Spectroscopy for Cold Atom Gases in Periodically Phase-Modulated Optical Lattices

Akiyuki Tokuno and Thierry Giamarchi

DPMC-MaNEP, University of Geneva, 24 Quai Ernest-Ansermet CH-1211 Geneva, Switzerland

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The response of cold atom gases to small periodic phase modulation of an optical lattice is discussed. For bosonic gases, the energy absorption rate is given, within linear response theory, by the imaginary part of the current autocorrelation function. For fermionic gases in a strong lattice potential, the same correlation function can be probed via the production rate of double occupancy. The phase modulation gives thus direct access to the conductivity of the system, as a function of the modulation frequency. We give an example of application in the case of bosonic systems at zero temperature and discuss the link between the phase and amplitude modulation.

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Cold atomic systems have proven to be remarkable laboratories to study several effects of strongly correlated systems. In particular, the control of parameters, kinetic energy in an optical lattice, and interaction using a Feshbach resonance, allows us to potentially use them as quantum simulators, with considerable success both for pure and disordered systems [1,2]. However, in addition to realizing the systems, the ability to probe it is important. Because of the electrical charge neutrality of cold atoms, unlike electron systems, they are insensitive to the usual electromagnetic probes. This makes it potentially difficult to probe correlations in such systems. To overcome this issue, several probes have been proposed besides the standard time of flight (TOF) experiment such as Bragg spectroscopy [3–6] to measure the dynamic structure factor, radio frequency spectroscopy measurement [7,8] to count the number of molecules formed by the Feshbach resonance, shot noise measurement [9–12] for the density-density correlation function, or momentum-resolved Raman spectroscopy [12,13] for the single-body spectrum function.

Among the various spectroscopic probes a particularly simple probe consists in changing periodically the amplitude of the optical lattice [14,15]. The energy absorbed by such a modulation can be estimated from the TOF image. The corresponding theory of the energy absorption rate (EAR) spectrum [16,17], was shown to give access both to the Mott-insulating (MI) gap and to the kinetic-energy correlations in the system. Although measuring the EAR by the TOF was possible for bosons, a similar measure was highly inconvenient for fermions. It was proposed [18] that a measurement of the doublon production rate (DPR) in response to the amplitude modulation would give access to the same information. Such a measure was successfully implemented for fermionic systems [19–22]. The amplitude modulation of the optical lattice coupled either to EAR or to DPR is thus a simple but powerful and versatile probe.

In this Letter we propose an alternative probe, based on a phase modulation of an optical lattice potential. Such a modulation is known to lead to a current [23–26] or to band narrowing [27,28]. Here we use the phase modulation in connection with EAR or DPR techniques, to analyze the spectrum of the system. We show that such a probe gives access to the current autocorrelation function and is thus analogous to optical conductivity measurements in condensed matter systems, allowing a very close comparison at the experimental level between the two domains. We illustrate the use of such a probe by some examples for bosonic gases and compare with the spectrum obtained by the amplitude modulation spectroscopy.

Let us first describe our proposed probe: The optical lattice potential is created by shining laser against a mirror. If the mirror is stationary, the created D-dimensional optical lattice is given as $V_{\mathrm{op}}(\boldsymbol{r}) = V_0 \sum_{\mu=1}^{D} \cos^2(Q_\mu r_\mu)$ where $\boldsymbol{Q} = (Q_1, \cdots, Q_D)$ is a wave vector of the optical lattice. One can modulate the phase by oscillating the mirror as shown in Fig. 1. The lattice potential in the laboratory frame is modified as $V_{\mathrm{op}}(\boldsymbol{r}, t) = \sum_{\mu=1}^{D} V_0 \cos^2[Q_\mu(r_\mu - F_\mu(t))]$ where $\boldsymbol{F}(t)$ represents the oscillation of the phase. It is convenient to switch to the comoving frame by the gauge transform $U(t) = \exp(iM\boldsymbol{F}(t) \cdot \boldsymbol{J}/\hbar)$ where M is a mass of the atoms and $\boldsymbol{J}$ the current operator [29]. In the comoving frame, the lattice becomes a stationary one, $V_{\mathrm{op}}(\boldsymbol{r})$, and an additional term, which reflects the inertial force, emerges in the Hamiltonian:

