Large Non-Gaussianity in Axion Inflation

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The inflationary paradigm has enjoyed phenomenological success; however, a compelling particle physics realization is still lacking. Axions are among the best-motivated inflaton candidates, since the flatness of their potential is naturally protected by a shift symmetry. We reconsider the cosmological perturbations in axion inflation, consistently accounting for the coupling to gauge fields $c\phi F\tilde{F}$, which is generically present in these models. This coupling leads to production of gauge quanta, which provide a new source of inflaton fluctuations, $\delta\phi$. For $c \gtrsim 10^2 M_p^{-1}$, these dominate over the vacuum fluctuations, and non-Gaussianity exceeds the current observational bound. This regime is typical for concrete realizations that admit a UV completion; hence, large non-Gaussianity is easily obtained in minimal and natural realizations of inflation.

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Introduction.—Primordial inflation is the dominant paradigm in current cosmology since (i) it resolves the conceptual difficulties of the standard big bang model, and (ii) it predicts primordial perturbations with properties in excellent agreement with those that characterize the cosmic microwave background (CMB) anisotropies. Despite these successes, there is still no compelling particle physics model of inflation, the key obstacle being the requirement of a sufficiently flat scalar potential $V(\phi)$. Even generic Planck-suppressed corrections may yield unacceptably large contributions to the slow-roll parameters $\epsilon \equiv \frac{M_p^2}{2}(\frac{V'}{V})^2$ and $\eta \equiv M_p^2 \frac{V''}{V}$, thus spoiling inflation (a prime denotes the derivative with respect to ϕ, while $M_p \cong 2.4 \times 10^{18}$ GeV is the reduced Planck mass). One of the simplest solutions to this problem is to assume that the inflaton ϕ is a pseudo Nambu-Goldstone boson (PNGB) [1–8]. In this case, the inflaton enjoys a shift symmetry $\phi \rightarrow \phi + \text{const}$, which is broken either explicitly or by quantum effects. In the limit of exact symmetry, the ϕ direction is flat, and thus dangerous corrections to ϵ and η are controlled by the smallness of the symmetry breaking. Moreover, PNGBs like the axion are ubiquitous in particle physics: They arise whenever an approximate global symmetry is spontaneously broken and are plentiful in string compactifications. Axion inflation is also phenomenologically desirable since the tensor-to-scalar ratio is typically large in such models.

The first explicit example of axion inflation was the natural inflation model [1] in which the shift symmetry is broken down to a discrete subgroup $\phi \rightarrow \phi + (2\pi)f$, resulting in a periodic potential

$$V_{\text{np}}(\phi) \cong \Lambda^4[1 - \cos(\phi/f)] \quad (1)$$

with f the axion decay constant. For such a potential, agreement with observations requires $f > M_p$, which may be problematic since it suggests a global symmetry broken above the quantum gravity scale, where effective field theory is presumably not valid. Moreover, $f > M_p$ does not seem possible in string theory [9]. More recently, several controlled realizations of axion inflation have been studied—including double-axion inflation [2], N-flation [3,4], axion monodromy [5], and axion–4-form mixing [8]—which have $f < M_p$ but nevertheless behave effectively as large-field inflation models ($\phi \gtrsim M_p$).

In axion inflation models, the inflaton couples to some gauge field as $\frac{\alpha}{f}\phi F^{\mu\nu}\tilde{F}_{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}/2$. The scale of this coupling is set by the axion decay constant f; the dimensionless parameter α is typically of the order of unity but can be $\gtrsim 1$ in multifield [2] or extra-dimensional models [7]. It is natural to explore the implications of this generic interaction for observables. In Ref. [7] it was shown that energy dissipation into gauge fields can slow the motion of ϕ, providing a novel new inflationary mechanism that operates at very strong coupling. Here, we point out that even in the conventional slow-roll regime, the coupling $\phi F\tilde{F}$ can have significant impact. The motion of the inflaton amplifies the fluctuations of the gauge field, which in turn produce inflaton fluctuations via *inverse decay* [10]: $\delta A + \delta A \rightarrow \delta\phi$. When $f \lesssim 10^{-2} M_p$, which is natural for realizations that admit an UV completion, we show that the inverse decay typically dominates over the usual vacuum fluctuations from inflation, and this has dramatic phenomenological consequences. Our results are quite general: In the spirit of effective field theory, a coupling $\phi F\tilde{F}$ should be included whenever ϕ is pseudoscalar [11].

