Engineering Nonclassicality in a Mechanical System through Photon Subtraction

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Nonclassical states of a mechanical mode at nonzero temperature are achieved in a scheme that combines radiation-pressure coupling to a light field and photon subtraction. The scheme embodies an original and experimentally realistic way to obtain mesoscopic quantumness by putting together two mature technologies for quantum control. The protocol is quasi-insensitive to mechanical damping.

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In the eye of the layman, quantumness is usually synonymous with atomic-size, well-protected, and weakly energetic systems. Yet nowadays, suggestions are abundant that, although certainly true, this picture might be too restrictive [1]. The idea that nonclassicality might extend its domains well beyond the boundaries of the microscopic world is, at the same time, fascinating and challenging. Demonstrating nonclassicality already at the mesoscopic level would be a major achievement in this respect. Here, such "mesoscopic nonclassicality" should be intended as the enforcement of a controllable quantum mechanical behavior in systems that, for their dimensions or intrinsic nature, go beyond the boundaries of the microscopic world. Among the systems explored for such a task, those involving mechanical modes coupled to electromagnetic fields offer unprecedented promises [2]. The control on nano- or micromechanical systems is such that the achievement of genuine quantum regimes in devices fairly away from microscopic conditions is now possible or so will be in the near future, although at the price of dealing with very low temperatures and quite "expensive" experimental conditions. Entanglement induced by the radiation-pressure coupling between a mechanical mode and a light field has been predicted [3–5] and significant steps towards its experimental demonstration have been recently performed [6]. However, such a rudimental light-matter interaction does not seem to be able to naturally generate nonclassical states of the mechanical mode itself [7]. In this Letter we propose an experimentally viable protocol for quantum state engineering of a massive mechanical mode based on the combination of radiation-pressure coupling [2] and photon subtraction from a light field [8,9]. We show a dynamical regime where nonclassical states of the mechanical oscillator can be in principle achieved under nondemanding conditions: cooling of the oscillator down to its ground-state energy is not required as the scheme prepares nonclassical states for operating temperatures in the range of 1 K and inefficiencies at the photon-subtraction stage do not hinder the effectiveness of the method. Our proposal thus embodies a new way to enforce mesoscopic nonclassicality through mature and well-understood technological tools.

We consider a prototypical optomechanical setting consisting of a cavity driven by an intense light field of frequency ω_l and endowed with a highly reflecting end mirror that can oscillate around an equilibrium position. The vibrating mirror is modeled as a mechanical harmonic oscillator (frequency ω_m). It experiences displacements in phase space dependent on the intensity of the cavity field (which has frequency ω_c) and due to the radiation-pressure coupling [10]. We use m to label the mechanical oscillator, while f indicates the field mode. For a cavity having a large enough free spectral range, such interaction is modeled by the Hamiltonian [11] $\hat{\mathcal{H}}_{mf} = -\hbar\chi\hat{n}_f\hat{Q}_m$ with $\chi = \omega_c L^{-1}$ the optomechanical coupling rate (L is the length of the cavity), $\hat{n}_f = \hat{f}^\dagger\hat{f}$ the photon-number operator of the cavity field [$\hat{f}$ ($\hat{f}^\dagger$) is the corresponding annihilation (creation) operator], and $\hat{Q}_m = \sqrt{\hbar/(2\mu\omega_m)}\hat{q}_m$ the position operator of the mechanical oscillator, whose associated dimensionless quadrature is $\hat{q}_m = (\hat{m} + \hat{m}^\dagger)/\sqrt{2}$ [μ is the mass of the oscillator, while $\hat{m}$ and $\hat{m}^\dagger$ are its bosonic operators]. An important parameter is the detuning Δ between the pumping field and the cavity mode. Solving the dynamics induced by $\hat{\mathcal{H}}_{mf}$ when photon leakage from the cavity, mechanical damping, and chaotic thermal motion of the mechanical oscillator at nonzero temperature are considered is conveniently done by linearizing the dynamics around steady-state values of field intensity and position of the mirror [3]. The resulting model is quadratic in the system's bosonic operators and is solved for the statistical properties of its parties. Starting with a coherent state of the pumping field and a thermal state of the mechanical mode, the linearized coupling maintains the Gaussian nature of the initial optomechanical state. The resulting dynamics is such that entanglement at nonzero temperature is possible [3,4].

