Optical Traps for Dark Excitons

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We propose a mechanism for optical trapping of dark excitons by linearly polarized unabsorbed standing waves, with a potential depth of the order of a few meV. Since this trapping, based on carrier exchanges with virtual excitons coupled to unabsorbed photons, equally acts on bright and dark states, Bose-Einstein condensation of excitons—which occurs in dark states—must appear as dark spots in a cloud of bright excitons, at the trap potential minima, when the temperature decreases.

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The observation of Bose-Einstein condensation (BEC) of excitons [1,2] is one of the grand challenges of semiconductor physics. This search started long ago, excitons seeming at first much better candidates than atoms to observe this remarkable quantum effect: their mass is much smaller than the atomic mass so that they must condense at a much higher temperature. Nevertheless, exciton BEC has not been observed yet, while atomic BEC [3,4] is routinely obtained despite critical temperatures as low as 100 nK. This achievement relies on two technics: laser cooling and optical trapping. Several methods have been proposed to trap excitons based on local strain field [5], metallic gates [6], repulsive potential by photoexcited excitons or polaritons [7–9], and acoustic waves [10,11]. Notably absent is the primary mechanism used for atoms based on the optical Stark effect. It has been recently proposed to trap electrons [12,13] and electron-hole plasma [14,15]. We here show how it can be adapted to bound excitons: the laser-based mechanism we propose, which uses the composite boson nature of the excitons, has the surprising remarkable feature to equally trap bright excitons and dark excitons in spite of the fact that dark excitons are not directly coupled to light.

This feature could be a key to evidence Bose-Einstein condensation of excitons. Indeed, excitons condense into a dark state [16], since bright excitons suffer additional repulsive valence-conduction Coulomb processes [17], which bring their energy above dark excitons. Although known long ago, this aspect of exciton BEC, which makes its direct observation through optical experiments impossible, has apparently been ignored, probably because excitons are created by photon absorption in bright states. Yet, carrier exchanges can transform two opposite-spin bright excitons into two dark excitons [18,19]. While standard optical experiments cannot directly evidence a dark exciton condensate, an indirect way to optically observe it could be through the temperature dependence of the bright state fraction in an unpolarized exciton gas. This, however, requires traps acting on both bright and dark excitons. Above critical temperature, recombination of bright excitons will make the trapping regions appear as bright spots. By lowering the temperature, excitons will undergo Bose-Einstein condensation into dark state at the trap center where the energy is the lowest. So, the center of bright spots must turn dark as the temperature decreases.

We have recently proposed two mechanisms to trap free electrons in semiconductors using spatially varying laser intensity. Mechanism (i) uses semivirtual trion made of a real electron and a virtual exciton coupled to an unabsorbed standing wave [12]. Since electrons in a trionic molecule are in a singlet state, the spins of the real and virtual electrons are then opposite. Mechanism (ii) relies on carrier exchange between the real electron and the electron of a virtual exciton [13], thus acting only for parallel spins. We here reconsider these ideas to construct an optical trap acting on both dark and bright excitons. This fundamentally eliminates the equivalent of mechanism (i) with coupling to semivirtual biexcitonic molecule [20], because the trap will only act on bright excitons. By contrast, mechanism (ii) based on carrier exchanges can trap dark excitons as efficiently as bright excitons, provided that the light is linearly polarized.

Physical idea.—We propose a spatial modulation of the exciton center-of-mass distribution, based on carrier exchanges with ($\pm\mathbf{Q}$) virtual excitons coupled to a standing wave made of unabsorbed ($\pm\mathbf{Q}$) photons. As carrier exchange conserves momentum, an exciton with center-of-mass momentum $\mathbf{K}$ transforms (see Fig. 1) into a superposition of $\mathbf{K}$, $\mathbf{K}+2\mathbf{Q}$, and $\mathbf{K}-2\mathbf{Q}$ states, thus leading to a sinusoidal modulation of its spatial distribution.