Recently, there has been considerable interest in non-Gaussian effects in the CMB (see the reviews [12] for references). Non-Gaussianity will be probed to unprecedented accuracy with the forthcoming Planck data and may provide a valuable tool to discriminate between models. Several constructions are known which can predict an observable signature; however, in the minimal cases (decoupled single field models of slow-roll inflation) non-Gaussianity is small [13], and obtaining an observable level usually requires either fine-tuning or unconventional field theories. Here we point out that the inverse decay contribution to $\delta\phi$ is highly non-Gaussian in axion models; observational bounds are easily saturated for modest values of f. Thus, the simplest and, perhaps, most natural models of inflation can lead to observable non-Gaussianity.

Cosmological perturbations.—We consider the theory

$$S = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\alpha}{4f}\phi F^{\mu\nu}\tilde{F}_{\mu\nu}, \quad (2)$$

where ϕ is the PNGB inflaton, $F_{\mu\nu}$ the field strength of the gauge field [for simplicity, a $U(1)$ gauge field is considered; the extension to non-Abelian groups is straightforward], and $\tilde{F}_{\mu\nu}$ its dual. The potential $V(\phi)$ may contain a periodic contribution of the form (1) due to nonperturbative effects and, perhaps, nonperiodic contributions from other effects (such as wrapped branes). In this section, we leave $V(\phi)$ arbitrary, except to suppose that it is sufficiently flat to support $N_e \gtrsim 60$ e-foldings of inflation. We assume a Friedmann-Robertson-Walker geometry $ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 = a(\tau)^2[-d\tau^2 + d\mathbf{x}^2]$.

Working in Coulomb gauge, we decompose $\vec{A}(t, \mathbf{x})$ into circular polarization modes obeying [7]

$$\left[\frac{\partial^2}{\partial\tau^2} + k^2 \pm \frac{2k\xi}{\tau}\right]A_\pm(\tau, k) = 0, \qquad \xi \equiv \frac{\alpha\dot{\phi}}{2fH}, \quad (3)$$

where a dot denotes differentiation with respect to t, $H \equiv \dot{a}/a$, $\xi \cong$ const. We observe that one of the polarizations of $\vec{A}$ experiences a tachyonic instability for $k/(aH) \lesssim 2\xi$. The growth of fluctuations is described by [7,14]

$$A_+(\tau, k) \cong \frac{1}{\sqrt{2k}}\left(\frac{k}{2\xi aH}\right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}} \quad (4)$$

in the interval $(8\xi)^{-1} \lesssim k/(aH) \lesssim 2\xi$ of phase space which accounts for most of the power in the produced gauge field (we take $\dot{\phi} > 0$ without loss of generality). This interval is nonvanishing only for $\xi \gtrsim \mathcal{O}(1)$, which we assume in the following. The production is uninteresting at smaller ξ. Notice that modes with higher momenta remain in their vacuum state (the same applies to all the modes A_-) and their effect is renormalized away [7,14].

The unstable growth of $A_+(\tau, k)$ yields an important new source of cosmological fluctuations, $\delta\phi$. The perturbations of the inflaton are described by [7,15]

$$\left[\frac{\partial^2}{\partial t^2} + 3H\frac{\partial}{\partial t} - \frac{\nabla^2}{a^2}\right]\delta\phi(t, \mathbf{x}) = \frac{\alpha}{f}F^{\mu\nu}\tilde{F}_{\mu\nu}, \quad (5)$$

where the source term is constructed from (4). Notice that metric perturbations are neglected in this calculation; it is shown in Ref. [14] that this is a proper approximation. The solution of (5) splits into two parts: the solution of the homogeneous equation and the particular solution which is due to the source. Schematically,

$$\delta\phi = \underbrace{\delta\phi_{\rm vac}}_{\rm homogeneous} + \underbrace{\delta\phi_{\rm inv.decay}}_{\rm particular}. \quad (6)$$

