Identifying the Inflaton with Primordial Gravitational Waves

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(Received 2 November 2010; revised manuscript received 25 February 2011; published 11 May 2011)

We explore the ability of experimental physics to uncover the underlying structure of the gravitational Lagrangian describing inflation. While the observable degeneracy of the inflationary parameter space is large, future measurements of observables beyond the adiabatic and tensor two-point functions, such as non-Gaussianity or isocurvature modes, might reduce this degeneracy. We show that, even in the absence of such observables, the range of possible inflaton potentials can be reduced with a precision measurement of the tensor spectral index, as might be possible with a direct detection of primordial gravitational waves.

DOI: 10.1103/PhysRevLett.106.191302 PACS numbers: 98.80.Cq, 98.80.Es

In the simplest realizations of the inflationary universe paradigm, acceleration expansion is generated by a single canonically normalized scalar inflaton field ϕ with a potential $V(\phi)$. Within this setting, there exists a unique mapping between the set of observables and the free parameters of the Lagrangian; in single field, slow roll inflation the observables are determined by the potential, $\mathcal{O}[V(\phi)]$. There are, however, a vast number of ways to achieve the desired acceleration including models involving multifields, nonstandard kinetic terms, and nontrivial gravitational couplings. In such elaborate settings the observables depend on modified or even additional degrees of freedom, $\mathcal{O}[V(\phi), F(\phi, \cdots)]$. Further, any process designed to "invert" a subset of observables to obtain the underlying free parameters of the Lagrangian will reveal a space of Lagrangians that is not observationally unique. This degeneracy problem is well known and is a formidable challenge for cosmologist attempting to identify the inflaton. Fortunately, many of the alternatives to canonical single field inflation produce unique observational signatures, such as non-Gaussian and/or isocurvature perturbations. Such observations beyond the two-point adiabatic and tensor power spectra can be used to distinguish between these models and break the degeneracy [1]. However, of particular concern is the future possibility that such discriminating observables are not detected. While this problem has been previously documented, there has been no attempt to systematically determine the size of the degeneracy, for example, by estimating the envelope of different potentials $V(\phi)$ within the larger class of inflation theories that yield the same observables.

In this work, we take an initial step in estimating the magnitude of this degeneracy by performing Monte Carlo potential reconstructions within the context of two broad classes of alternatives to canonical single field inflation, in the absence of discriminating observations. First we consider the case in which the perturbation spectra are generated by degrees of freedom that are decoupled from the inflationary dynamics, and second, the case where the inflationary dynamics are extended beyond the paradigm of single field, canonical inflation by altering dynamical degrees of freedom. As representative examples of such models we examine, respectively, the curvaton scenario, in which a noninflationary degree of freedom generates the primordial perturbations, and Dirac-Born-Infeld (DBI) inflation, in which a noncanonical kinetic term contributes to the inflationary dynamics.

In canonical single field models, the reconstruction program reveals that inflationary potentials can be grouped into three distinct classes based on their observable predictions for n_s, the spectral index of scalar perturbations, and r, the tensor/scalar ratio; i.e., vast numbers of inflationary potentials organize themselves into a few observational families (cf. Fig. 1). This classification scheme is commonly referred to as the "zoology" of inflation models [4,5]. "Hybrid" models include potentials that evolve asymptotically to their minima, requiring an auxiliary field to end inflation. However, they are effectively single field models with nonvanishing energy density at the minimum, and have the common form $V(\phi) \propto 1 + (\phi/\mu)^p$, where μ is

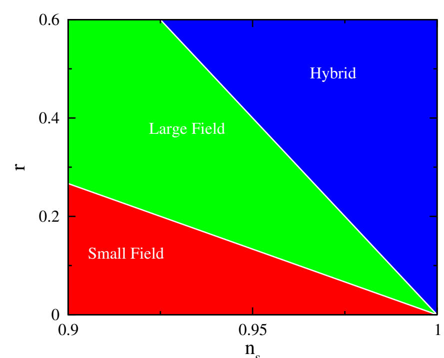

FIG. 1 (color online). Zoology of inflation models.