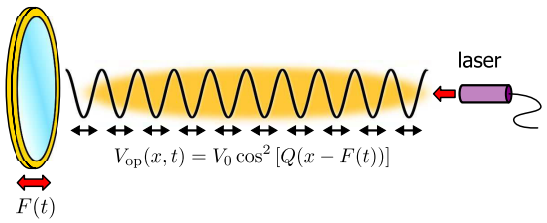

FIG. 1 (color online). A schematic showing the setup of the periodic phase modulation of an optical lattice. The incident laser and the reflected one forms the standing wave corresponding to an optical lattice. The lattice potential follows mirror oscillation, and, consequently, the phase is modulated.

$$H(t) = H_0 - M\dot{\boldsymbol{F}}(t)\cdot\boldsymbol{J}, \quad (1)$$

where H_0 is the Hamiltonian of the interacting system in the optical lattice $V_{\rm op}(\boldsymbol{r})$. Carrying out the further gauge transform $U'(t) = \exp[iNM\int^t dt'\dot{\boldsymbol{F}}^2(t')/2\hbar]$ where N is the total atom number, we can find an expression identical to that of a charged particle in an electromagnetic field, the atomic mass corresponding to the charge. The vector potential is given as $\boldsymbol{A}_{\rm ext} = \dot{\boldsymbol{F}}(t)$ with $\boldsymbol{\nabla}\cdot\boldsymbol{A}_{\rm ext} = 0$ and the scalar potential is zero $\phi_{\rm ext} = 0$ [23]. The system thus behaves as charged particles under an electric field $\boldsymbol{E}_{\rm ext}(t) = -\ddot{\boldsymbol{F}}(t)$. Hereafter we set $\boldsymbol{F}(t) = \boldsymbol{f}\cos(\omega t)$.

Let us first consider bosonic atom cases. One can then measure the EAR by similar techniques than for the amplitude modulation [14]. The EAR is given by the time average of the absorbed energy: $R(\omega) = \frac{\omega}{2\pi}\int_T^{T+2\pi/\omega} dt\langle\dot{H}(t)\rangle$, where $\langle\cdots\rangle$ denotes the statistical average with the Hamiltonian (1). The EAR within the linear response theory is given by

$$R_{\rm PM}(\omega) = -\frac{M^2\omega^3}{2\hbar}\sum_{\mu,\nu=1}^{D} f_\mu f_\nu \Im\tilde{\Pi}^R_{\mu\nu}(\omega), \quad (2)$$

where $\tilde{\Pi}^R_{\mu\nu}(\omega)$ is the Fourier transform of the retarded current correlation function for the Hamiltonian H_0. Note that for the EAR, due to $\dot{H}(t) = M\omega^2\cos(\omega t)\boldsymbol{f}\cdot\boldsymbol{J}$, one can automatically derive the second-order response of $R(\omega)$ in terms of f_μ within the first order perturbation theory. Note, as shown in Eq. (1), that in order to stay within linear response a small modulation is necessary. In particular one needs $|F_\mu(t)| \ll Q_\mu^{-1}$, thus a modulation amplitude smaller than a lattice constant. This is something difficult but achievable with the current experimental technique [30]. For the higher frequency the smaller amplitude needed to stay within the linear response and away from the dynamically induced phase transition [31,32]. We will confine our analysis in the following to such a regime for which the EAR gives direct access to $\tilde{\Pi}^R_{\mu\nu}(\omega)$.

Since the EAR directly gives $\tilde{\Pi}^R_{\mu\nu}(\omega)$, it is immediately related to the "optical conductivity" [33]. While the zero frequency part of the Drude peak in the conductivity will be suppressed in the EAR due to the factor ω^3 in Eq. (2), all the other features, at finite frequency, are perfectly reproduced. It is thus a particularly useful quantity to make comparison with similar phenomena in condensed matter systems, or to probe the physics of disordered systems, for which transport is the prime probe.