The initial point of our analysis is such a correlated field-oscillator state, whose properties are conveniently characterized by its covariance matrix $\boldsymbol{\sigma}$ having elements $\sigma_{ij} = \langle\hat{q}_i\hat{q}_j + \hat{q}_j\hat{q}_i\rangle/2$ ($i, j = 1, \ldots, 4$) with $\mathbf{q} = (\hat{q}_m, \hat{p}_m, \hat{x}_f, \hat{y}_f)$. In this expression we have introduced the

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field quadratures $\hat{x}_f = (\hat{f} + \hat{f}^\dagger)/\sqrt{2}$, $\hat{y}_f = -i(\hat{f} - \hat{f}^\dagger)/\sqrt{2}$ and the mechanical out-of-phase quadrature $\hat{p}_m$ such that $[\hat{q}_m, \hat{p}_m] = i$. The covariance matrix of the system can in general be written as

$$\boldsymbol{\sigma} = \begin{pmatrix} \boldsymbol{m} & \boldsymbol{c} \\ \boldsymbol{c}^T & \boldsymbol{f} \end{pmatrix} \tag{1}$$

with $\boldsymbol{m} = \mathrm{Diag}[m_{11}, m_{22}]$ a diagonal matrix encompassing the local properties of the mechanical mode and $\mathbf{j} = \left(\begin{smallmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{smallmatrix}\right)$ embodying either the field's properties (for $j = f$) or the correlations between the two subsystems (when $j = c$). The analytical expressions for the entries of $\boldsymbol{\sigma}$ are given in Ref. [12].

We now pass to the description of the scheme proposed in this Letter. The field reflected by the mechanical mirror undergoes a single photon-subtraction process (that correspondingly stops the cavity-pumping process). A sketch of the proposed system is given in Fig. 1. The idea behind our proposal is that the correlations (not necessarily quantum [12]) set between the mechanical oscillator and the field are enough to "transfer" the nonclassicality induced in the conditional state of the field by the photon-subtraction process to the state of the mechanical mode. In this respect, our proposal is along the lines of the scheme by Dakna *et al.* [13], where a photon-number measurement on one arm of an entangled two-mode state projects the other one into a highly nonclassical state. However, we should remark again that no assumption on an initially entangled optomechanical state will be necessary [12]. While here we are interested in the formal aspects of the mechanism behind our proposal, a physical protocol will be addressed later on. Given the covariance matrix of the bipartite state of the system, we calculate the Weyl characteristic function as $\chi(\eta, \lambda) = e^{-(1/2)\tilde{\mathbf{q}}\boldsymbol{\sigma}\tilde{\mathbf{q}}^t}$ with $\eta = \eta_r + i\eta_i$, $\lambda = \lambda_r + i\lambda_i$ and $\tilde{\mathbf{q}} = (\eta_r, \eta_i, \lambda_r, \lambda_i)$ the vector of complex phase-space variables. With this, the density matrix of the joint file-oscillator system can be written as $\varrho_{fm} = \pi^{-2}\int \chi(\eta, \lambda)\hat{D}_m^\dagger(\eta) \otimes \hat{D}_f^\dagger(\lambda) d^2\eta d^2\lambda$ [14]. Here, $\hat{D}_j(\alpha) = \exp[\alpha\hat{a}_j^\dagger - \alpha^*\hat{a}_j]$ is the displacement operator of mode $j = m, f$ of amplitude $\alpha \in \mathbb{C}$. The mechanical state resulting from the subtraction of a single quantum from the field is then described by [15]

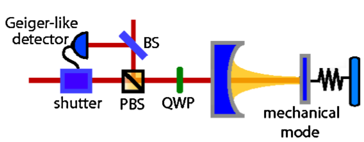

FIG. 1 (color online). Sketch of the thought experiment. A laser field (with a set polarization) enters a cavity and drives the oscillations of an end mirror embodying a mechanical mode. The field is then photon subtracted by a high-transmittivity beam splitter (BS) and a Geiger-like photodetector. A click at the latter triggers a shutter (such as an electrically driven half-wave-plate) that blocks the pumping process. We show the symbols for a polarizing beam splitter (PBS) and a quarter-wave-plate (QWP) used to direct the field to the cavity or the photosubtraction unit.

