# Dynamics of Coupled Vortices in a Pair of Ferromagnetic Disks

Satoshi Sugimoto,[1] Yasuhiro Fukuma,[2] Shinya Kasai,[3] Takashi Kimura,[1,*] Anjan Barman,[4] and YoshiChika Otani[1,2,†]
[1]*Institute for Solid State Physics, University of Tokyo, 5-1-5 Kashiwa-no-ha, Kashiwa, Chiba 277-8581, Japan*
[2]*Advanced Science Institute, RIKEN, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan*
[3]*National Institute for Material Sciences, Sengen, Tsukuba 1-2-1, Japan*
[4]*Department of Material Sciences, S. N. Bose National Centre for Basic Sciences,*
*Block JD, Sector III, Salt Lake, Kolkata 700 098, India*
(Received 17 September 2010; published 13 May 2011)

We here experimentally demonstrate that gyration modes of coupled vortices can be resonantly excited primarily by the ac current in a pair of ferromagnetic disks with variable separation. The sole gyration mode clearly splits into higher and lower frequency modes via dipolar interaction, where the main mode splitting is due to a chirality sensitive phase difference in gyrations of the coupled vortices, whereas the magnitude of the splitting is determined by their polarity configuration. These experimental results show that the coupled pair of vortices behaves similar to a diatomic molecule with bonding and antibonding states, implying a possibility for designing the magnonic band structure in a chain or an array of magnetic vortex oscillators.

DOI: 10.1103/PhysRevLett.106.197203 PACS numbers: 85.75.−d, 75.40.Gb, 75.75.−c, 75.78.−n

Magnetic vortex structure [1,2] is one of the fundamental spin structures observed in submicron-sized ferromagnetic elements. It is well characterized by 2 degrees of freedom: one is "chirality" ($c = \pm 1$), the direction of the in-plane curling magnetization along the disk circumference, and the other is "polarity" ($p = \pm 1$), the direction of the out-of-plane core magnetization. Particularly in the case of the disk, only the core region whose typical size is $\sim$10 nm generates the stray field; the static interaction between vortices in an array is thus negligibly small in the ground state. However, in the low frequency excitation state, called the gyration (or translational) mode [3,4], the surface magnetic charges appear with the core motion that brings about the dynamic dipolar interaction between vortices.

A coupled pair of magnetic vortices can be considered as a vortex molecule bound via dipolar interaction, which is a mimic of diatomic molecule with the van der Waals bonding [5]. The bonding or antibonding state, respectively, corresponds to in-phase or out-of-phase gyration of coupled vortices whose detailed energy levels are decided by combination of chiralities and polarities. This dynamic coupling is also effective in a two-dimensional array system [6,7], and thus allows us to design the density of states of the eigenfrequencies, the so-called "magnonic band structure," by arranging the core polarizations in a two-dimensional array. At the moment, there are few experimental reports for the coupled vortices via direct exchange interaction [8,9] and also dipolar interaction in physically separated vortices [10–13]. However the problem is still wide open in terms of experimental determination of the detailed condition for the mode splitting and the magnitude of the dipolar coupling in interacting vortices.

Herein we demonstrate the experimental evidence of the resonant excitation of a magnetostatically coupled pair of vortices as a clear mode splitting of spectra. The partial excitation using ac current causes the energy transfer via dipolar interaction between two vortices, which results in a collective excitation of coupled gyrations. The observed mode splitting is reproduced by both micromagnetic simulation and analytical calculation. The different combinations of core polarities and phase difference in gyrations give rise to four distinct modes. The magnitude of the vortex coupling is also discussed.

Figure 1(a) shows a scanning electron microscope (SEM) image of the sample together with the measurement circuit. Each disk has the same dimension, 500 nm in disk radius $r$ and 50 nm in thickness. Two neighboring Permalloy ($Fe_{19}Ni_{81}$; Py) disks with variable edge to edge separation $d$ from 75 to 250 nm and electrical leads are fabricated on thermally oxidized Si (100) by means of electron beam lithography combined with electron beam evaporation techniques. The polarities of vortices were confirmed by means of magnetic force microscopy prior to all the electrical measurements. Hereafter, the number 1 or 2, respectively, represents the vortex confined in the left or right disk shown in the SEM image of Fig. 1. One of the paired vortices is excited by a radio frequency ac current $I_{\mathrm{ac}}$ [14–18], and the resulting dc voltage $V_{\mathrm{dc}}$ through a bias tee is synchronously detected by the same electrical contact probes [15,19]. In order to set the configuration of polarities $p_1 p_2 = 1$ or $p_1 p_2 = -1$, $p_1$ of the excited vortex is switched by applying high $I_{\mathrm{ac}}$ of about 20 mA ($3.5 \times 10^{11}$ A/m$^2$) at the resonant frequency [20].