an energy scale and p a positive integer. They are characterized by the conditions $V''(\phi) > 0$ and $(\log V(\phi))'' > 0$. The simplest models of tree-level hybrid inflation [6,7] belong to this class. "Small field" and "large field" models are differentiated by their initial field values. Large field models, for example $m^2\phi^2$ inflation [8], are characterized by a field initially displaced far from its minimum, with the general form $V(\phi) \propto (\phi/\mu)^p$, satisfying $V''(\phi) > 0$ and $(\log V(\phi))'' < 0$. Conversely, small field models are characterized by a field initially close to the origin, with general form $V(\phi) \propto 1 - (\phi/\mu)^p$ (near the maximum), satisfying $V''(\phi) < 0$ and $(\log V(\phi))'' < 0$; "new" inflation and models based on spontaneous symmetry breaking belong to this class. If the simplest implementation of canonical single field inflation is assumed, we may hope to determine which class of the above potentials is ultimately responsible for driving inflation. To what degree is our ability to reconstruct the physics of inflation threatened by relaxing this assumption?

In the curvaton scenario [9–11], the central assumption of traditional reconstruction—that the inflaton generates the primordial spectra—is relaxed. Without knowledge of how the spectra were generated, whether by the inflaton or by some other means, a unique inversion of observables is clearly impossible. The curvaton field, σ, is weakly coupled and relatively light during inflation, $m^2 \ll H^2$. It influences the primordial power spectrum, $P_\Phi(k) = k^3|\Phi|^2/2\pi^2$, via the final curvature perturbation

$$\Phi = -\frac{1}{2}\frac{H}{M_{\rm Pl}^2 H'}\delta\phi - \frac{\tilde{f}(\sigma)}{\sqrt{2}M_{\rm Pl}}\delta\sigma, \tag{1}$$

where $\delta\phi$ and $\delta\sigma$ are the inflaton and curvaton vacuum fluctuations, and $\tilde{f}(\sigma)$ controls the contribution of the curvaton to the overall perturbation. After inflation ends, the curvaton rolls to its minimum where it begins to oscillate during the postinflationary phase. These oscillations set up a small isocurvature perturbation that grows with time. After the curvaton decays, the perturbation is converted to an adiabatic mode and structure begins to evolve according to the standard model. Measurements of the adiabatic perturbation spectrum derived from Eq. (1) and the tensor/scalar ratio r do not uniquely determine the potential $V(\phi)$ unless $\tilde{f}(\sigma)$ can be constrained, leading to the possibility of the same potential giving rise to a wide range of observables. For example, the first two derivatives of $V(\phi)$ can be written

$$V'_{\rm curv} = \sqrt{2}\frac{V_0}{M_{\rm Pl}}\left(\frac{r}{16 - \tilde{f}^2 r}\right)^{1/2}, \tag{2}$$

$$V''_{\rm curv} = \frac{V_0}{M_{\rm Pl}^2}\left[\frac{8(n_s - 1) + 3r}{16 - \tilde{f}^2 r}\right]. \tag{3}$$

However, depending on the thermal history and the energy density of the curvaton at the time of decay, there may be residual isocurvature modes or primordial "local"-type non-Gaussianity large enough to be detected in future experiments; these additional observables might enable a determination of $\tilde{f}(\sigma)$. We consider the effects of such observations on reconstruction in [2]—in this analysis we assume that they are not detected.

A degeneracy problem might also arise in the context of noncanonical models [3,12]. In noncanonical models, the inflaton field Lagrangian includes nonstandard kinetic terms $\mathcal{L}(X, \phi)$, where $2X \equiv \partial^\mu\phi\partial_\mu\phi$. We study the most well-motivated case—that of the nonlinear Lorentz invariant DBI action $\mathcal{L} = -f^{-1}(\phi)\sqrt{1 + 2f(\phi)X} + f^{-1}(\phi) - V(\phi)$, where $f(\phi)$ is the "warp factor" [13,14]. In the DBI model, the inflaton speed is bounded from above by a generalized Lorentz factor $\gamma^{-1} \equiv \sqrt{1 - f(\phi)\dot{\phi}^2}$, which can lead to a new type of slow roll inflation even with steep potentials. As a result, cosmological fluctuations travel with sound speed less than unity, $c_s = \gamma^{-1} \le 1$, leading to a curvature perturbation that depends on γ,