The quantity of interest is the primordial curvature perturbation on uniform density hypersurfaces, $\zeta = -\frac{H}{\dot{\phi}}\delta\phi$. We computed the two-point $\langle\zeta(\mathbf{x})\zeta(\mathbf{y})\rangle$ and three-point $\langle\zeta(\mathbf{x})\zeta(\mathbf{y})\zeta(\mathbf{z})\rangle$ correlation functions by using the formalism of Refs. [7,15]. The two-point function defines the power spectrum

$$\langle\zeta(\mathbf{x})\zeta(\mathbf{y})\rangle = \int\frac{dk}{k}\frac{\sin[k|\mathbf{x}-\mathbf{y}|]}{k|\mathbf{x}-\mathbf{y}|}P_\zeta(k). \quad (7)$$

We find the result (see [14] for details)

$$P_\zeta(k) = \mathcal{P}\left(\frac{k}{k_0}\right)^{n_s-1}\left[1 + 7.5\times10^{-5}\mathcal{P}\frac{e^{4\pi\xi}}{\xi^6}\right], \quad (8)$$

where $\mathcal{P}^{1/2} \equiv \frac{H^2}{2\pi|\dot{\phi}|}$, n_s is the spectral index, and the pivot scale is $k_0 = 0.002\ \mathrm{Mpc}^{-1}$. The two terms in (8) are the power spectra of the homogeneous and inhomogeneous parts of (6), respectively. There is no "mixed term" since the two contributions (6) are uncorrelated. [The gauge fluctuations that source $\delta\phi_{\rm inv.decay}$, and that are amplified according to (3), are not correlated with the vacuum inflaton fluctuations.] The sourced term has identical scale dependence to the vacuum one, since the free modes $\delta\phi_{\rm vac}$ enter in the Green function of (5) [14]. The power spectrum is probed by CMB and large scale structure observations. It is found to be nearly scale invariant ($n_s \simeq 1$; the precise value depends on the data set assumed [16]) and have amplitude $P_\zeta(k) \cong 25\times10^{-10}$ [17] [the so-called Cosmic Background Explorer (COBE) normalization]. When inverse decay fluctuations are subdominant, we have the standard result $\mathcal{P}^{1/2} = 5\times10^{-5}$; however, at large ξ, the value of $\mathcal{P}$ must be modified.

The three-point correlation function encodes departures from Gaussianity. Non-Gaussian effects from inverse decays are maximal when all three modes have comparable wavelength (the equilateral configuration). The intuitive reason is that a mode of $\delta\phi_{\rm inv.decay}$ is mostly sourced by two modes A_+ of comparable wavelength $\lambda \sim 1/H$ [14], and causality considerations suppress the convolution $\langle\prod_{i=1}^3 \delta\phi_{k_i}\rangle \propto \langle\prod_i \int d^3p_i A_{p_i}A_{k_i-p_i}\rangle$ whenever the external momenta are too different from each other [14]. The magnitude of the three-point function is conventionally quantified by using the parameter $f_{\rm NL}$ [16]. We find

$$f_{\rm NL}^{\rm equil} \cong 4.4\times10^{10}\mathcal{P}^3\frac{e^{6\pi\xi}}{\xi^9}. \quad (9)$$

Schematically, $f_{\rm NL} \sim \langle\delta\phi^3\rangle/(\langle\delta\phi^2\rangle)^2$. The result (9) includes the full value of the two-point function, but does not include the negligible contribution from $\delta\phi_{\rm vac}$ to the three-point function, and is accurate as long as $|f_{\rm NL}| \gtrsim 1$.

From Eqs. (8) and (9) we can see that $f_{\rm NL}^{\rm equil} \simeq 8400$ at large ξ [notice from (8) that $\mathcal{P}$ decreases exponentially with large ξ]. The current WMAP bounds are $-214 < f_{\rm NL}^{\rm equil} < 266$ (95% C.L.), while the Planck satellite, and planned missions, will constrain $f_{\rm NL}^{\rm equil}$ to $\mathcal{O}(10)$ [18].

The results (8) and (9) depend only on the two dimensionless combinations ξ and $\mathcal{P}^{1/2}$, shown in Fig. 1. The solid red curve indicates the parameter values which reproduce the COBE normalization of the power spectrum. In the region below, and above the dashed black line, the power spectrum is dominated by $\delta\phi_{\rm vac}$ and by $\delta\phi_{\rm inv.decay}$, respectively. Notice that the bound on $f_{\rm NL}^{\rm equil}$ implies that $\delta\phi_{\rm vac}$ must dominate the power spectrum.