Photons with momentum $\mathbf{Q}$ and circular polarization σ_+ are coupled to virtual excitons $|\nu,\mathbf{Q},S\rangle = B^\dagger_{\nu,\mathbf{Q},S}|0\rangle$ with same center-of-mass momentum $\mathbf{Q}$, same total spin $S=+1$, but arbitrary relative motion index ν. The exciton Rabi coupling to photon depends on ν as $\Omega_\nu = \Omega L^{D/2}\langle\nu|\mathbf{r}=0\rangle$ where Ω is the free pair coupling, L the sample size, and D the space dimension. $2D$ excitons with total spin $S=+1$ are made of $s=-1/2$ electrons and $m=+3/2$ heavy holes. In bulk, they can also be made of $s=+1/2$

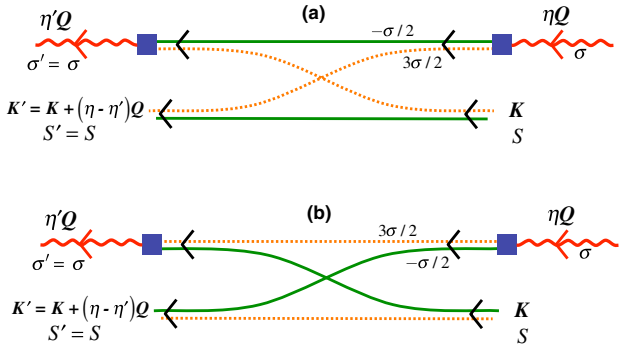

FIG. 1 (color online). Pauli scattering for (a) hole and (b) electron exchange between a real exciton ($\mathbf{K}$, S) and a virtual exciton coupled to an unabsorbed photon. Momentum can be transferred from photon to exciton while polarization stays unchanged. For heavy-hole exciton with total spin S, hole (respectively, electron) exchange requires photons with polarization $\sigma = S/|S|$ (respectively, $\sigma = -2S + 3S/|S|$).

electrons and $m = +1/2$ light holes. However, since the photon coupling to light holes is weaker than to heavy holes, we will ignore them here.

Four scenarios are possible for a $\mathbf{K}$ exciton with spin $S = (\pm 1, \pm 2)$ in the presence of a σ_+ standing wave: (i) If $S = 1$, the bright exciton $\mathbf{K}$ can exchange its two carriers with the $S = 1$ virtual excitons coupled to the σ_+ standing wave ($\pm \mathbf{Q}$). These exchanges generate two components $\mathbf{K} \pm 2\mathbf{Q}$ to its center-of-mass wave function. (ii) If $S = -1$, carrier exchanges with $S = 1$ virtual excitons are impossible. (iii) If $S = 2$, the dark exciton $\mathbf{K}$, made of $s = +1/2$ electrons and $m = 3/2$ holes, can only exchange its hole with the $S = 1$ virtual excitons [see Fig. 1(a)]. (iv) If $S = -2$, the dark exciton $\mathbf{K}$ can only exchange its electron [see Fig. 1(b)].

If we now consider standing wave with linear polarization, $S = \pm 2$ dark excitons can exchange their hole through the $\sigma_\pm$ component of the light, and their electron through the $\sigma_\mp$ component. $S = \pm 1$ bright excitons can exchange their two carriers through the same $\sigma_\pm$ component. So, in the end, a standing wave with linear polarization induces the same amount of exchanges with any of the $S = (\pm 1, \pm 2)$ excitons, trapping them with the same efficiency.

Calculation of the trapping probability.—The semiconductor coupling to a linealy polarized standing wave [21] made of ($\pm \mathbf{Q}$) photons with frequency ω_P, reads in the rotating frame as $W = U + U^\dagger$, with

$$U^\dagger = \frac{1}{\sqrt{2}} \sum_{\eta=\pm 1} \sum_{\sigma=\sigma_\pm} U^\dagger_{\eta\mathbf{Q},\sigma} \tag{1}$$

where $U^\dagger_{\eta\mathbf{Q},\sigma} = \sum_\nu \Omega_\nu B^\dagger_{\nu,\eta\mathbf{Q},\sigma}$.