$$\varrho_m = \mathcal{N}\pi^{-2}\int \chi(\eta, \lambda)\hat{D}_m^\dagger(\eta)\mathrm{tr}[\hat{f}\hat{D}_f^\dagger(\lambda)\hat{f}^\dagger]d^2\eta d^2\lambda \tag{2}$$

with $\mathcal{N}$ a normalization constant. Equation (2) can be manipulated so as to get rid of the degrees of freedom of the cavity field by using the transformation rule of $\hat{f}$ induced by $\hat{D}_f^\dagger(\lambda)$ and the closure relation $\pi^{-1}\int d^2\alpha|\alpha\rangle_f\langle\alpha| = \hat{\mathbb{1}}_f$, where $|\alpha\rangle$ is a coherent state [14]. After some algebraic manipulations, one gets

$$\mathrm{tr}[\hat{f}\hat{D}_f^\dagger(\lambda)\hat{f}^\dagger] = \pi^{-1}\int(|\alpha|^2 - |\lambda|^2 + \lambda^*\alpha - \lambda\alpha^* + 1)e^{-(1/2)|\lambda|^2 + \lambda^*\alpha - \lambda\alpha^*}d^2\alpha. \tag{3}$$

We now first perform the integration over λ, introduce the function $\mathcal{C}(\alpha, \eta, \lambda) = \chi(\eta, \lambda)(|\alpha|^2 - |\lambda|^2 + \lambda^*\alpha - \lambda\alpha^* + 1)e^{-(1/2)|\lambda|^2}$, and cast the state of the mechanical mode as $\varrho_m = \mathcal{N}\pi^{-3}\int \hat{D}_m^\dagger(\eta)\mathcal{F}[\mathcal{C}(\eta, \lambda)]d^2\eta d^2\alpha$ with $\mathcal{F}[\mathcal{C}(\eta, \lambda)]$ the Fourier transform of $\mathcal{C}(\alpha, \eta, \lambda)$. Such a function encompasses any effects that the photon subtraction might have on the state of the mechanical system. As discussed before, the idea behind our proposal is that the correlations shared by the field and the mechanical mode are sufficient for the latter to experience the effects of the de-Gaussification induced by the photon subtraction. In what follows, we show that this is indeed the case.

In order to determine the features of ϱ_m, we address its Wigner function (WF) $W(\delta_r, \delta_i) = \pi^{-2}\int \Xi(\gamma)e^{\gamma^*\delta - \gamma\delta^*}d^2\gamma$ (with $\delta = \delta_r + i\delta_i$), which is calculated using the characteristic function $\Xi(\gamma) = \mathrm{tr}[\hat{D}_m(\gamma)\varrho_m]$ evaluated at the phase-space point $\gamma \in \mathbb{C}$. A lengthy calculation leads to

$$W(\delta_r, \delta_i) = \pi\mathcal{N}\mathcal{A}(\boldsymbol{\sigma})\exp[-2(\delta_i^2/m_{11} - \delta_r^2/m_{22})] \tag{4}$$

with $\mathcal{N} = 2/[(\det\boldsymbol{m})^{5/2}(f_{22} + f_{11} - 2)]$ and

$$\mathcal{A}(\boldsymbol{\sigma}) = m_{22}^2[(f_{11} + f_{22} - 2)m_{11}^2 + (4\delta_i^2 - m_{11})(c_{11}^2 + c_{12}^2)] + m_{11}^2(4\delta_r^2 - m_{22}) \times (c_{22}^2 + c_{21}^2) - 8m_{11}m_{22}(c_{11}c_{21} + c_{12}c_{22})\delta_r\delta_i. \tag{5}$$

The polynomial dependence of $\mathcal{A}(\boldsymbol{\sigma})$ on δ entails the non-Gaussian nature of the reduced mechanical state. We now seek evidences of nonclassicality. A rather stringent criterion for deviations from classicality is the negativity of the WF associated with a given state. This embodies the failure to interpret it as a classical probability distribution, which is instead possible whenever the WF is positive. Building on the so-called Hudson theorem [16], which proves that only multimode coherent and squeezed-vacuum states have non-negative Wigner functions, measures of nonclassicality based on the negativity of the WF have been formulated. More recently, operational criteria for inferring quantumness through the negative regions in the WF have been proposed [17]. By inspection, we find that

