Cooling of Nanomechanical Resonators by Thermally Activated Single-Electron Transport

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We show that the vibrations of a nanomechanical resonator can be cooled to near its quantum ground state by tunneling injection of electrons from a scanning tunneling microscope tip. The interplay between two mechanisms for coupling the electronic and mechanical degrees of freedom results in a bias-voltage-dependent difference between the probability amplitudes for vibron emission and absorption during tunneling. For a bias voltage just below the Coulomb blockade threshold, we find that absorption dominates, which leads to cooling corresponding to an average vibron population of the fundamental bending mode of 0.2.

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Remarkable steps are now being taken towards achieving the experimental conditions under which macroscopic degrees of freedom (such as the center of mass of systems composed by a large number of atoms) manifest quantum behavior [1]. This is encouraging both for the prospects of realizing a plethora of applications that depend on our ability to control and monitor the coherent dynamics of nanometer-scale mechanical oscillators and for shedding light on purely fundamental issues, such as the nature of the crossover from classical to quantum physics [2].

However, cooling down mechanical resonators to temperatures at which the quantum features of their dynamics should be observable (which requires $k_B T \ll \hbar\omega$, where ω is the eigenfrequency of the resonator) is still challenging if $\omega \lesssim 1$ GHz.

In order to overcome this problem, a number of proposals have been put forward [3]. Some of them are based on the well-established principles of laser cooling for atoms and molecules, whereas alternative approaches show that, in the case of nanowire-based resonators, it is possible to achieve ground-state cooling by coupling them with a suitable nonequilibrium electronic environment.

A common feature of all these proposals is that they rely, in one form or another, on resonant transitions between coherent states of the refrigerant. Basically, they exploit the energy conservation constraint in order to suppress those processes that involve emission of vibrational energy quanta (vibrons) with respect to those that lead to the absorption of such quanta.

Here we present a fundamentally new mechanism for cooling a suspended carbon nanotube-based resonator. This mechanism does not rely on the energy conservation constraint and allows one to approach the ground state of the mechanical vibrations by simply exploiting incoherent, thermally excited states of the nonequilibrium electronic subsystem used as a refrigerant.

To be specific, we consider the system sketched in Fig. 1, where electrons are injected from the tip of a scanning tunneling microscope (STM) into a suspended metallic carbon nanotube. Low-temperature tunneling spectroscopy studies on a similar device have shown that inelastic electron tunneling can create a non-thermal-equilibrium population of vibronic states in the nanotube [4]. Below, we will show that the probability for vibron emission can be suppressed as a result of destructive interference between two different mechanisms for coupling the mechanical and electronic degrees of freedom of the system. One of these mechanisms is the nanotube-position-dependent probability amplitude for electron tunneling from the STM tip to the nanotube, and the other is the electrostatic force on the nanotube when it is charged.

It turns out that the effect of the interference depends on the voltage bias between the STM tip and the leads. Our analysis shows that the destructive interference is maximal for a bias voltage slightly below the threshold voltage for lifting the Coulomb blockade of electron tunneling through the system. If the nanotube is weakly enough coupled to the environment, the suppression of vibron emission is strong enough to drive the nanotube to near its vibrational

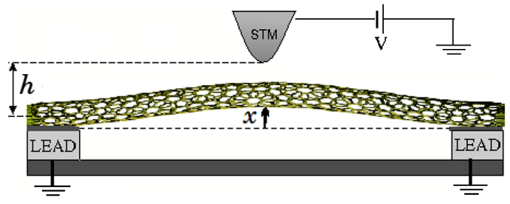

FIG. 1 (color online). Sketch of the model system considered. A metallic carbon nanotube is suspended over a trench between two grounded metallic leads, while an STM tip placed a distance h above the nanotube is biased at a negative voltage $-V$.

ground state and hence effectively "cool" the mechanical degrees of freedom.