Figure 1(b) shows the measured $V_{\mathrm{dc}}/I_{\mathrm{ac}}$ as a function of the frequency of $I_{\mathrm{ac}}$ for the Py disk pair with $d = 75$ nm. The reference spectrum for a single disk shown by green

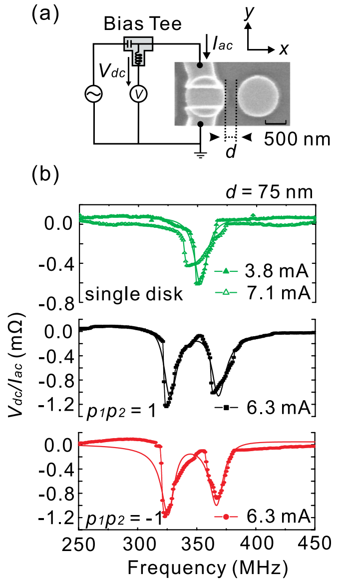

FIG. 1 (color online). (a) Schematic diagram of the measurement circuit and a SEM image of the sample. Two copper electrodes are attached to one of the disks in the Permalloy disk pair and the enveloped core is excited by a radio frequency current. Dynamics of the cores can be detected as dc voltages through the spin-torque diode effect utilized by a resistance oscillation associated with the core gyration. A lock-in technique is adopted at room temperature. (b) Frequency dependence of the normalized dc voltage $V_{\mathrm{dc}}/I_{\mathrm{ac}}$ measured for an isolated disk (green triangles) and for the paired disks with different polarities; black squares for $p_1p_2 = 1$, and red circles for $p_1p_2 = -1$. The ac current amplitudes $I_{\mathrm{ac}}$ used for the measurements are $I_{\mathrm{ac}} = 3.8$ mA and 7.1 mA for the single disk, and $I_{\mathrm{ac}} = 6.3$ mA for the paired disks with the edge to edge distance $d = 75$ nm. Solid curves in each spectrum represent the best fit to the data points using Eq. (1), thereby the dipolar coupling is evaluated.

symbols (top) exhibits a sole dip at 352 MHz, which corresponds to the resonant frequency of the vortex core gyration. When $I_{\mathrm{ac}}$ is applied to one of the two neighboring Py disks, clear mode splitting takes place as can be seen in the black and red (bottom) spectra in the figure. It is important to note that the gyration mode of the single vortex with $I_{\mathrm{ac}} = 7.1$ mA only shows lower frequency shift and asymmetric broadening due to nonlinear effects [21] as plotted by open symbols. Therefore, the observed mode splitting is primarily due to the effect of the dipolar coupling between the disks mediated by magnetic side charges. The magnitude of the mode splitting for $p_1p_2 = -1$ is slightly enhanced by several MHz compared to that for $p_1p_2 = 1$.

To gain insight into the dynamics of magnetostatically coupled vortices, micromagnetic simulations based on the Landau-Lifshitz-Gilbert equation [22] were performed on pair of Py disks with identical physical dimensions. Typical material parameters for Py are used: the saturation magnetization $M_s = 1$ T, the exchange stiffness constant $A = 1.05 \times 10^{-11}$ J/m, the spin polarization $P = 0.4$, and the damping coefficient $\alpha = 0.01$. The disk is divided into rectangular prisms of $5 \times 5 \times 50$ nm$^3$ for the simulation. A uniform $I_{\mathrm{ac}}$ of $1.2 \times 10^{11}$ A/m$^2$ is applied only to the left disk.

After several nanoseconds from the start of excitation, the core gyration settles in an almost circular orbit and its amplitude is strongly enhanced at the resonance frequency [14,15]. This induces the core gyration in the neighboring disk, which also settles in the steady circular orbit, and the collective gyration of the two vortices becomes fully synchronized and the eigenfrequencies of these modes appear in the spectra as characteristic resonance frequencies. Figure 2(a) represents the time evolution of the core deviations $\delta y$ at lower and higher resonance frequencies with respect to the single vortex for parallel polarities $p_1p_2 = 1$ and the same chiralities $c_1c_2 = 1$. At the lower frequency (365 MHz) both left and right cores rotate almost in phase, whereas at the high frequency (390 MHz) the phase of the right core is retarded by approximately half a period. This is very much in analogy with covalent bonding in diatomic molecules or other forms of coupled oscillators. In the case