It is interesting to compare this result to another one obtained for the amplitude modulation [16], in the same linear response regime. In the later case the EAR is either given by the density correlation function for weak optical lattices or by the kinetic energy one for strong optical lattices [16,22]. The different representation of perturbation operator comes from the fact that the energy scale of the amplitude modulation also goes beyond the chemical potential as the energy scale of the lattice potential $V_{\rm op}(\boldsymbol{r})$ increases. On the contrary, in the case of the phase modulation, the perturbation in Eq. (1) does not follow the energy scale of the lattice potential. Therefore, Eq. (2) is independent of the strength of the lattice potential. This is a definite advantage of the phase modulation, which is always related to the same physical quantity irrespectively of the strength of the optical lattice. Another important difference between the phase and amplitude modulation comes from their symmetries. Indeed, for example, in the case of a 1D strong lattice the amplitude modulation would correspond to the kinetic-energy correlation function which is given by $T \propto \sum_k \cos(k) b_k^\dagger b_k$ while the current is given by $J \propto \sum_k \sin(k) b_k^\dagger b_k$. For the parity inversion, $\boldsymbol{J} \to -\boldsymbol{J}$ while T is invariant. This affects the selection rules. The phase modulation perturbation probes the transitions from a state to an opposite parity state. In contrast, the parity is preserved in the transition due to the amplitude modulation. Thus, both modulations complement each other in the way they probe excited states, and lead in general to different results.

For fermions, as for the amplitude modulation, the EAR is not a convenient way to probe the consequences of the modulation. We follow here the same approach as in [18] and show that for the phase modulation the measurement of the DPR gives essentially the same information as the EAR. We assume that H_0 in Eq. (1) is described by the Hubbard model, $H_0 = -t_H\sum_{\sigma,\langle i,j\rangle} c_{i\sigma}^\dagger c_{j\sigma} + U\sum_j n_{j\uparrow}n_{j\downarrow}$. The number of doubly occupied sites is defined as $N_D(t) = \langle V\rangle/U$, and can be rewritten as $N_D(t) = \frac{1}{U}[\langle H(t)\rangle - \langle T\rangle + M\dot{\boldsymbol{F}}\cdot\langle\boldsymbol{J}\rangle]$, where T and V are the kinetic energy and interaction terms in the above Hamiltonian. The production rate is defined as the time average of $\dot{N}_D(t)$ for a period: $P(\omega) = \frac{\omega}{2\pi}\int_t^{t+2\pi/\omega} dt'\dot{N}_D(t')$. A second-order perturbation expansion in $\boldsymbol{F}(t)$ gives for the productions rate

$$P_{\rm PM}(\omega) = R_{\rm PM}(\omega)/U, \quad (3)$$

which shows the direct relation between the EAR and the DPR for the phase modulation. This shows that DPR gives also access to the optical conductivity for these system.

The results (2) and the equivalence of the DPR to the EAR (3) are thus our main proposal to use the phase modulation of the optical lattice to measure the optical conductivity of interacting systems in a cold atom context.

Let us now examine an example of the phase modulation technique. For the sake of simplicity we take a repulsively interacting 1D bosonic atom system at zero temperature. The unperturbed Hamiltonian in Eq. (1) is written as

$$H_0 = \int dx\left[\psi^\dagger\left(-\frac{\hbar^2}{2M}\partial_x^2 - \mu + V_{\rm op}(x)\right)\psi + \frac{g}{2}\rho^2\right], \quad (4)$$

where $V_{\rm op}(x) = V_0\cos^2 Qx$ is a 1D lattice potential, and g an interaction parameter. The field $\psi(x)$ and $\rho(x)$ are, respectively, the annihilation and density operators.

For a shallow lattice potential, $V_0 \ll \mu$, $V_{\rm op}(x)$ can be regarded as a perturbation, and then the Hamiltonian (4) can be rewritten via the bosonization [34,35], $\psi^\dagger(x) \sim \sqrt{\bar\rho} e^{-i\theta(x)}$ and $\rho(x) \sim \bar\rho - \partial_x\varphi(x)/\pi + 2\bar\rho\cos[2\pi\bar\rho x - 2\varphi(x)]$, where $\bar\rho$ is the mean density of the system. Retaining only the most relevant term generated by the presence of $V_{\rm op}(x)$, one can obtain

$$H_{\rm eff} = H_{\rm TL} + \lambda \int dx \cos[2(Q - \pi\bar\rho)x + 2\varphi(x)], \quad (5)$$

where $H_{\rm TL} = \frac{\hbar v}{2\pi}\int dx[K(\partial_x\theta(x))^2 + K^{-1}(\partial_x\varphi(x))^2]$ is the Tomonaga-Luttinger (TL) Hamiltonian. The phase fields $\theta(x)$ and $\varphi(x)$ represent, respectively, the phase and density fluctuations of bosons. For an arbitrary repulsion K runs from ∞ to unity as the interaction increases, and $K = 1$ and ∞ correspond to the Tonks gas and the noninteracting bosons, respectively. Thus in the boson systems the low-energy physics is mainly governed by the cosine term in Eq. (5). Furthermore, the current is written as $J = vK\int dx\Pi(x)$ where $\Pi(x)$ is the canonically conjugate momentum to $\varphi(x)$. In the incommensurate case, i.e., $\pi\bar\rho \neq Q$, the cosine term in Eq. (5) vanishes, and the effective theory is the TL liquid. For $\pi\bar\rho = Q$, the model (5) becomes a sine-Gordon model for which the cosine term is relevant for $K < 2$, leading to a MI gap in the excitation spectrum [35]. We will consider this last case in what follows.