$$\Phi = -\frac{1}{2}\frac{H\gamma}{M_{\rm Pl}^2 H'}\delta\phi. \tag{4}$$

Despite the formal distinction between the curvaton and DBI reconstructions, the treatment of the two cases is the same: a determination of $V(\phi)$ requires observations of more than simply the adiabatic density perturbation and tensor/scalar ratio. The potential in DBI inflation gives

$$V'_{\rm DBI} = \frac{V_0}{M_{\rm Pl}}\sqrt{\frac{r}{8}}\gamma, \tag{5}$$

$$V''_{\rm DBI} = \frac{V_0}{2M_{\rm Pl}^2}\gamma\left(n_s - 1 + \frac{3}{8}r\gamma\right). \tag{6}$$

In the case of curvatons, the function $\tilde{f}(\sigma)$ needs to be measured; in the case of DBI inflation, the γ factor must be constrained. While large equilateral non-Gaussianities might be produced in DBI inflation, we assume that future missions fail to detect them.

We consider only the minimal set of observational parameters describing the primordial scalar and tensor power spectra: $P_\Phi(k)$ and r. To ascertain the size of the degeneracy, we employ the flow formalism [15,16], which is a Monte Carlo approach to potential reconstruction [17]. The inflationary model space is stochastically sampled and models of interest can be selected out. We first seek to determine the constraints that can be imposed on $V(\phi)$ at Planck precision [18], in the absence of discriminating observations: we consider 68% C.L. detections of r ($r \gtrsim 0.01$, $\Delta r \sim 0.03$) [19], n_s ($\Delta n_s \sim 0.0038$), and $dn_s/d\ln k$ ($\Delta dn_s/d\ln k \sim 0.005$) [20]. Since $\Delta n_T \sim 0.1$ with a Planck B mode detection, the tensor spectral index will not be well resolved and will not be included in the reconstruction. This worst-case reconstruction therefore makes use of only the adiabatic and tensor two-point functions on CMB scales. We perform separate analyses for curvatons, DBI, and canonical single field inflation. We collect only models that support at least 10 e-foldings of inflation and satisfy the above observational constraints at $k = 0.01$ Mpc^{-1}. We present the constraints on the first two derivatives of $V(\phi)$ in Fig. 2(a): magenta or light gray (BW), black, and blue or dark gray (BW) points denote single field, curvaton, and DBI models, respectively. The constraints depend only weakly on the fiducial observables chosen [2]: in Figs. 2(a) and 2(c) we choose $r = 0.15$, $n_s = 0.97$, and $dn_s/d\ln k = 0$ for the potential reconstructions. If r is not measured ($r \lesssim 0.05$ with Planck) then the uncertainty in $V(\phi)$ extends to $V'/V = 0$, but is of the same order of magnitude as in the case $r = 0.15$ [2,3]. Even an improved measurement of r by next-generation CMB experiments like CMBPol will scarcely improve constraints on $V(\phi)$ in the presence of the degeneracy [2,3].

We next examine the effects of the unresolved degeneracy on the zoology by sorting the curvaton and DBI models by (n_s, r) according to the potential classification: small field, large field, and hybrid. We find that all observables that are compatible with canonical single large field models are also consistent with curvaton and/or DBI hybrid models. Furthermore, we find that all observations compatible with canonical single small field models are also consistent with both large field and hybrid curvaton and/or DBI models. Only those hybrid models existing in the single field "hybrid" region can be correctly classified in the presence of the degeneracy; i.e., they must satisfy $r > 8(1 - n_s)$. We present the zoology in Fig. 2(b) indicating in gray regions in which at least two classes of model overlap.