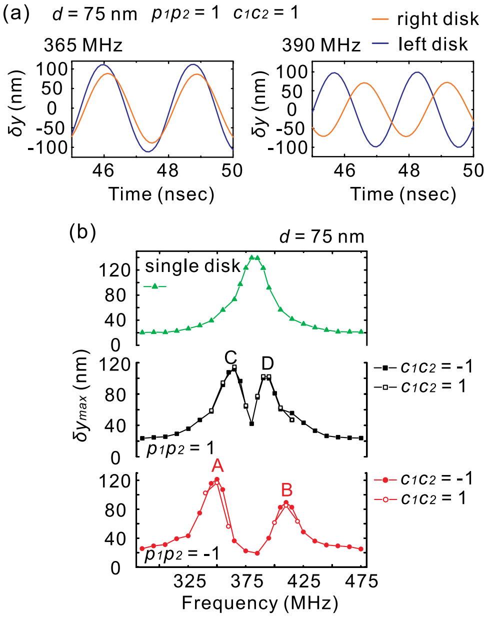

FIG. 2 (color online). (a) Simulated time evolutions of vortex cores at resonance frequencies for $(p_1p_2, c_1c_2) = (1, 1)$ under ac currents (365 and 390 MHz). Blue solid lines show the motions of the current-excited core in the left disk and red lines correspond to those of the indirectly excited core in the right disk. (b) Dispersion relations of amplitude of steady gyrations. Values of $\delta y_{\mathrm{max}}$ show the radii of steady gyrations (50 nsec after beginning of the current flow). Black squares correspond to the parallel polarities $p_1p_2 = 1$ and red circles to antiparallel polarities $p_1p_2 = -1$ for opposite chiralities $c_1c_2 = -1$. Results of same chiralities $c_1c_2 = 1$ are plotted by open symbols. Simulation results for a single vortex are also presented by green triangles for comparison.

of in-phase excitation, magnetic charges at side surfaces form magnetic dipoles resulting in attractive force between two cores, which corresponds to a bonding orbital. On the other hand, the side charges of the disks repel each other for the out-of-phase excitation, in analogy with an antibonding orbital.

Figure 2(b) shows the maximum deviation of the core from the center ($\delta y_{\max}$) of the steady orbital for the current-excited vortex in the left disk as a function of the ac frequency for the sample with $d = 75$ nm. The single vortex has a clear resonance peak at around 380 MHz and exhibits a small discrepancy with the experimental result [Fig. 1(b)] caused by self-reduced magnetization [20]. For the coupled vortices, two clear resonance peaks are observed on both higher and lower frequencies relative to the sole peak for the single vortex. The magnitude of mode splitting for $p_1p_2 = -1$ is larger than that for $p_1p_2 = 1$, as experimentally observed in Fig. 1(b). At both low and high frequencies, the in-phase and out-of-phase modes are degenerate with respect to the chirality. Figure 3 summarizes all four resonance modes from $A$ to $D$ characterized by rotational directions and the phase difference in gyrations. The lower frequency modes ($A$ and $C$) stabilize with the help of attractive interaction due to the magnetic charges appearing along the disk circumferences. On the other hand, the higher frequency modes ($B$ and $D$) stabilize with the help of repulsive interaction due to the opposite magnetic charges. Therefore the appearance of either in-phase or out-of-phase gyration in Fig. 2(a) depends only on the polarity configuration $p_1p_2$.

In both experimental and simulated results, the splitting amplitude tends to be enhanced with the decrease in the normalized separation distance $d_n = d/r$, as shown in Fig. 4. To check the effect of the current induced Oersted field on the excitation of the gyration of the right Py disk, the simulation was performed by replacing the left Py disk with an electrode where the current flows with the identical condition. The excited gyration amplitude due to the Oersted field is much smaller than that for the coupled vortices, implying that the magnetic dipolar interaction is the dominant factor for the indirect excitation of the right Py disk.

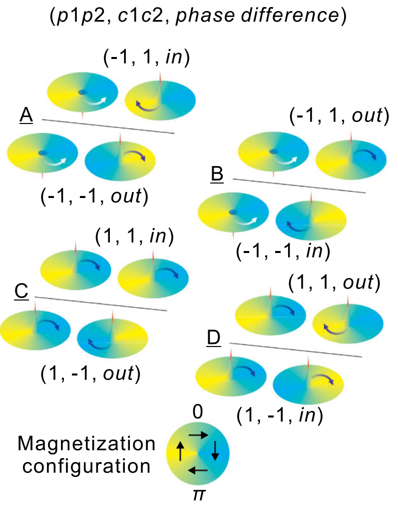

FIG. 3 (color online). Schematic diagrams of four different resonance modes. Each mode is identified with the sign of polarities $p_1p_2$, chiralities $c_1c_2$, and the phase difference of the gyrations.