In this frame, we do have $(H_{\rm SC} - E^X_{\nu,\mathbf{K}})|\nu, \mathbf{K}, S\rangle = 0$ with $E^X_{\nu,\mathbf{K}} = E_{\rm gap} - \omega_P + \epsilon_\nu + K^2/2(m_e + m_h)$. The time evolution of a ground state exciton ν_0, projected on the single pair subspace, reads

$$|\nu_0, \mathbf{K}, S\rangle_t = \sum_{\nu', \mathbf{K}', S'=(\pm 1,\pm 2)} |\nu', \mathbf{K}', S'\rangle \times \langle \nu', \mathbf{K}', S'| e^{-i(H_{\rm SC} + W - E^X_{\nu_0,\mathbf{K}})t} |\nu_0, \mathbf{K}, S\rangle. \tag{2}$$

To evaluate this matrix element, we use the integral representation of the exponential and expand $1/(z - H_{\rm SC} - W)$ in terms of W. The zero order term gives $\delta_{\nu'\mathbf{K}'S',\nu_0\mathbf{K}S}$; the first order term cancels, while the second order term reads for $\mathbf{K} \neq \pm\mathbf{Q}$ as

$$\langle \nu', \mathbf{K}', S'| U G_t(E^X_{\nu',\mathbf{K}'} - E^X_{\nu_0,\mathbf{K}}, H_{\rm SC} - E^X_{\nu_0,\mathbf{K}}) U^\dagger |\nu_0, \mathbf{K}, S\rangle, \tag{3}$$

where $G_t(E', E) = [F_t(E') - F_t(E)]/(E' - E)$, with $F_t(E) = (e^{-iEt} - 1)/E$ being the standard function for second order optical processes [22].

For Et and $E't$ both small, $G_t(E', E)$ tends to $-t^2/2$, while for $E't \ll 1 \ll Et$, it reduces to it/E. Since $U^\dagger$ creates a virtual exciton with momentum $\eta\mathbf{Q}$, while U destroys a $\eta'\mathbf{Q}$ exciton, momentum conservation imposes $\mathbf{K}' + \eta'\mathbf{Q} = \mathbf{K} + \eta\mathbf{Q}$; so $|\mathbf{K} - \mathbf{K}'|$ is at most equal to $2\mathbf{Q}$, making $E^X_{\nu'\mathbf{K}'} \simeq E^X_{\nu_0\mathbf{K}}$ for $\nu' = \nu_0$. By contrast, $H_{\rm SC}$ in Eq. (3) acts on two pairs; so $H_{\rm SC} - E^X_{\nu_0\mathbf{K}}$ gives contribution of the order of the photon detuning, not small for unabsorbed photons. Thus, experimental conditions correspond to the second time regime, so that

$$|\nu_0, \mathbf{K}, S\rangle_t \simeq |\nu_0, \mathbf{K}, S\rangle + it \sum_{S',\eta',\eta} \beta_{\eta',\eta} |\nu_0, \mathbf{K} + (\eta - \eta')\mathbf{Q}, S'\rangle, \tag{4}$$

where the prefactor is given by

$$\beta_{\eta',\eta} = \frac{1}{2} \sum_{\sigma,\sigma'} \langle \nu_0, \mathbf{K} + (\eta - \eta')\mathbf{Q}, S'| U_{\eta'\mathbf{Q},\sigma'} \times \frac{1}{H_{\rm SC} - E^X_{\nu_0,\mathbf{K}}} U^\dagger_{\eta\mathbf{Q},\sigma} |\nu_0, \mathbf{K}, S\rangle. \tag{5}$$