We have obtained the worst-case degeneracy by utilizing only the two-point adiabatic and tensor spectra on CMB scales in the reconstructions. It is certainly possible that these will be the only detected observables: canonical single field inflation could be the true underlying model, curvatons need not generate detectable isocurvature modes or non-Gaussianity, and DBI inflation will fail to generate observable non-Gaussianity if the sound speed $c_s \gtrsim 0.1$. However, we need not restrict ourselves to observables on CMB scales only: primordial gravitational waves on scales $k_* \sim 10^{14}$ Mpc^{-1} can be used to measure the tensor spectral index, n_T, at a precision surpassing that possible with a detection of B modes on CMB scales. Future space-based laser interferometers, like Big Bang Observer (BBO) [21] and Japan's Deci-hertz Interferometer Gravitational Wave Observatory (DECIGO) [22], will detect gravitational waves if B modes on CMB scales give $r \gtrsim 10^{-3}$ and $r \gtrsim 10^{-6}$, respectively. This range includes a substantial portion of the inflationary model space. In comparison with an ideal B mode detection on CMB scales, a direct detection with BBO will yield comparable constraints ($\Delta n_T \sim 10^{-2}$) while DECIGO gives the best measurement: $\Delta n_T \sim 10^{-3}$ or better [23,24].

The tensor index turns out to be highly valuable to the reconstruction program, since, while canonical single field inflation predicts the consistency condition $r = -n_T/8$, alternative theories typically yield modified relations: curvatons predict $r = -16n_T/(2 - \tilde{f}^2(\sigma)n_T)$ and DBI

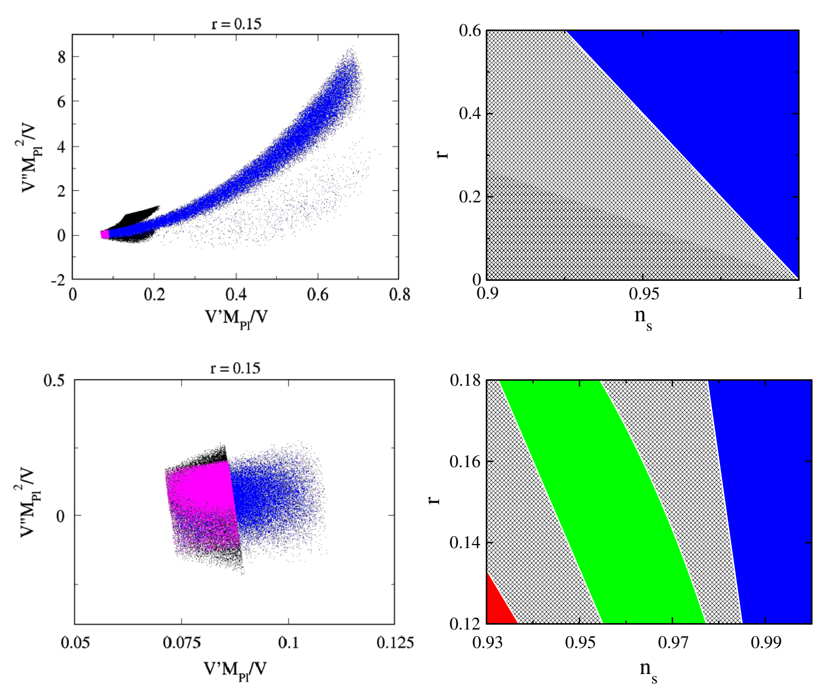

FIG. 2 (color online). (a) Monte Carlo results for the worst-case degeneracy at Planck-precision using r, n_s, and $dn_s/d\ln k$ in the canonical single field [magenta or light gray (BW)], curvaton (black), and DBI [blue or dark gray (BW)] reconstructions. (b) Zoology of the worst-case degeneracy (a). Gray areas denote regions in which multiple classes overlap: only hybrid models can be uniquely classified. (c) Monte Carlo results for the best-case degeneracy utilizing a direct detection of primordial gravitational waves at DECIGO precision to constrain n_T with the same observables as in (a). (d) Zoology of models that results from the best-case degeneracy (c).