In order to analyze the dynamics of the nanotube and of the tunneling electrons in the quantum regime, we introduce a model Hamiltonian: $H = H_e + H_m + H_t + H_C$, where

$$H_e = \sum_{q,\alpha} E_{q,\alpha} a^\dagger_{q,\alpha} a_{q,\alpha} + \sum_q \xi_q c^\dagger_q c_q, \tag{1}$$

$$H_m = \hbar\omega(b^\dagger b + 1/2), \tag{2}$$

$$H_t = \sum_{q,q'} e^{i\hat{\varphi}} c^\dagger_{q'} [t_S(\hat{X}) a_{q,S} + t_L a_{q,L}] + \text{H.c.}, \tag{3}$$

$a^{(\dagger)}_{q,\alpha}$ and $c^{(\dagger)}_q$ being the annihilation (creation) operators for electrons in the STM tip ($\alpha = S$), in the leads ($\alpha = L$), and in the nanotube, respectively.

The first term in the total Hamiltonian, H_e, describes the STM tip, the leads, and the nanotube as reservoirs of noninteracting electrons. The second term, H_m, describes the nanotube's mechanical degrees of freedom, which we restrict to the fundamental bending mode considered as a simple harmonic oscillator with angular frequency ω, $b^{(\dagger)}$ being the annihilation (creation) operator for an elementary excitation (vibron) of this mode.

Electron tunneling between the STM tip and the nanotube and between the nanotube and the leads is described by H_t, the third term of the Hamiltonian, in terms of the tunneling amplitudes t_S and t_L. Here the operator $e^{i\hat{\varphi}}$ changes the number N of excess electrons on the nanotube by one: $e^{-i\hat{\varphi}} \hat{N} e^{i\hat{\varphi}} = \hat{N} + 1$. Since t_S depends on the overlap between electronic states in the STM tip and the nanotube, it depends on the deflection of the tube through the operator $\hat{X} = \Delta x_{\text{gs}}(b^\dagger + b)$, where $\Delta x_{\text{gs}} \equiv \sqrt{\hbar/(2M\omega)}$ is the displacement uncertainty in the vibrational ground state and M is an effective oscillator mass ($\Delta x_{\text{gs}} \sim 10^{-11}$ m for $\omega \sim 10^9$ s^{-1} and $M \sim 10^{-22}$ kg). In contrast, the distance between the nanotube and the leads is fixed, so that t_L does not depend on the nanotube deflection.

For simplicity, we assume that the STM tip is positioned above the midpoint of the nanotube (see Fig. 1) and model the deflection dependence of the tunneling amplitude as $t_S(\hat{X}) \equiv t_S \exp(\hat{X}/\lambda)$, where λ is the characteristic tunneling length of the barrier ($\lambda \simeq 10^{-10}$ m). Such a form of the tunneling amplitude accounts for the fact that tunneling of electrons between the STM tip and the nanotube generates fluctuations in the nanotube momentum of the order of $\Delta p \sim \hbar/\lambda$ [5]. In what follows, we will refer to this mechanism of interaction between mechanical and electronic degrees of freedom as a tunneling electromechanical (TEM) coupling.

The last term in the total Hamiltonian, H_C, describes the electrostatic interactions in the system, which we will treat in the framework of the capacitance model. In this approximation H_C depends only on the total charge on the nanotube and on the voltages applied to the bulk electrodes. Assuming the supporting leads to be grounded and that a negative electrostatic potential $-V$ ($V > 0$) is applied to the STM electrode, we restrict our analysis to the Coulomb blockade regime in which at most one extra electron may reside on the nanotube. Under such conditions, H_C can be written as [6]

$$H_C = e\left[\frac{C_g(V_C - V)}{C_\Sigma} + V\right]\hat{N} - \frac{C_S C_g V^2}{2C_\Sigma}, \tag{4}$$

where $C_\Sigma = C_S + C_g$, C_S ($\sim 10^{-18}$ F from the data reported in Ref. [4]) is the mutual capacitance between nanotube and STM tip, C_g is the total capacitance between nanotube and ground, $V_C = e/2C_g$ is the threshold value of V for lifting the Coulomb blockade ($V_C \simeq$ a few mV for $C_g \sim 10^{-17}$ F), and $-e$ is the electronic charge.