The results (8) and (9) have been obtained by disregarding two backreaction effects of the produced gauge quanta. Such quanta are produced at the expense of the kinetic energy of ϕ, so that, if the instability is sufficiently strong, then it will affect the inflaton dynamics. The region of parameter space where this occurs is above the black solid line ($\mathcal{P}^{1/2} > 13\xi^{3/2}e^{-\pi\xi}$) shown in Fig. 1. We have also disregarded the impact of the energy density of the produced quanta on the expansion rate H. This is justified provided $e^{2\pi\xi}/\xi^3 \ll 2\times 10^4 M_p^2/H^2$. (In practice, these two conditions amount to disregarding the produced gauge quanta in the background equations for ϕ and H [7,14]). This constraint is not expressed in terms of ξ and $\mathcal{P}^{1/2}$, so we have not included it in Fig. 1. However, it can be studied case by case.

The gauge quanta also source gravity waves (GWs). It is customary to normalize the power of GWs to that of the density perturbations. Proceeding analogously to the computation of the density perturbations, we find [14]

$$r \equiv \frac{P_{\rm GW}}{P_\zeta} = 8.1\times 10^7 \frac{H^2}{M_p^2}\left[1 + 4.3\times 10^{-7}\frac{H^2}{M_p^2}\frac{e^{4\pi\xi}}{\xi^6}\right] \tag{10}$$

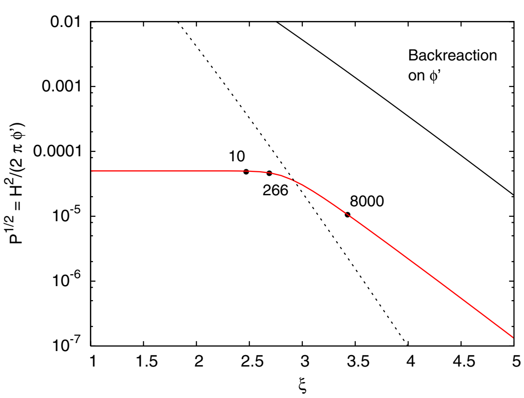

FIG. 1 (color online). Values of parameters leading to the observed COBE normalization of the power spectrum (red line) and reference values for the non-Gaussianity parameter $f_{\rm NL}^{\rm equil} = 10, 266, 8000$ along this curve. See the main text for details.

(there the denominator has been normalized to the observed value). The tensor-to-scalar ratio r is an important quantity to discriminate between different inflationary models. The current observational limit is $r \lesssim 0.2$ [16], and activity is underway to probe $r \gtrsim 0.01$ [17].

Predictions for specific models.—We now consider the power-law potential $V(\phi) = \mu^{4-p}\phi^p$, which subsumes many interesting scenarios. Inflation proceeds at large-field values $\phi \gtrsim M_p$ and ends when $\phi \sim M_p$. For this model, the values of H, $\dot\phi$, and n_s are uniquely determined by the number of e-foldings of observable inflation N_e, according to the standard slow-roll inflaton evolution ($\epsilon, \eta \ll 1$). In the following, we fix $N_e = 60$, which is the typical value taken in large-field models. Once we do so, we are left with the two parameters f/α and μ. For any given value of f/α, the mass scale μ is uniquely determined by fixing the power spectrum (8) to the COBE value. We can then plot the other observational predictions as a function of f/α only. We do so in Fig. 2, where we take $p = 1, 2$ for illustration. In both cases, backreaction effects can be neglected.

Figure 2 shows that large non-Gaussianity is rather generic for large-field axion inflation. The current bound is violated for decay constants $f/\alpha \lesssim 10^{-2}M_p$, which is natural in a model that admits a UV completion. Current limits on non-Gaussianity therefore provide an upper bound on the *strongest* couplings of the type $\phi F\tilde F$ between the inflaton and *any* gauge field.

Natural inflation.—The original natural inflation model [1] was based on the potential (1). If we require $n_s \gtrsim 0.95$, as suggested by recent data [16], then the model requires a large decay constant $f \gtrsim 5M_p$ [19]. Hence inverse decay is negligible unless $\alpha \gtrsim 200$. On the other hand, $f \gtrsim M_p$ may be problematic, and it seems that a UV completion of axion inflation requires $f < M_p$. We now turn our attention to such scenarios.