To evaluate $\beta_{\eta',\eta}$ we note that $H_{\rm SC}$ acting on $B^\dagger_{\nu,\eta\mathbf{Q},\sigma}|\nu_0, \mathbf{K}, S\rangle$ gives $E^X_{\nu,\eta\mathbf{Q}} + E^X_{\nu_0,\mathbf{K}}$ to the lowest order in Coulomb scattering divided by the detuning. We then write the scalar products of exciton states in terms of Pauli scatterings for carrier exchanges, according to Ref. [23]. Figure 1 shows that, if we restrict to heavy-hole excitons, carrier exchange imposes $\sigma = \sigma'$ and $S = S'$. Moreover, for $S = \pm 1$ bright states, electron or hole exchanges occur for $\sigma = S$ only, while for $S = \pm 2$ dark states, hole exchange is possible for $\sigma = S/2$ and electron exchange for $\sigma = -S/2$. As a result, the total number of exchanges induced on bright or dark excitons by linearly polarized photons is indeed the same.

For nondiagonal process, $\eta' = -\eta$, momentum $2\mathbf{Q}$ is transferred between the light field and the exciton as a result of exchange. The nondiagonal coefficients are given by

$$\beta_{-\eta,\eta} = -\delta_{S,S'} \sum_\nu \frac{\Omega_\nu}{2E^X_{\nu,\eta\mathbf{Q}}} \gamma_{\eta\mathbf{Q}}(\mathbf{K}), \tag{6}$$

where

$$\gamma_{\eta\mathbf{Q}}(\mathbf{K}) = \sum_{\nu'} \Omega^*_{\nu'} \left[\lambda_h \begin{pmatrix} \nu', -\eta\mathbf{Q} & \nu, \eta\mathbf{Q} \\ \nu_0, \mathbf{K} + 2\eta\mathbf{Q} & \nu_0, \mathbf{K} \end{pmatrix} + (h \leftrightarrow e) \right]. \tag{7}$$

This includes the hole exchange of Fig. 1(a) through λ_h, and the electron exchange of Fig. 1(b) through λ_e. Next, we write the orbital Pauli scattering amplitudes $\lambda_{h,e}\binom{n\ \ j}{m\ \ i}$ in terms of the in and out wave functions (see Ref. [23]), and we sum over ν' through closure relation using $\Omega^*_{\nu'} = \Omega^* L^{D/2} \langle \mathbf{r} = 0 | \nu' \rangle$. Since $\mathbf{Q}$ is very small on the exciton scale, $\gamma_{\eta\mathbf{Q}}(\mathbf{K}) \simeq \gamma_{\mathbf{0}}(\mathbf{K})$. For laser tuned close to the heavy-hole exciton resonance, we can keep only the $\nu = \nu_0$ state; so, $\gamma_{\mathbf{0}}(\mathbf{K})$ reduces to $\Omega^*[\zeta(\alpha_h \mathbf{K}) + \zeta(\alpha_e \mathbf{K})]$ where $\alpha_h = 1 - \alpha_e = m_h/(m_h + m_e)$ and

$$\zeta(\mathbf{K}) = \sum_{\mathbf{p}} |\langle \nu_0 | \mathbf{p} \rangle|^2 \langle \mathbf{p} + \mathbf{K} | \nu_0 \rangle. \tag{8}$$

To make a connection with atomic optical traps, let us now derive the effective potential $\tilde{V}$ associated with this Pauli blocking mechanism. Adding an external potential to H_{SC} leads to

$$e^{-i(H_{\mathrm{SC}} + \tilde{V} - E^X_{\nu_0,\mathbf{K}})t} \simeq 1 + F_t(H_{\mathrm{SC}} - E^X_{\nu_0,\mathbf{K}})\tilde{V}, \tag{9}$$

which reproduces the time evolution given by Eq. (4) provided $\tilde{V} = \sum_{S=\pm1,\pm2} \tilde{V}_S$ with $\tilde{V}_S$ given by

$$\tilde{V}_S = \frac{\Omega_{\nu_0}}{2E^X_{\nu_0,\mathbf{0}}} \sum_{\eta=\pm1,\mathbf{K}} \gamma_{\mathbf{0}}(\mathbf{K}) |\nu_0, \mathbf{K} + \eta\mathbf{Q}, S\rangle\langle \nu_0, \mathbf{K} - \eta\mathbf{Q}, S|. \tag{10}$$