Eq. (4) can indeed be nonpositive and achieves its most negative value for $\delta_{r,i} = 0$. Assuming $k_{ii} \geq 1$ (with $i = 1, 2$ and $k = m, f$), we have $W(0, 0) < 0$ for $m_{11}/m_{22} > [(f_{11} + f_{22} - 2)m_{11} - (c_{11}^2 + c_{12}^2)]/(c_{22}^2 + c_{21}^2)$, which is quite an interesting finding. First it shows that the nonclassicality of the mechanical mode depends on its initial degree of squeezing given by the ratio m_{11}/m_{22} [18]. Second, we expect a $\boldsymbol{\sigma}$-based criterion for nonclassicality to be less prone to artifacts (such as large error bars) that would mask negativity and thus erroneously make the WF consistent with a classical probability distribution (the reconstruction of the matrix $\boldsymbol{\sigma}$ can be performed as described in [3,4,17] via high-precision all-optical procedures [19]). Finally, the condition above is handy to gauge the quality of the parameters of a given experiment with respect to the achievement of a nonclassical mechanical state. Figure 2 shows that $W(\delta_r, \delta_i)$ becomes negative for proper choices of the parameters and quite a large temperature.

Depending on the parameters being used, optomechanical entanglement can persist up to temperatures of about 20 K [3,4]. We now wonder whether the conditional state-engineering scheme proposed here enjoys this very same feature. First, we notice that by subtracting a single photon from mode 2 of a two-mode squeezed vacuum $|\zeta\rangle = (\cosh\zeta)^{-1}\sum_{n=0}^{\infty}\lambda^n|n, n\rangle_{12}$ with squeezing factor $\zeta < 1$ and $\lambda = \tanh\zeta$, the Wigner function of the unmeasured mode 1 is basically identical to $W(\delta_r, \delta_i)$ in the limit of small temperature ($T \sim 1$ mK). This is quantitatively illustrated in Figs. 3(a) and 3(b), where the WF of mode 1 for $\zeta = 0.4$ is shown to be indistinguishable from the analogous function of mode m after the application of our scheme. Such a similarity is understood as follows: the high-quality mechanical mode, large-finesse cavity, and low-temperature limit used here make a unitary approach to the time evolution of the optomechanical system quite appropriate. The dynamics, in such case, involve two-mode squeezing of modes m and f [20], which explains the similarity seen in Fig. 3 and discussed here. Such an analogy is illuminating as it is straightforward to see that the effects experienced by mode 1 in the (unnormalized) unilaterally photon-subtracted state $\hat{a}_2|\zeta\rangle_{12}\langle\zeta|\hat{a}_2^\dagger$ can be interpreted as the addition of a photon, which is the origin for nonclassicality of the resulting state (as signaled by

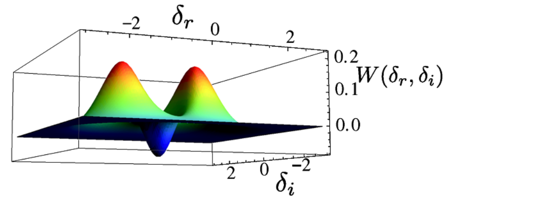

FIG. 2 (color online). WF of m for $\Delta/\omega_m = 0.05$, $T = 0.4$ K, and $\mu = 5 \times 10^{-12}$ kg. We have taken a cavity of length $L = 1$ mm, frequency $\omega_c/2\pi \simeq 4 \times 10^{14}$ Hz, and finesse 10^4, pumped with 20 mW. The mechanical damping rate is as small as $\sim$10 Hz [2].

the negativity of its WF). When T is increased, however, the rotational invariance of $W(\delta_r, \delta_i)$ is progressively lost. As a result of the loss of coherence, $W(\delta_r, \delta_i)$ splits into two localized peaks, which become progressively Gaussian shaped as the temperature grows and represent the thermal average of displaced states in the phase space. This effect is clearly illustrated in Figs. 3(c)–3(f), where a snapshot of the phase-space dynamics against T is shown. As anticipated above, for the parameters chosen in our analysis, quite large negative values are observed in $W(\delta_r, \delta_i)$ for $T \ll 1$ K and the WF remains negative up to 1 K [see Fig. 3(g)]. In Ref. [12] we estimate the lifetime of the enforced nonclassicality.