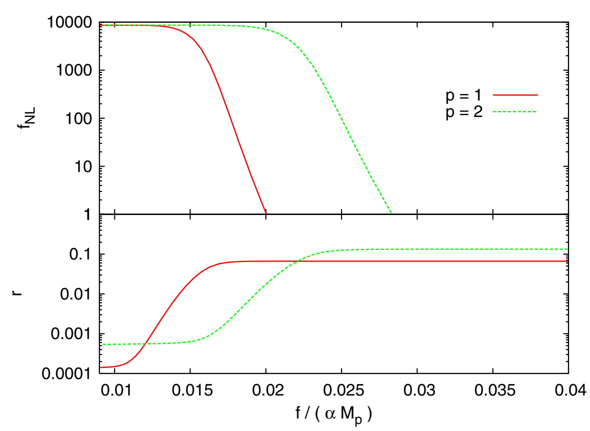

FIG. 2 (color online). Observational predictions for the large-field power-law inflation model $V \propto \phi^p$, with $p = 1, 2$ and assuming $N_e \cong 60$. The spectral index is $n_s = 0.975, 0.967$ for $p = 1, 2$. At small f/α the coupling of ϕ to $F\tilde F$ is stronger and non-Gaussianity is large. The tensor-to-scalar ratio decreases at strong coupling; however, the decrease is important only at values of f/α ruled out by the current bound on $f_{\rm NL}^{\rm equil}$.

Axion monodromy.—In Ref. [5] an explicit, controlled realization of axion inflation was obtained from string theory. The potential has the form $V(\phi) = \mu^3 \phi + \Lambda^4 \cos(\phi/f)$ where the linear contribution arises because the shift symmetry is broken by wrapping an $NS5$-brane on an appropriate 2-cycle, and the periodic modulation is due to nonperturbative effects. The former typically dominates [5,6], so we have the potential $V \propto \phi$, to first approximation. The decay constant is bounded [5] as $0.06 \mathcal{V}^{-1/2} g_s^{1/4} < f/M_p < 0.9 g_s$ with $g_s < 1$ the string coupling and $\mathcal{V} \gg 1$ the compactification volume in string units. From Fig. 2, we see that large non-Gaussianity is easily obtained for $\alpha = \mathcal{O}(1)$. Periodic modulation of $V(\phi)$ can also lead to resonant non-Gaussianity [20] for $f \lesssim 10^{-2} M_p$ and Λ sufficiently large [6,21].

Multiaxion inflation.—Reference [2] proposed a model characterized by two axions θ and ρ, with potential $V \propto \cos(\frac{\theta}{f_i} + \frac{\rho}{g_i})$ which arises from the coupling of the two axions to two different gauge groups: $\frac{\theta}{f_i} F_i \tilde{F}_i$ and $\frac{\rho}{g_i} F_i \tilde{F}_i$ (up to numerical coefficients). For $f_1/g_1 = f_2/g_2$, one linear combination of the two axions becomes a flat direction of V. This relation can be ascribed to a symmetry of the theory, and the curvature of the potential along this direction can be made controllably small if this symmetry is only slightly broken. In this case, one obtains an effective large-field inflaton, with a potential of the type (1), and with an effective axion constant $> M_p$, even if all the f_i and g_i are sub-Planckian. In Ref. [3], it was then noted that the collective motion of N axions ϕ_i, each with its own broken shift symmetry, can support inflation when $f_i < M_p$, via the assisted inflation mechanism [22]. This scenario is quite natural in string theory, where generic compactifications may contain exponentially large numbers of axions [3,4]. For $\phi_i \lesssim f_i$ we can expand the potential near the minimum to obtain $V \cong \sum_i m_i^2 \phi_i^2/2$. The dynamics of the collective field $\Phi \equiv \sqrt{\sum_i \phi_i^2}$ are well-described by the single field potential $V \propto \phi^2$ [3,4]. Sufficient inflation requires $\Phi > M_p$; sub-Planckian ϕ_i (and f_i) are possible for sufficiently large $\sqrt{N}$. The coupling to gauge fields is discussed in Ref. [23]

Axion mixing.—Reference [8] realizes $p = 2$ via axion–4-form mixing. Here $f < M_p$, so $f_{\rm NL}^{\rm equil} \gg 1$ is possible.

In summary, we have shown that large non-Gaussianity is possible for many explicit axion inflation models which admit a UV completion. Our qualitative results will carry over to any inflation model with a PNGB dynamically important during inflation, including multifield models, such as Refs. [24,25]. It would be interesting to study the value of α in concrete string theory realizations.

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