In general, C_S and C_g both depend on the geometry of the system and therefore on the nanotube deflection. Here we will take only the dominant deflection dependence of the STM-nanotube capacitance into account. Hence $C_S = C_S(h - \hat{X})$, where h is the distance between the STM and the straight nanotube. For small displacements of the nanotube, we may linearize the interaction Hamiltonian (4) and use an approximation that for $C_S \ll C_g$ takes the form

$$H_C = U_C(V)\hat{N} - \Im\hat{X}\hat{N} - \alpha(\hat{X})V^2, \tag{5}$$

where $\Im \equiv 2(\partial C_S/\partial x)_0 V_C \delta V$ and $\delta V \equiv V_C - V$. The first term of (5) determines the Coulomb blockade effect in the absence of nanotube deflections, while the second is a deflection-dependent electromechanical interaction term. Because of a formal analogy with the interaction term in the model Hamiltonian for the polaron problem, we will refer to the origin of this term as a polaronic electromechanical (PEM) coupling. The last term of (5) is a contribution that does not depend on whether the nanotube is charged or not.

It is important for what follows that the sign of the polaronic force constant $\Im$ in (5) depends on the bias voltage. If the bias voltage is below the Coulomb blockade threshold, so that only thermally activated transport is possible, i.e., if $\delta V > 0$, then $\Im > 0$ and hence if charged by an electron the nanotube will be attracted to the STM tip. On the other hand, if $\delta V < 0$, then $\Im < 0$ and the charged nanotube is repelled from the STM.

Note that the possibility to change the sign of $\Im$ by varying the bias voltage crucially relies on the discrete nature of the tunneling charge. If this charge could be arbitrarily small, then $V_C \to 0$ and hence $\Im \propto -V$. For any (positive) value of V, the polaronic force would therefore be negative and push the charged nanotube away from the STM tip, decreasing the tunneling matrix element [7].

As we have seen above, the electromechanical interaction is described by two separate terms in the Hamiltonian,

one due to what we call TEM coupling and the other due to PEM coupling. The cooling mechanism to be discussed below results from the interplay between these different types of coupling. In order to analyze this interplay, it is convenient to apply a unitary transformation that removes the polaronic term from the Hamiltonian and instead makes the tunneling amplitudes dependent on both the midpoint position $\hat{X}$ of the nanotube and its conjugate momentum $\hat{P} = i\hbar(b^\dagger - b)/2\Delta x_{\rm gs}$. This is achieved by the transformation $H \to \tilde{H} = UHU^\dagger$, where $U \equiv \exp(i\Delta x_e \hat{P}\hat{N}/\hbar)$. Here $\Delta x_e = (x_{e,1} - x_{e,0})/2 = \Im/2M\omega^2$ is the difference between the equilibrium positions of the charged and neutral nanotube. To leading order in the small dimensionless parameters $\varepsilon_t = \Delta x_{\rm gs}/\lambda$ and $\varepsilon_p = \Delta x_e/\Delta x_{\rm gs}$, the transformed tunneling Hamiltonian (3) is

$$\tilde{H}_t = t_S \sum_{k,q} [1 - (\varepsilon_t + \varepsilon_p)b + (\varepsilon_t - \varepsilon_p)b^\dagger] c_q^\dagger a_{k,S} + t_L \sum_{k,q} (1 - \varepsilon_p b + \varepsilon_p b^\dagger) a_{k,L}^\dagger c_q + \text{H.c.} \quad (6)$$

From Eq. (6), it follows that in the Born approximation the rate of inelastic single-electron tunneling from the STM tip to the nanotube accompanied by the absorption (+) or emission (−) of a vibron is

$$\Gamma_{S,\pm} = \Gamma_S(\varepsilon_t^2 + \varepsilon_p^2 \pm 2\varepsilon_t\varepsilon_p), \quad (7)$$

where $\Gamma_S = (\delta V/eR_S)[\exp(e\delta V/k_BT) - 1]^{-1}$ is the rate of elastic electron tunneling across the STM-nanotube junction, R_S being the tunnel resistance of the junction.

The first (second) term of (7) gives the probability for tunneling assisted by either absorption or emission of a vibron due to the TEM (PEM) coupling alone, while the third term corresponds to the ''interference'' between these two mechanisms in the case of vibron emission (−) and absorption (+). Clearly, the probability for vibron-assisted electron tunneling is different depending on whether a vibron is absorbed or emitted, and the difference can be controlled by the bias voltage since $\varepsilon_p \propto \Delta x_e \propto \delta V$.