Our approach so far was to consider photon subtraction at a formal level. Although, as we will demonstrate shortly, the accuracy of the quantitative results achieved in this way is excellent we now go beyond such an abstract description and assess a close-to-reality version of our proposal. In a real experiment, the non-Hermitian operation of subtracting a photon is realized by superimposing, at a high-transmittivity beam splitter (BS), mode f to an ancilla A prepared in the vacuum state [8,21]. This makes ours a three-body system characterized by the variance matrix $\boldsymbol{\sigma}' = (\mathbb{1}_m \oplus \mathbf{B}_{fA}^t)(\boldsymbol{\sigma} \oplus \mathbb{1}_A)(\mathbb{1} \oplus \mathbf{B}_{fA})$, where we have introduced the symplectic BS transformation $\mathbf{B}_{fA} = \sigma_0 \otimes (\tau\mathbb{1}_A) - i\sigma_2 \otimes (r\mathbb{1})$. Here τ is the transmittance of the BS ($r^2 + \tau^2 = 1$), $\sigma_0 = \mathbb{1}$ and σ_2 is the y Pauli matrix. The characteristic function of such correlated three-mode state is $\tilde{\chi}(\eta, \lambda, \xi) = \exp[-\tilde{\mathbf{q}}\boldsymbol{\sigma}'\tilde{\mathbf{q}}^t/2]$, where $\tilde{\mathbf{q}} = (\eta_r, \eta_i, \lambda_r, \lambda_i, \xi_r, \xi_i)$ is the vector of phase-space variables of the three modes and $\xi = \xi_r + i\xi_i$. The

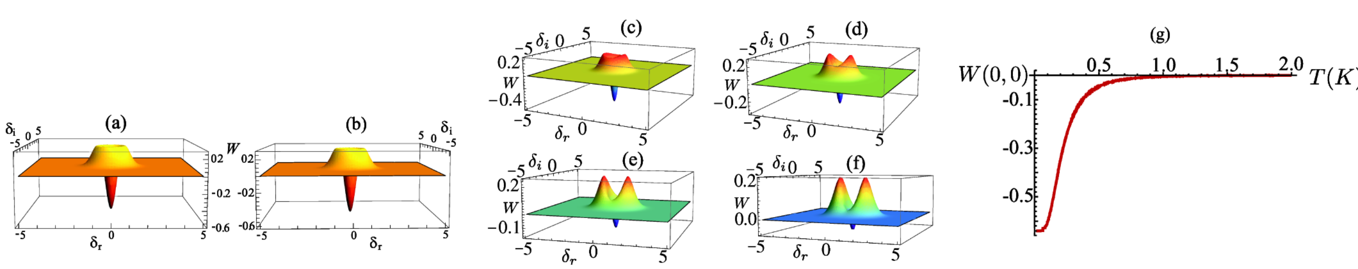

FIG. 3 (color online). (a) Conditional Wigner function of mode m after photon subtraction for $T = 4 \times 10^{-3}$ K and $\mu = 5 \times 10^{-12}$ kg. (b) Same as panel (a) but assuming that modes m and f are initially in a pure two-mode squeezed-vacuum state of squeezing factor $\zeta = 0.4$. (c)–(f) Snapshot of the Wigner function of the mechanical mode for $T = 0.1, 0.2, 0.3, 0.4$ K [in going from panel (c) to (f)]. (g) Negativity of $W(0, 0)$ against temperature T for $\Delta/\omega_m = 0.05$. Other parameters as in Fig. 2.

corresponding density matrix is thus given by $\tilde{\varrho} = \pi^{-3} \int \tilde{\chi}(\eta, \lambda, \xi)\hat{D}_m^\dagger(\eta)\hat{D}_f^\dagger(\lambda)\hat{D}_A^\dagger(\xi)d^2\eta d^2\xi d^2\lambda$. We postselect the event where a single click is obtained at a photo-resolving detector measuring the state of mode A, thus projecting its state onto $|1\rangle_a$. This gives the conditional state $\tilde{\varrho}_m = \tilde{\mathcal{N}}\pi^{-2} \int \tilde{\chi}(\eta, 0, \xi)\hat{D}_m^\dagger(\eta)(1-|\xi|^2)\times e^{-(1/2)|\xi|^2} d^2\eta d^2\xi$ with $\tilde{\mathcal{N}}$ the normalization factor and where the formula ${}_A\langle 1|\hat{D}^\dagger(\xi)|1\rangle_A = e^{-(|\xi|^2/2)}(1-|\xi|^2)$ has been used [14]. The calculation of the WF $\tilde{W}(\delta_r, \delta_i)$ of the mechanical mode then proceeds along the lines sketched above. The resulting analytic expression is, however, very involved and can only be managed numerically. A thorough analysis shows that already at $\tau^2 = 0.8$, $\tilde{W}(\delta_r, \delta_i)$ reproduces very accurately the behavior of $W(\delta_r, \delta_i)$. For instance, at the value of Δ used to produce the figures in this Letter, we get $|\tilde{W}(0,0) - W(0,0)| \simeq 10^{-8}$. Clearly, the quality of the agreement depends crucially on the BS transmittivity. It is enough to take $\tau \sim 0.9$ to get full agreement between $\tilde{W}(\delta_r, \delta_i)$ and its formal counterpart over the range $\Delta \in [0, 0.1]\omega_m$, where nonclassicality is observed.