This effective potential is nonlocal, being given in coordinate representation by

$$\tilde{V}_S = \int d\mathbf{R} d\mathbf{R}' \tilde{V}(\mathbf{R}', \mathbf{R}) |\nu_0, \mathbf{R}', S\rangle\langle S, \mathbf{R}, \nu_0|, \tag{11}$$

where

$$\tilde{V}(\mathbf{R}', \mathbf{R}) = \frac{|\Omega|^2}{E^X_{\nu_0,\mathbf{0}}} K(\mathbf{R}' - \mathbf{R}) \cos(\mathbf{Q} \cdot (\mathbf{R}' + \mathbf{R})). \tag{12}$$

The kernel $K(\mathbf{R})$, which comes from carrier exchanges, splits as $K_{\alpha_h}(\mathbf{R}) + K_{\alpha_e}(\mathbf{R})$ with

$$K_\alpha(\mathbf{R}) = \sum_{\mathbf{K},\mathbf{p}} \langle \mathbf{R} | \mathbf{K} \rangle \langle \alpha\mathbf{K} + \mathbf{p} | \nu_0 \rangle \langle \nu_0 | \mathbf{r} = 0 \rangle |\langle \mathbf{p} | \nu_0 \rangle|^2, \tag{13}$$

which has a finite spatial extent on the order of the exciton Bohr radius.

Discussion.—To provide a quantitative estimate of the trapping potential, let us neglect the nonlocality of the potential and replace $K(\mathbf{R})$ by $\chi\delta(\mathbf{R})$ with χ obtained by matching the $\mathbf{R}$ integral of $K(\mathbf{R})$. This gives $\chi = 2\zeta(\mathbf{0}) L^{D/2} \langle \nu_0 | \mathbf{r} = 0 \rangle$, which is equal to 7 in 3D and 32/7 in 2D. The effective diagonal potential is then

$$\tilde{V}(\mathbf{R}) = U_0 \cos(2\mathbf{Q} \cdot \mathbf{R}), \tag{14}$$

with $U_0 = \chi |\Omega|^2 / E^X_{\nu_0\mathbf{0}}$.

The half-depth $|U_0|$ of the trapping potential is shown in Fig. 2 as a function of laser power. The electric field E_0 is related to the laser power P through the intensity $I = ncE_0^2/8\pi = P/\pi(l/2)^2$, where c is the speed of light, l the laser spot diameter taken equal to 100 μm, and n the refractive index taken equal to 3.5. The square of the Rabi energy reads $|\Omega|^2 = d_0^2 E_0^2/4 = d_0^2(8\pi/nc)P/\pi l^2$, where for GaAs, the d_0 dipole is $\sim$6.2 eÅ from Ref. [24]. Note that in order to remain within the validity of the theory, we must avoid saturating the exciton transition, which requires $\epsilon = \chi |\Omega|^2 / (E^X_{\nu_0\mathbf{0}})^2 < 1$: the lines in the plot end at this limit.

We see that for a few meV detunings, potentials as deep as a few meV can be obtained for laser powers within the validity of the theory.