In particular, $\Delta x_e > 0$ if $\delta V > 0$ so that the interference is destructive (constructive) for tunneling accompanied by vibron emission (absorption). If $\delta V < 0$, the situation is reversed in the sense that $\Delta x_e < 0$ and the interference is constructive (destructive) for emission (absorption) processes.

The case of constructive interference for emission processes has been analyzed in Ref. [8], where it was shown that a promotion of emission over absorption processes may lead to an electromechanical instability of the system if V exceeds a certain dissipation-dependent threshold. Here we will focus on the reverse situation.

A complete suppression of the emission processes would eventually drive the mechanical subsystem to its ground state. However, two more types of electronic transitions that may generate vibron emission remain to be considered. The first is the tunneling of an electron from the nanotube to the STM. By virtue of time reversal symmetry, the mechanism responsible for the suppression of vibron emission during tunneling from the STM to the nanotube stimulates the emission of vibrons during tunneling in the reverse direction.

In order to make the effect of such transitions negligible in the energy balance for the mechanical subsystem, an electron that has tunneled from the STM should escape from the nanotube to the leads before it can tunnel back to the STM by an inelastic transition. This requires that the tunnel resistance of the nanotube-leads junction, R_L, must be much smaller than R_S and that $k_BT \ll eV_C$, where the latter constraint ensures an exponential suppression of the probability for electrons to tunnel from the leads to the nanotube. These conditions together with the Coulomb blockade regime of electronic transport can be achieved experimentally if $R_S \sim 1\ \text{M}\Omega$.

In addition to the ''backward'' tunneling transitions, vibrons can also be emitted when electrons tunnel from the nanotube to the leads, but then only by virtue of the polaronic coupling mechanism [see Eq. (6)].

From Eqs. (7) and (6) and the definitions of ε_t and ε_p, it follows that the ratio between the total rate of vibron emission and the total rate of vibron absorption reaches an absolute minimum for the bias voltage $V^* = V_c - \delta V^*$ that verifies the condition $\Im(\delta V^*) = \hbar\omega/\sqrt{2}\lambda$. From the above considerations, we conclude that cooling of the nanotube vibrations can occur only for bias voltages below the Coulomb blockade threshold ($\delta V > 0$).

However, below the Coulomb blockade threshold voltage, charge transport is blocked at zero temperature. The temperature needed to overcome the Coulomb blockade is determined by $k_BT \geq e\delta V$. On the other hand, the temperature cannot be too high, since otherwise backward transitions from the leads to the nanotube would no longer be negligible and possibly compensate for the vibrons absorbed during the ''forward'' transitions. These conditions restrict the range of possible temperatures to the interval $eV_C \gg k_BT \geq e\delta V \cong e\delta V^*$.

The order of magnitude of the lower bound can be found by means of the condition of minimum vibron rate emission and by estimating the capacitance between the STM tip and the nanotube as $C_S \simeq 10^{-18}$ F, so that the temperature required in order to overcome the Coulomb blockade at δV^* turns out to be about 0.1 K.

For a quantitative analysis of the cooling mechanism described above, we followed the standard procedure to derive a generalized master equation for the reduced density matrix that describes the nanotube degrees of freedom [9]. After tracing out the charge degrees of freedom and applying a perturbation approach with respect to the small parameters $\epsilon_{t,p}$, one gets a set of equations for the probabilities p_n to find the nanotube in the Fock state $|n\rangle$

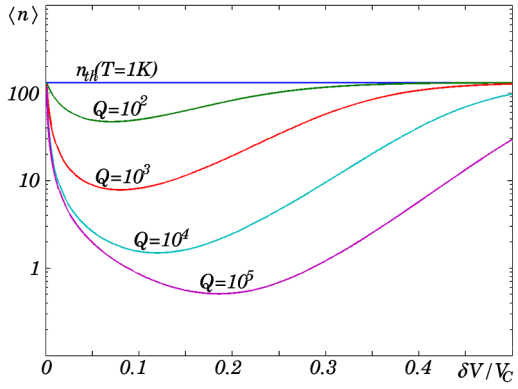

FIG. 2 (color online). Average number of vibrons $\langle n\rangle$ plotted against the difference $\delta V = V_C - V$ between the Coulomb blockade threshold voltage V_C and the bias voltage V. Each curve corresponds to a different quality factor of the oscillator, while the straight line gives the thermal average number of vibrons at the temperature of $T = 1$ K. Other parameters used were $V_C = 2$ mV, $R_S = 2.5$ MΩ, $R_L = 250$ kΩ, $\varepsilon_t = 0.27$, and $\varepsilon_p (\propto \delta V) = 0.0$–$0.3$.