We here discuss the experimental results analytically on the basis of Thiele's equation [23,24], which describes the gyrations of vortex cores $r_i = (X_i, Y_i)$ with $i = 1, 2$. It should be noted that our calculations take into account only the adiabatic spin transfer torque as the excitation force of vortex 1 in the left disk to simplify the analyses. The nonadiabatic and the Oersted field terms for the excitation are neglected. The effect of the nonadiabatic term fundamentally appears as the phase and the rotational radius shifts of the gyroscopic motion of the vortex core. One would point out the current induced Oersted field as the other origin of core excitation. Both terms would affect the dynamics of vortex 1 [16–18,25]; however, they may not influence the coupled modes identified in Fig. 3. Herein, the detected dc voltage can be evaluated by the following equation [19]:

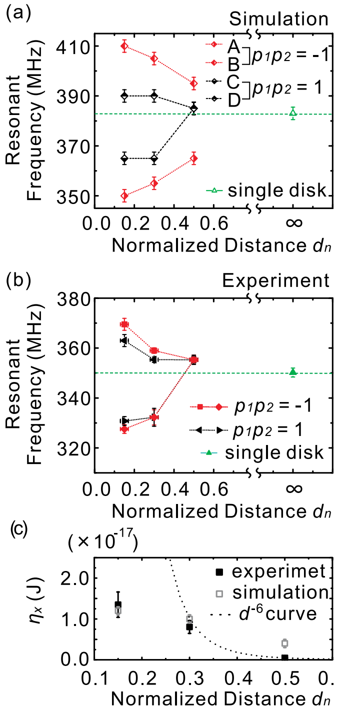

FIG. 4 (color online). Eigenfrequencies of the coupled gyration modes as a function of the separation distances. The simulation and experimental results are shown in (a) and (b), respectively. The separation distance is given by a dimensionless value $d_n = d/R$. (c) Estimated values of coupling strength $\eta_x$ from both experiment and simulation with different separation distances. A $d^{-6}$ curve from the rigid vortex model is also plotted as a dotted line.

$$V_{dc} = I_{ac}C/2(\mathrm{Re}[X_1]\cos\delta_1 + \mathrm{Re}[Y_1]\sin\delta_1), \quad (1)$$

where $C$ is a constant and $\delta_1$ is the phase difference between core and resistance oscillations in the left disk. The model [5] assumes the magnetostatic interaction energy term in Thiele's equation as

$$U_{\mathrm{int}} = c_1c_2/R^2(\eta_x X_1 X_2 - \eta_y Y_1 Y_2) + O(|r/R^3|). \quad (2)$$

According to the rigid vortex model [23], the values of $\eta_x$ and $\eta_y$ are decided only by the shape of the ferromagnetic element and the separation distance, independent of the excitation amplitude. As a simple solution for these problems, $\eta_x$ and $\eta_y$ should be treated as phenomenological fitting parameters including the influence of volume charges under the assumption of complete linear response with the fixed parabolic potential $U = 1/2k_xX_i^2 + 1/2k_yY_i^2$ ($k_x, k_y \equiv$ const) and the interaction energy $U_{\mathrm{int}}$. The best-fit curves to the experimental data using the above equation are shown in Fig. 1(b) by solid curves. The values of $k_x$ and $k_y$ are decided by a result of the single vortex with $I_{ac} = 3.8$ mA as $(k_x, k_y) = (4.5 \times 10^{-4}\ \mathrm{J/m^2}, 6.4 \times 10^{-4}\ \mathrm{J/m^2})$. These results agree well with the previous work [19], implying adequacy of Thiele's equation for the present analyses. The obtained values from experiments with 75 nm separation are $(\eta_x, \eta_y) = (1.4 \times 10^{-17}\ \mathrm{J}, 4.9 \times 10^{-22}\ \mathrm{J})$, and those from simulations performed under identical conditions are $(\eta_x, \eta_y) = (1.2 \times 10^{-17}\ \mathrm{J}, 8.1 \times 10^{-18}\ \mathrm{J})$. While the values of $\eta_x$ agree well with each other, a large discrepancy of $\eta_y$ is observed. This is possibly due to anisotropy along the $y$ direction in the experiment due to the presence of attached Cu electrodes, which causes deviation of stray field from the simulation, resulting in closer resonance frequencies for coupled vortices with $p_1p_2 = \pm 1$ [gray (red) and black symbols] in Fig. 4(b). The separation dependence of $\eta_x$ is shown in Fig. 4(c). It is clear that the coupling strength increases with the decrease in the separation distance, which can be clearly reproduced by the two-dimensional micromagnetic simulation. It should be noted that the $d$ dependence of $\eta_x$ does not follow the $d^{-6}$ dependence expected from a rigid vortex model [5,6]. A strong magnetostatic coupling modifies the trajectories of the gyrations and the magnetization configuration near the edge region thus causes a deviation from the model, where a circular magnetization around the core is assumed.