Several effects could compete with the optical trapping proposed here. In 2D, high-quality quantum wells with rather large width of the order of 20 nm are required to minimize the disorder potential due to monolayer fluctuations at the interfaces [25]. Effects due to laser heating and photon cooling were already studied in Ref. [12] in the case of electron trapping. They depend on the photon recoil energy associated with the trapped particles. For samples held at 300 mK, phonon cooling dominates, while photon absorption and reemission, occurring with a probability that scales with ϵ, does not produce appreciable heating. Exciton trapping is favorable with respect to electron trapping because its recoil energy is smaller. In contrast to extended systems, exciton scatterings by acoustic phonons in a trap do not conserve the total crystal momentum; so

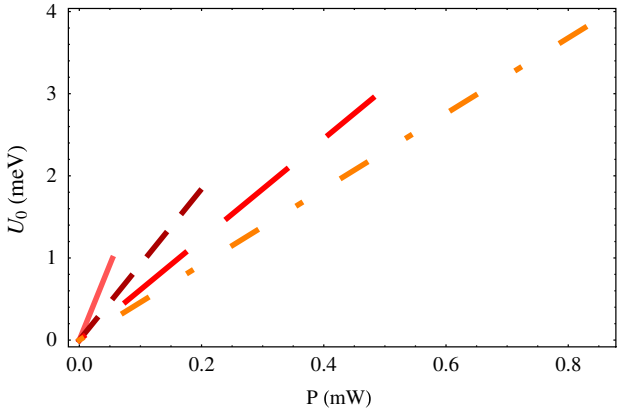

FIG. 2 (color online). Trapping potential half-depth U_0 in 3D as a function of the laser power, for a 100 μm laser spot and detunings of 1 meV (solid line), 2 meV (dashed line), 3 meV (long dashed line), and 4 meV (dot-dashed line). Lines end in correspondence to the validity of the theory. Materials parameters for the calculations are from Ref. [24]. The values of U_0 for 2D systems can be obtained by rescaling the vertical axis by 32/49, which is the ratio of the coefficients χ in two and three dimensions.

cooling by phonon emission at low energy is more effective. Another effect is carrier ionization. Because of phase matching conditions, its rate, estimated from the Fermi golden rule, remains very small, the absorption coefficient for this process being 5 orders of magnitude smaller than for interband absorption [12,26]. Similarly, we have checked that two-photon absorption, estimated using rates calculated for quantum wells [27], does not appreciably affect the trapping scheme. Note that the few high-energy pairs created by two-photon or exciton ionization will quickly relax by optical phonon emission as excitons to the bottom of the band. However, due to their low creation rate, the exciton temperature in the trap will not be substantially affected.

In contrast to atoms, the exciton effective potential is repulsive ($U_0 > 0$) for red detuning and attractive for blue detuning. This comes from the change in energy of the semiconductor ''vacuum'' induced by light. As for atoms, the semiconductor vacuum lowers its energy for red detunings, and increases it for blue detunings. However, if excitons are present, Pauli blocking tends to block the creation of virtual excitons. Therefore, for red detuning, the total energy shift of the vacuum is increased by the presence of excitons, which are therefore weak-field seeking. The opposite occurs for blue detuning, excitons being then strong-field seeking. Consequently, attractive traps, with negative U_0, require lasers tuned in the transparency region between the ground and first excited exciton states. Actually, in this spectral region, a residual absorption exists due to second order processes involving phonons. These processes create carriers at a rate of the order of 1 exciton every few ns (see supplemental materials in Ref. [12]), adding a few excitons in the traps. So, this slow rate should not considerably raise the temperature of the trapped excitons nor substantially affect the overall efficiency of the trapping mechanism. Alternatively, one could use a red-detuned Laguerre-Gaussian beam focused onto a 2D quantum well to create a single ring-of-light trap, with excitons confined to the dark center spot of the beam. We emphasize that the periodicity of the optical lattices can be controlled using different excitation geometries. For instance, a QW can be excited by sending two counterpropagating beams from the lateral sides, along the QW plane, or the two beams can come from the top of the sample at an angle $\pm\theta$ with respect to the growth axis: the obtained lattice constant is then much bigger.

