|^2$ is an even function of the position, and $J_1 = J_{-1}$. As a result, the momentum distribution is always symmetric. This also applies to the intensity balance case with the local-field effect (see Fig. 6). However, in the intensity imbalance case with the local-field effect, $|\mathbf{d}\cdot\mathbf{E}|^2$ has no definite parity (see Fig. 7), and $J_1 \neq J_{-1}$, such that the momentum transfer from the zero momentum

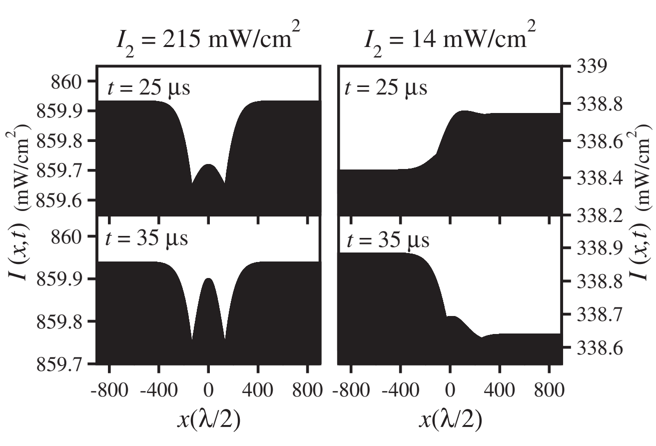

FIG. 5. The detailed envelope of the optical intensity is shown in the condensate filled regime.

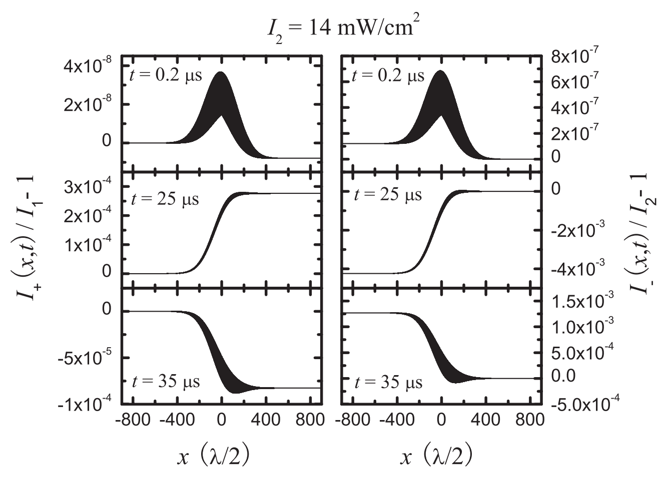

FIG. 7. The intensities of the right- and left-propagating optical fields in the intensity imbalance case are shown.

component to the diffracted component Φ_1 is not equal to that of Φ_{-1}, and thus the asymmetric diffraction is induced.

In summary, we have shown that the diffraction of a BEC by two counterpropagating optical fields can be strongly influenced by the backaction of the local field. We explain the asymmetric momentum distribution of the diffracted atoms and the matter-wave self-imaging self-consistently for the first time. We also found that the self-imaging could be induced in the intensity balance case when the light-condensate interaction time is sufficiently long, suggesting that the appearance of the self-imaging is irrelevant to the intensity difference of the two optical fields.

Our analysis shows that the atomic gas is not a perfect photonic crystal, and the optical lattice formed in the atomic gas is slightly deformed. However, this small deformation produces a remarkable effect on the cold atom dynamics. These results will shed new light on the precise quantitative analysis of other optical lattice experiments such as the demonstration of the quantum phase transition theory [6], superexchange [12], the crossover between quantum tunneling, and the thermal activation of phase slips [13], collective mode, and transport [14]. The Bose- or Fermi-Hubbard models have been used in analysis of these experimental observations; however, precise quantitative analysis has not yet been achieved. The difficulty of the temperature measurement of the atomic gas is now investigated as a main factor that hampers the quantitative analysis of the experiments [15]. However, an assumption of a perfect optical lattice with a uniform amplitude, used for realizing these models is in question, due to the deformation of the optical lattice by local-field effects. Thus, the Bose- or Fermi-Hubard model with this deformation should be applied to give a better quantitative analysis of cold atoms within an ''optical lattice.'' Moreover, local-field effect may offer a new way to control quantum dynamics of an atomic gas.

Similar analysis of Bragg amplification of a weak beam interfering with a strong laser beam in a thermal molecular gas was presented [16]. The experiment in Ref. [10] and our analysis as well as [16] show that the intensity difference of two counterpropagating optical fields would be a new tool for tailoring the dynamics of atomic or molecular gases and optical propagation. This issue has just been initiated, leading to many questions. For example, if the intensity of one beam is further weakened to quantum level, what effect should be expected? Can the quantum reflection or splitting of a cold atomic beam be controlled in this way? Is the local-field effect a new decoherence source for precise measurement with an optical lattice [7]? These outstanding questions are worthy of further theoretical and experimental investigation.

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