characterized by n vibrons. If the rate Γ_L of tunneling from the nanotube to the leads is much larger than the rate Γ_S of tunneling from the STM to the nanotube, these equations reduce to

$$[(4n+2)\varepsilon_p^2 + (2n+1)\varepsilon_t^2 - 2\varepsilon_p\varepsilon_t]p_n - \frac{\mathcal{L}_\gamma[p_n]}{\Gamma_S} = [(\varepsilon_p+\varepsilon_t)^2+\varepsilon_p^2](n+1)p_{n+1} + [(\varepsilon_p-\varepsilon_t)^2+\varepsilon_p^2]np_{n-1}, \tag{8}$$

where $\mathcal{L}_\gamma$ describes the interaction with the environment, which takes the standard form [10] $\mathcal{L}_\gamma[p_n] \equiv \gamma(n+1)\times[(n_{\rm th}+1)p_{n+1} - n_{\rm th}p_n] - \gamma n[(n_{\rm th}+1)p_n - n_{\rm th}p_{n-1}]$, where $\gamma \equiv \omega/Q$, Q being the quality factor of the nanotube resonator, and $n_{\rm th} = (e^{\hbar\omega/k_BT}-1)^{-1}$ is the thermal average number of vibrons.

Equation (8) can be solved for the stationary probability distribution p_n with the result

$$p_n = (1-r)r^n, \qquad r = \frac{\varepsilon_p^2 + (\varepsilon_t-\varepsilon_p)^2 + (\gamma/\Gamma_S)n_{\rm th}}{\varepsilon_p^2 + (\varepsilon_t+\varepsilon_p)^2 + (\gamma/\Gamma_S)(n_{\rm th}+1)}. \tag{9}$$

The average number of vibrons, $\langle n\rangle = \sum_m m p_m = r/(1-r)$, is plotted as a function of the bias voltage for several values of the quality factor in Fig. 2. The theoretical limit of the cooling efficiency occurs for the bias voltage defined by $\varepsilon_p(\delta V^*) = \varepsilon_t/\sqrt{2}$ in the limit $Q\to\infty$. Equation (9) implies that the corresponding average number of excitations is $\langle n\rangle_{\rm min} = (\sqrt{2}-1)/2 \approx 0.2$.

In order to investigate the signatures of the cooling mechanism in a directly measurable property, we have calculated the current I perturbatively to second order in $\varepsilon_{t,p}$ with the result $I = I_0[1+\varepsilon_t^2(1+2\langle n\rangle)]$. Here $I_0 = e\Gamma_S$ with $\Gamma_S \approx k_BT/e^2R_S$ if $k_BT \gg e\delta V^*$ and Γ_S remains independent of voltage in a certain voltage interval, where the differential conductance will be completely determined by the derivative of the average number of vibrons with respect to voltage, i.e., $\partial I/\partial V \cong 2I_0\varepsilon_t^2\partial\langle n\rangle/\partial V$. Therefore, the cooling effect will be reflected in the structure of the $dI/dV - V$ curves and accessible for experimental investigation.

In conclusion, we have proposed a novel mechanism for ground-state cooling of nanomechanical resonators based on the injection of a tunneling current from a voltage-biased STM tip. For the model system considered we have shown that, by varying the voltage bias, it is possible to control the interference between two distinct contributions to the quantum probability amplitudes for vibron absorption and emission during electron tunneling. For a bias voltage slightly below the Coulomb blockade threshold voltage, the probability amplitude for vibron emission becomes very small. At this bias a thermally activated current therefore leads to a cooling of the nanomechanical vibrations. Our analysis shows that the effective temperature that can be reached may correspond to an average vibron population of the fundamental bending mode as low as 0.2. The cooling mechanism is crucially dependent on the Coulomb blockade phenomenon and hence on the quantization of electric charge.

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