The conductivity can be calculated using the methods in Refs. [35,36]. To determine the large frequency behavior of the 1D current correlation function $\tilde\Pi_J^R(\omega)$, we use the memory function method, which gives correctly $\tilde\Pi_J^R(\omega)$ for large frequency compared to the MI gap. The memory function $M(\omega) \equiv \omega\tilde\Pi_J^R(\omega)/[\tilde\Pi_J^R(0) - \tilde\Pi_J^R(\omega)]$ can be approximated as $M(\omega) \approx [\tilde\Pi_F^R(0) - \tilde\Pi_F^R(\omega)]/\omega\tilde\Pi_J^R(0)$ where $\tilde\Pi_F^R(\omega)$ is the retarded correlation function of $F(t) = [H_0, J(t)]$. From the Hamiltonian (5), F is given by $F = i2vK\lambda\int dx\sin[2(Q - \pi\bar\rho) + 2\varphi(x)]$, and $M(\omega) \sim \omega^{2K-3}$ is immediately computed. In the gapless ($K > 2$) case, negligible $M(\omega)/\omega$ for small ω leads to $\tilde\Pi_J^R(\omega) \propto \omega^{2K-5}$. As a result, it is found that the EAR spectrum for small ω and $K > 2$ behaves as $R_{\rm PM}(\omega) \propto \omega^{2K-2}$. This is to be compared with the amplitude modulation result [16] $R_{\rm AM}(\omega) \propto \omega^{2K-1}$ for weak lattices. A similar result is obtained in the large-ω limit for the massive case $K < 2$. In the gapful case the cosine is relevant, and the conductivity, i.e., the phase modulation response will be zero below the gap [35]. This example thus shows differences between the phase and amplitude modulation. This difference has two origins: one is the trivial different prefactors of the correlation functions (ω^3 for the phase modulation and ω for the amplitude one). More importantly, and as discussed above, the main difference comes from the perturbation coupling to two different operators: namely the current for the phase modulation and the density for the amplitude modulation in the shallow lattice limit.

We now consider a strong lattice potential. Then, the system is well described by a lattice model: H_0 is given by the Bose-Hubbard Hamiltonian $H_{\rm eff} = T + V$ where $T = -t_H\sum_j[b_{j+1}^\dagger b_j + \text{H.c.}]$ and $V = \frac{U}{2}\sum_j n_j(n_j - 1)$. $b_j^\dagger$ and n_j are, respectively, the creation and number operators for a bosonic atom at the jth site, and t_H and U are the hopping parameter and on-site interaction, respectively. For an incommensurate filling, the ground state is in the gapless superfluid (SF) phase. For filling of one particle per site, the SF-MI transition occurs at $U/t_H = 1.92$ [37]. In the SF phase, the low-energy physics is governed by the TL liquid [35]. In the MI phase, an energy gap opens, and the low-energy physics is no longer described by the TL liquid. The lowest energy excitation above the gap is formed by a pair of atoms at the same site (doublon) and an empty site (holon). In the limit $t_H/U \to 0$, the pair excitations are $\mathcal{N}(\mathcal{N} - 1)$-fold degenerate where $\mathcal{N}$ is the number of lattice sites. For finite but small t_H/U, the degenerate energy levels split, and an energy band whose width is about t_H is formed. This band leads in the phase and amplitude modulation spectrums to a peak around $\omega \approx U/\hbar$ as shown in Fig. 2.