In summary, we have experimentally demonstrated the resonant excitation of coupled gyration modes in paired vortices by means of local excitation by an ac current passing through one of the disks in the pair. Excited coupled modes are identified by rotational directions and a phase difference as four different eigenmodes. The unique property in this system gives us a guiding principle for designing the magnonic crystal in further expanded systems such as one-dimensional chains and two-dimensional arrays and is a candidate for novel tunable oscillators using vortices [6,9,26].

We would like to thank Y. Nakatani for fruitful discussions. This work is supported in part by a Grant-in-Aid for Scientific Research in Priority Area "Creation and Control of Spin Current" (Grant No. 19048013) from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

---

*Present address: Inamori Frontier Research Center, Kyushu University, 744 Motooka, Nishi-ku, Fukuoka 819-0395, Japan.
†yotani@issp.u-tokyo.ac.jp

[1] T. Shinjo, T. Okuno, R. Hassdorf, K. Shigeto, and T. Ono, Science **289**, 930 (2000).
[2] A. Wachowiak *et al.*, Science **298**, 577 (2002).
[3] V. Novosad *et al.*, Phys. Rev. B **72**, 024455 (2005).
[4] S.-B. Choe *et al.*, Science **304**, 420 (2004).
[5] J. Shibata, K. Shigeto, and Y. Otani, Phys. Rev. B **67**, 224404 (2003).
[6] J. Shibata and Y. Otani, Phys. Rev. B **70**, 012404 (2004).
[7] A. Y. Galkin, B. A. Ivanov, and C. E. Zaspel, Phys. Rev. B **74**, 144419 (2006).
[8] K. S. Buchanan *et al.*, Nature Phys. **1**, 172 (2005).
[9] A. Ruotolo *et al.*, Nature Nanotech. **4**, 528 (2009).
[10] A. Vogel, A. Drews, T. Kamionka, M. Bolte, and G. Meier, Phys. Rev. Lett. **105**, 037201 (2010).
[11] A. A. Awad *et al.*, Appl. Phys. Lett. **97**, 132501 (2010).
[12] A. Barman, S. Barman, T. Kimura, Y. Fukuma, and Y. Otani, J. Phys. D **43**, 422001 (2010).
[13] H. Jung *et al.*, Appl. Phys. Lett. **97**, 222502 (2010).
[14] J. Shibata, Y. Nakatani, G. Tatara, H. Kohno, and Y. Otani, Phys. Rev. B **73**, 020403 (2006).
[15] S. Kasai, Y. Nakatani, K. Kobayashi, H. Kohno, and T. Ono, Phys. Rev. Lett. **97**, 107204 (2006).
[16] M. Bolte *et al.*, Phys. Rev. Lett. **100**, 176601 (2008).
[17] T. Kamionka *et al.*, Phys. Rev. Lett. **105**, 137204 (2010).
[18] Y. Nakatani *et al.* (private communication). The vortex core should be dominantly excited by the spin transfer torque due to the cancellation of the in-plane current induced Oersted field in their three-dimensional micromagnetic simulations.
[19] R. Moriya *et al.*, Nature Phys. **4**, 368 (2008).
[20] K. Yamada *et al.*, Nature Mater. **6**, 270 (2007).
[21] K. Y. Guslienko, R. H. Heredero, and O. Chubykalo-Fesenko, Phys. Rev. B **82**, 014402 (2010).
[22] LLG Micromagnetic Simulator, http://llgmicro.home.mindspring.com, 1997.
[23] A. A. Thiele, Phys. Rev. Lett. **30**, 230 (1973).
[24] D. L. Huber, Phys. Rev. B **26**, 3758 (1982).
[25] L. Heyne *et al.*, Phys. Rev. Lett. **105**, 187203 (2010).
[26] S. Barman, A. Barman, and Y. Otani, J. Phys. D **43**, 335001 (2010).