inflation predicts $r = -8c_s n_T$. With the modified conditions, we find that $\tilde{f}^2(\sigma)$ drops out of the curvaton reconstruction, Eqs. (2) and (3), giving $V'_{\rm curv} \propto V_0\sqrt{-n_T}$ and $V''_{\rm curv} \propto -V''_{\rm csf} n_T/r$, where $V''_{\rm csf}$ is the canonical single field reconstruction. Likewise for DBI, γ vanishes from Eqs. (5) and (6) giving $V'_{\rm DBI} \propto -V_0 n_T/\sqrt{r}$ and $V''_{\rm DBI} \propto -V_0(n_s - 1 - 3n_T)n_T/r$.

We stress that we are not considering cases in which the values of r and n_T violate one or more of the above consistency conditions; i.e., we are assuming that the degeneracy remains intact. The range of $V(\phi)$ is reduced despite the unbroken degeneracy because the consistency conditions constrain precisely the degrees of freedom that are necessary for an inversion of the potential: $\tilde{f}^2(\sigma)$ for curvatons and c_s for DBI inflation. We need not know *a priori* which condition to impose—we impose each one that agrees with the fiducial r and n_T to within experimental error and perform the reconstruction.

We assume that the tensor spectrum is of the form

$$P_h(k) = P_h(k_0)\left(\frac{k}{k_0}\right)^{n_T + (1/2)\alpha_T \ln(k/k_0)}, \qquad (7)$$

where $\alpha_T = dn_T/d\ln k$ is the tensor index running and $k_0 = 0.01\ \mathrm{Mpc}^{-1}$. The challenge is that direct detection experiments determine $n_T(k_*)$, while the consistency relations are functions of $n_T(k_0)$. In principle, the spectrum Eq. (7) provides the mapping from k_* to k_0; however, α_T is unlikely to be reliably constrained by these experiments. Although likely small, our ignorance of α_T limits the accuracy of the extrapolation from k_0 to k_* and contributes to the error on n_T [25,26],

$$\Delta n_T = \left\{\left[\frac{6\times 10^{-18}}{XA_{\rm GW}P_h(k_*)}\right]^2 + \left[\frac{1}{2}\alpha_T \ln\left(\frac{k_*}{k_0}\right)\right]^2\right\}^{1/2}, \qquad (8)$$

where $A_{\rm GW} = 2.74\times 10^{-6}$ and X characterizes the experiment: $X = 0.25$ for BBO and $X = 100$ for DECIGO. In Fig. 2(c) we present the best-case reconstruction including a direct detection of n_T at DECIGO precision with a fiducial value of $n_T = -r/8 = -0.018\,75$, in agreement with all three consistency conditions. We find that the curvaton models (black) are almost as well constrained as canonical single field inflation [magenta or light gray (BW)], while for DBI [blue or dark gray (BW)] we find $V''_{\rm DBI} \approx V''_{\rm csf}$ and $V'_{\rm DBI} \approx 2V'_{\rm csf}$. In addition, with a measurement of n_T the zoology can be partially recovered compared to the worst-case degeneracy, Fig. 2(b). In the event of a future detection with DECIGO together with a moderate amplitude of tensors, $r = 0.15$, the zoology possesses regions occupied uniquely by small field, large field, and hybrid models, shown in Fig. 2(d). Constraints on models with smaller fiducial r are also improved although to a lesser degree. We note that measurements of n_T at BBO precision also offer improvements over the worst-case degeneracy, but for brevity we present only the best-case reconstruction here (see [2,3]).

In conclusion, while we may never know the true underlying theory of inflation, we have found that it is nonetheless still possible to vastly improve our understanding of the inflaton potential despite this.

The work of D. A. E. is supported in part by the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan and by a Grant-in-Aid for Scientific Research (21740167) from the Japan Society for Promotion of Science (JSPS), and by the Arizona State University Cosmology Initiative.

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[1] Once the degeneracy is broken, the inflaton potential may be reconstructed, but depending on the model the observable-to-potential mapping might still not be unique. We investigate this inversion problem in detail for curvatons [2] and DBI inflation [3].
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