Using degenerate perturbation theory [16], the EAR spectrum can be calculated. Let $|d_R h_r\rangle$ be a pair state of the doublon and holon at Rth and rth site, respectively, which is an exact eigenstate of V. We represent T in the Hilbert space spanned by $|d_R h_r\rangle$. The eigenstate of T is $|l, l'\rangle = \frac{\sqrt{2}}{\mathcal{N}}\sum_{R=1}^{\mathcal{N}}\sum_{r=1}^{\mathcal{N}-1}\times e^{i(p_l R + \arg[w_{p_l}]r)}\sin(q_{l'}r)|d_R h_{R+r}\rangle$ where $w_{p_l} = 1 + 2e^{ip_l}$, $p_l = 2\pi l/\mathcal{N}$, and $q_{l'} = \pi l'/\mathcal{N}$ ($l = 1, \cdots, \mathcal{N}$ and $l' = 1, \cdots, \mathcal{N} - 1$). The corresponding eigenenergy is $E_{l,l'} = U - t_H|w_{p_l}|\cos q_{l'}$. The retarded correlation

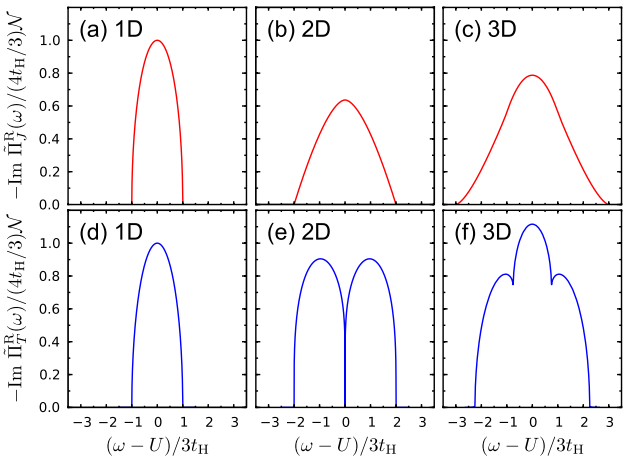

FIG. 2 (color online). The imaginary part of the current (a)–(c), and the kinetic-energy (d)–(f) correlation functions in the bosonic Mott insulator for $t_H/U = 0.01$ at zero temperature. The lattice constant and $\hbar$ have been taken to be unity. For 1D [(a) and (d)], these two correlation functions are identical while they are qualitatively different for 2D [(b) and (e)] and for 3D [(c) and (f)].

function of $\mathcal{O}$ for $\omega > 0$ is expressed as $\Im\tilde{\Pi}^R_{\mathcal{O}}(\omega) = -\pi\hbar\sum_n |\langle n|\mathcal{O}|0\rangle|^2\delta(\hbar\omega - E_n)$ where $|n\rangle$ and $|0\rangle = \prod_j b_j^\dagger|\text{vac}\rangle$, respectively, denotes an intermediate state and the MI ground state. Restricting the intermediate states onto $|l, l'\rangle$, both the amplitude and the phase modulation can be computed. In the $\mathcal{N} \to \infty$ limit they turn out to be identical and can be written as $\Im\tilde{\Pi}^R_J(\omega) = -\mathcal{N}\frac{\pi^2 t_H}{\hbar Q^2}\pi(\omega)$ and $\Im\tilde{\Pi}^R_T(\omega) = -\mathcal{N}\hbar t_H\pi(\omega)$ [16] where $\pi(\omega) = \frac{4}{3} \times \sqrt{1 - [(\hbar\omega - U)/3t_H]^2}$, as shown in Fig. 2. Therefore, the appropriately scaled EAR, i.e., $\omega^{-2}R_{\text{PM}}(\omega)$, is identical to $R_{\text{AM}}(\omega)$. This is a peculiar feature of the 1D MI excitation spectrum, linked to the fact that in 1D the hole and doublon cannot cross each other during their motion. Thus qualitative difference must appear in the 2D and 3D cases. We thus compute $\tilde{\Pi}^R_J(\omega)$ and $\tilde{\Pi}^R_T(\omega)$ for 2D and for 3D by using an diagrammatic approach. We consider a doublon and holon with an infinite repulsive interaction which implements the constraint that the two particles cannot be at the same point except when they recombine. For 1D this method is in full agreement with [16,38]. The result is also shown in Fig. 2. One clearly sees the difference between the two modulations. Note that the anomalous structures in the amplitude modulations are related to the Van Hove singularities in the density of states.

In summary, we have proposed in this Letter to use small periodic phase modulation of an optical lattice to probe for the current autocorrelation function. The consequences of the modulation can be measured either by probing the absorbed energy of the system (for bosons) or by measuring the production rate of doubly occupied sites (for fermions). Such a phase modulation probe gives direct access to the frequency dependent conductivity of the system.

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