For the sake of a practical implementation, it is important to assess the role that imperfections play in the performance of our scheme. The most relevant one for our tasks is the inability of discriminating the number of photons impinging on the detector used to subtract a single photon from f. We thus consider a finite-efficiency Geiger-like detector modeled by the positive operator valued measurement $\{\hat{\Pi}_A^{\rm nc}, \mathbb{1}_A - \hat{\Pi}_A^{\rm nc}\}$ with $\hat{\Pi}_A^{\rm nc} = \sum_{j=0}^\infty (1-\epsilon)^j |j\rangle_a\langle j|$ the projection operator accounting for "no-click" at the detector. Because of the finite efficiency $\epsilon \in [0, 1]$, a photonic state with j photons has a probability $(1-\epsilon)^j$ to be missed. It is straightforward to see that the WF corresponding to the state of mode m is then given by $\mathcal{W}(\delta_r, \delta_i) = \pi^{-2}\mathcal{F}[\Xi(\gamma, \epsilon)]$ with $\Xi(\gamma, \epsilon) \propto \tilde{\chi}(\gamma, 0, 0) - \frac{1}{\pi}\sum_{j=0}^\infty (1-\epsilon)^j \int \tilde{\chi}(\gamma, 0, \xi)e^{-(|\xi|^2/2)}\mathcal{L}_j(|\xi|^2)d^2\xi$ and $\mathcal{L}_j(|\xi|^2)$ the Laguerre polynomial of order j. By means of straightforward algebra we have $\sum_{j=0}^\infty (1-\epsilon)^j \mathcal{L}_j(|\xi|^2) = e^{-(2-\epsilon/2\epsilon)|\xi|^2}/\epsilon$, so that $\Xi(\gamma, \epsilon) \propto \tilde{\chi}(\gamma, 0, 0) - \Phi(\gamma, \epsilon)$ with $\Phi(\gamma, \epsilon) = -(\pi\epsilon)^{-1} \int \tilde{\chi}(\gamma, 0, \xi)e^{-(2-\epsilon/2\epsilon)|\xi|^2}d^2\xi$. The effects of detection inefficiency are thus quantified by considering that $\Phi(\gamma, \epsilon)$ is the only term that depends on ϵ in $\Xi(\gamma, \epsilon)$. Therefore, $|\Phi(\gamma, 1) - \Phi(\gamma, \epsilon)|$ provides a quantitative estimate of the differences due to a nonideal detector. Numerically, for $\epsilon \geq 0.7$ we have found negligible values of this quantity ($\sim 10^{-2}$), almost uniformly with respect to τ: Fig. 3(g) is reproduced without noticeable differences. Moreover, the performance of our scheme is not affected by even smaller detection efficiency. In line with what holds for photon-subtraction processes, detection inefficiencies only lower the success probability of the scheme without affecting the fidelity of the process itself [8,21]. The dark count rate of photodetectors can generally be neglected in photon-subtraction experiments [8,9].

We have put forward a scheme for the preparation of nonclassical states of a mechanical mode achieved by combining the paradigm for photon subtraction and a cavity-optomechanical setup. By using parameters currently achievable in the lab, we have demonstrated nonclassicality (as given by negativity of the WF) robust to both the effects of a nonzero operating temperature and imperfections at the photon-subtraction stage. The latter could be performed either intracavity, exploiting the interaction between the cavity field and a two-level system [9] (such as an atom trapped within the cavity volume, as in some proposals put forward recently [22]), or extracavity (using the proposals in [2–4,17]). It will also be interesting to quantitatively study the regime suggested in [4] for effective bilateral subtraction of excitations from both f and, indirectly, the mechanical mode. The realistic nature of our proposal and the fundamental character of the problem addressed here adhere very well with the current quest for quantumness at the mesoscopic level and could represent a useful strategy for its achievement.

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