Conclusion.—We propose an optical trapping mechanism, based on carrier exchanges with virtual excitons coupled to a linearly polarized unabsorbed standing wave. This mechanism, which equally acts on bright and dark excitons—in spite of the fact that dark excitons are not directly coupled to light—is able to produce both attractive and repulsive potentials, depending on the photon detuning. Trapping depths of the order of a few meV can be obtained.

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[1] J. M. Blatt, K. W. Böer, and W. Brandt, Phys. Rev. **126**, 1691 (1962).
[2] L. V. Keldysh and A. N. Kozlov, Sov. Phys. JETP **27**, 521 (1968).
[3] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
[4] K. B. Davis, M. -O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[5] R. Balili, V. Hartwell, D. Snoke, L. Pfeiffer, and K. West, Science **316**, 1007 (2007).
[6] C. W. Lai, N. Y. Kim, S. Utsunomiya, G. Roumpos, H. Deng, M. D. Fraser, T. Byrnes, P. Recher, N. Kumada, T. Fujisawa, and Y. Yamamoto, Nature (London) **450**, 529 (2007).
[7] A. T. Hammack, M. Griswold, L. V. Butov, L. E. Smallwood, A. L. Ivanov, and A. C. Gossard, Phys. Rev. Lett. **96**, 227402 (2006).
[8] A. Amo, S. Pigeon, C. Adrados, R. Houdré, E. Giacobino, C. Ciuti, and A. Bramati, Phys. Rev. B **82**, 081301 (2010).
[9] E. Wertz, L. Ferrier, D. Solnyshkov, R. Johne, D. Sanvitto, A. Lemaitre, I. Sagnes, R. Grousson, A. V. Kavokin, P. Snellart, G. Malpuech, and J. Bloch, Nature Phys. **6**, 860 (2010).
[10] A. L. Ivanov and P. B. Littlewood, Phys. Rev. Lett. **87**, 136403 (2001).
[11] M. M. de Lima, M. van der Poel, P. V. Santos, and J. M. Hvam, Phys. Rev. Lett. **97**, 045501 (2006).
[12] M. A. Schuetz, M. G. Moore, and C. Piermarocchi, Nature Phys. **6**, 919 (2010).
[13] M. Combescot, M. G. Moore, and C. Piermarocchi, Europhys. Lett. **93**, 47012 (2011).
[14] M. Lindberg and R. Binder, J. Phys. Condens. Matter **15**, 1119 (2003).
[15] R. Binder and M. Lindberg, J. Phys. Condens. Matter **18**, 729 (2006).
[16] M. Combescot, O. Betbeder-Matibet, and R. Combescot, Phys. Rev. Lett. **99**, 176403 (2007).
[17] M. Combescot and M. N. Leuenberger, Solid State Commun. **149**, 567 (2009).
[18] C. Ciuti, V. Savona, C. Piermarocchi, A. Quattropani, and P. Schwendimann, Phys. Rev. B **58**, 7926 (1998).
[19] M. Combescot and O. Betbeder-Matibet, Eur. Phys. J. B **27**, 505 (2002).
[20] We disregard biexciton made of heavy and light hole excitons because their coupling to light is 3 times weaker.
[21] Such standing waves lead to traps in the form of stripes in 2D and planes in 3D. More elaborate trapping patterns are easy to obtain with more than two beams.
[22] M. Combescot J. Phys. A **34**, 6087 (2001).
[23] M. Combescot, O. Betbeder-Matibet, and F. Dubin, Phys. Rep. **463**, 215 (2008).
[24] *Semiconductors: Intrinsic Properties of Group IV Elements and III–V, II–VI, and I–VII Compounds*, edited by O. Madelung, Landolt Börnstein New Series, Group III, Vol. 22a (Springer-Verlag, Berlin, 1987).
[25] W. Langbein and J. M. Hvam, Phys. Rev. B **61**, 1692 (2000).
[26] M. J. A. Schuetz, MS thesis, Michigan State U., 2009.
[27] A. Pasquarello and A. Quattropani, Phys. Rev. B **38**, 6206 (1988).